A Feedback Stabilization and Collision Avoidance Scheme for Multiple Independent Nonholonomic Non-point Agents

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***Abstract*— A navigation functions' based methodology, established previously for decentralized navigation of multiple holonomic agents, is extended to address the problem of decentralized navigation of multiple nonholonomic agents. In contrast to our previous work, each agent does not require any knowledge about the velocities and the desired destinations of the other members of the team. Furthermore, the control inputs are the acceleration and rotational velocity of each vehicle, coping in this way with realistic dynamics of classes of mechanical systems. Asymptotic stability is guaranteed by LaSalle's Invariance Principle for nonsmooth systems. The collision avoidance and global convergence properties are verified through simulations.**

I. Introduction

Multi-agent Navigation is a field that has recently gained increasing attention both in the robotics and the control communities, due to the need for autonomous control of more than one mobile vehicle in the same workspace. While most efforts in the past have focused on centralized planning, specific real-world applications have led researchers throughout the globe to turn their attention to decentralized concepts. Examples include decentralized conflict resolution in air traffic management ([18]), automated highway systems, communication networks and the field of micro robotics, where a team of autonomous micro robots must cooperate to achieve manipulation precision in the sub micron level.

Decentralized navigation approaches are more appealing to centralized ones, due to their reduced computational complexity and increased robustness with respect to agent failures. The main focus of work in this domain has been cooperative and formation control of multiple agents, where so much effort has been devoted to the design of systems with variable degree of autonomy ([9],[17], [25]). There have been many different approaches to the decentralized motion planning problem. Open loop approaches use game theoretic and optimal control theory to solve the problem taking the constraints of vehicle motion into account; see for example [2], [10], [24]. On the other hand, closed loop approaches use tools from classical Lyapunov theory and graph theory to design control laws and achieve the convergence of the distributed system to a desired configuration both in the concept of cooperative ([6], [12], [13],) and formation control ([1], [8], [23]).

Closed loop strategies are apparently preferable to open loop ones, mainly because they provide robustness with respect to modelling uncertainties and agent failures and guaranteed convergence to the desired configurations. However, a common point of most work in this area is devoted to the case of point agents. Although this allows for variable degree of decentralization, it is far from realistic in real world applications. For example, in conflict resolution in Air Traffic Management, two aircraft are not allowed to approach each other closer than a specific "alert" distance. The construction of closed loop methods for distributed non-point multi-agent systems is both evident and appealing.

A closed loop approach for single robot navigation was proposed by Koditschek and Rimon [11] in their seminal work. This navigation functions' framework had all the sought qualities but could only handle single, point-sized, robot navigation. In [14] this method was successfully extended to take into account the volume of each robot while a decentralized version of this work has been presented by the authors in [26],[5].

The first extension of the latter work to the case of nonholonomic agents has appeared in [15]. The decentralization factor in this work lied in the fact that each agent had no specific knowledge of the destinations of the others, however it treated a spherical region around the target of each other agent as a static obstacle. In this work we modify the proposed control law in order to allow each agent to neglect any knowledge about the others' destinations. Furthermore, each agent had to have knowledge of the others' velocities. In this paper, we take advantage of the boundedness of the workspace, and design a decentralized controller that does not take the velocities of the other agents into account. Finally, the control inputs are the acceleration and rotational velocity of each vehicle, coping in this way with realistic dynamics of classes of mechanical systems.

The rest of the paper is organized as follows: section II introduces the decentralized multiagent navigation function used in this paper for multiple nonholonomic vehicles. Section III states the problem and the related assumptions. In section IV, we review the necessary mathematical tools for the stability analysis of section V. In section V we present the proposed control scheme and provide stability guarantees. Section VI presents simulation results while in section VII conclusions and issues for further research are discussed.

II. Decentralized Navigation Functions

In previous work [5],[14],[26] the authors presented an extension to the navigation function methodology with applications to multiple robot navigation. In this section we present how this novel class of potential functions can be enhanced with a dipolar structure [21] to provide trajectories suitable for nonholonomic navigation.

Let us assume the following situation: We have N mobile robots, and their workspace $W \subset R^2$. Each robot R_i, $i = 1 \dots N$ occupies a disk in the workspace: $R_i = \left\{ q \in R^2 : \|q - q_i\| \leq r_i \right\}$ where $q_i \in R^2$ is the center of the disk and r_i is the radius of the robot. The position vector of the robots is represented by $q = [q_1 \dots q_N]$. The orientation vector of the robots is represented by $\theta = [\theta_1 \dots \theta_N]$ where θ_i represents the orientation of each robot . Let $\mathcal{W}_i \subseteq R^2 \times (-\pi, \pi]$ represent each robot's workspace. The configuration of each robot is represented by $p_i = \begin{bmatrix} q_i & \theta_i \end{bmatrix} \in \mathcal{W}_i$ and it's target by $p_{d_i} = \begin{bmatrix} q_{d_i} & \theta_{d_i} \end{bmatrix} \in \mathcal{W}_i$. The following figure shows a three-agent conflict situation:

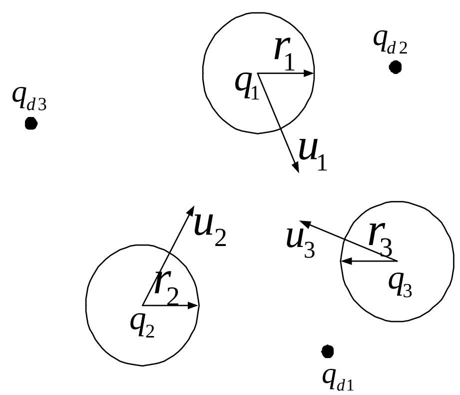

Fig. 1. A conflict scenario with three agents.

As it was shown in [5] the function: $\varphi_i = \frac{\gamma_{di}+f_i}{((\gamma_{di}+f_i)^k+G_i)^{1/k}}$ with a proper selection of G_i can be used for decentralized motion planning of multiple holonomic robots and can be made a navigation function by an appropriate choice of k. The function γ_{di} represents agent i's objective which is convergence to a desired destination while the function G_i encodes the possible collision schemes in which agent i could be involved. The function f_i encodes some form of cooperation between the moving agents and in particular, guarantees that an agent will cooperate with the rest in the collision avoidance procedure, even it has already reached its destination. Details for the construction of the function G_i can be found in [26],[5], while the construction of the f_i function is described in subsection IIb.

A. Decentralized Dipolar Navigation Functions

To be able to produce a dipolar potential field, φ_i must be modified as follows:

$$\varphi_i = \frac{\gamma_{di} + f_i}{\left((\gamma_{di} + f_i)^k + H_{nh_i} \cdot G_i \cdot \beta_{0_i}\right)^{1/k}} \qquad (1)$$

where H_{nh_i} has the form of a pseudo - obstacle. A possible selection of H_{nh_i} would be:

$$H_{nh_i} = \varepsilon_{nh} + \eta_{nh_i}$$

with $\eta_{nh_i} = \|(q_i - q_{di}) \cdot \eta_{d_i}\|^2$, where $\eta_{d_i} = [\cos(\theta_{d_i}) \quad \sin(\theta_{d_i})]^T$. Subscript d denotes destination. Moreover $\gamma_{di} = \|q_i - q_{d_i}\|^2$, i.e. the angle is not incorporated in the distance to the destination metric. $\beta_{0_i} = r_{world}^2 - \|q_i - q_{d_i}\|^2$ is the workspace bounding obstacle and r_{world} is the workspace radius. Figure 2 shows a 2D dipolar navigation function. Following the recipe of [5],[16], it can

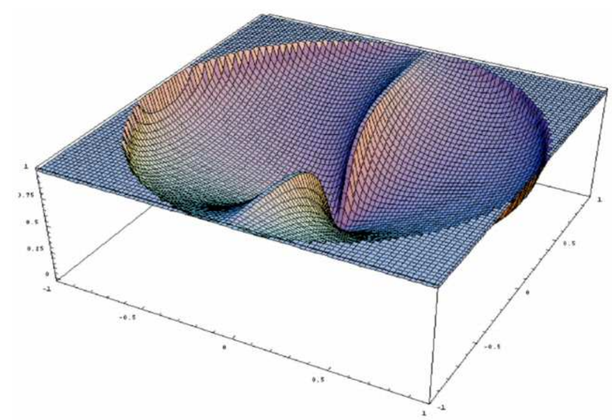

Fig. 2. 2D dipolar navigation function

be shown that the proposed modifications of the potential function do not affect its navigation properties, as long as the workspace is bounded and $\varepsilon_{nh} > \varepsilon(k)$.

B. The f function

The key difference of the decentralized method with respect to the centralized case is that the control law of each agent ignores the destinations of the others. By using $\varphi_i = \frac{\gamma_{di}}{((\gamma_{di})^k+G_i)^{1/k}}$ as a navigation function for agent i, there is no potential for i to cooperate in a possible collision scheme when its initial condition coincides with its final destination. In our previous work [15], the other agents goal configurations where considered as obstacles by the other ones. Clearly, this is a limiting factor for the level of decentralization that we aim to achieve.

In order to overcome this limitation,we add a function f_i to γ_i so that the cost function φ_i attains positive values in proximity situations even when i has already reached its destination. A preliminary definition for this function was given in [26]. Here, we modify the previous definition to ensure that the destination point is a non-degenerate local minimum of φ_i with minimum requirements on assumptions. We define the function f_i by:

$$f_i(G_i) = \begin{cases} a_0 + \sum_{j=1}^{3} a_j G_i^j, \ G_i \leq X \\ 0, \ G_i > X \end{cases} \tag{2}$$

where $X, Y = f_i(0) > 0$ are positive parameters the role of which will be made clear in the following. The parameters a_j are evaluated so that f_i is maximized when $G_i \to 0$ and minimized when $G_i = X$. We also require that f_i is continuously differentiable at X. Therefore we have:

$$a_0 = Y, a_1 = 0, a_2 = \frac{-3Y}{X^2}, a_3 = \frac{2Y}{X^3}$$

The parameter X serves as a sensing parameter that activates the f_i function whenever possible collisions are bound to occur. The only requirement we have for X is that it must be small enough to guarantee that f_i vanishes whenever the system has reached its equilibrium, i.e. when everyone has reached its destination. In mathematical terms:

$$X < G_i\left(q_{d1}, \ldots, q_{dN}\right) \ \forall i \tag{3}$$

That's the minimum requirement we have regarding knowledge of the destinations of the team.

The resulting navigation function is no longer analytic as required by the classic definition in [11], but merely C^1 at $G_i = X$. However, by choosing X large enough, the resulting function is analytic in a neighborhood of the boundary of the free space so that the characterization of its critical points can be made by the evaluation of its Hessian. Hence, the parameter X must be chosen small enough in order to satisfy (3) but large enough to include the region described above. Clearly, this is a tradeoff the control design has to pay in order to achieve decentralization. Intuitively, the destinations should be far enough from one another.

III. System and Problem Definition

We consider the following system of N nonholonomic agents with the following dynamics

$$\begin{aligned} \dot{x}_i &= v_i \cos\theta_i \\ \dot{y}_i &= v_i \sin\theta_i \\ \dot{\theta}_i &= \omega_i \\ \dot{v}_i &= u_i \end{aligned} \quad , i \in \{1, \ldots, N\} \tag{4}$$

where v_i, ω_i are the translational and rotational velocities of agent i respectively, and u_i its acceleration.

The problem we treat in this paper can be now stated as follows: "*Given the N nonholonomic agents (4),consider the rotational velocity ω_i and the acceleration u_i as control inputs for each agent and derive a control law that steers every agent from any feasible initial configuration to its goal configuration avoiding, at the same, collisions.*"

We make the following assumptions:

- Each agent has global knowledge of the position of the others at each time instant.
- Each agent has knowledge only of its own desired destination but not of the others.
- We consider spherical agents.
- The workspace is bounded and spherical.

Our assumption regarding the spherical shape of the agents does not constrain the generality of this work since it has been proven that navigation properties are invariant under diffeomorphisms ([11]). Arbitrarily shaped agents diffeomorphic to spheres can be taken into account. Methods for constructing analytic diffeomorphisms are discussed in [22] for point agents and in [19] for rigid body agents.

The second assumption makes the problem decentralized. Clearly, in the centralized case a central authority has knowledge of everyones goals and positions at each time instant and it coordinates the whole team so that the desired specifications (destination convergence and collision avoidance) are fulfilled. In the current situation no such authority exists and we have to deal with the limited knowledge of each agent.

IV. Elements from Nonsmooth Analysis

In this section, we review some elements from nonsmooth analysis and Lyapunov theory for nonsmooth systems that we use in the stability analysis of the next section.

We consider the vector differential equation with discontinuous right-hand side:

$$\dot{x} = f(x) \tag{5}$$

where $f : R^n \to R^n$ is measurable and essentially locally bounded.

Definition 4.1 [7] *In the case when n is finite, the vector function $x(.)$ is called a solution of (4) in $[t_0, t_1]$ if it is absolutely continuous on $[t_0, t_1]$ and there exists $N_f \subset R^n, \mu(N_f) = 0$ such that for all $N \subset R^n, \mu(N) = 0$ and for almost all $t \in [t_0, t_1]$*

$$\dot{x} \in K[f](x) \equiv \overline{co}\{\lim_{x_i \to x} f(x_i) | x_i \notin N_f \cup N\}$$

The above definition along with the assumption that f is measurable guarantees the uniqueness of solutions of (4)[7].

Lyapunov stability theorems have been extended for nonsmooth systems in [20],[3]. The authors use the concept of *generalized gradient* which for the case of finite-dimensional spaces is given by the following definition:

Definition 4.2 [4]*Let $V : R^n \to R$ be a locally Lipschitz function. The generalized gradient of V at x is given by*

$$\partial V(x) = \overline{co}\{\lim_{x_i \to x} \nabla V(x_i) | x_i \notin \Omega_V\}$$

where Ω_V is the set of points in R^n where V fails to be differentiable.

Lyapunov theorems for nonsmooth systems require the energy function to be *regular*. Regularity is based on the concept of *generalized derivative* which was defined by Clarke as follows:

Definition 4.3 [4] *Let f be Lipschitz near x and v be a vector in R^n. The generalized directional derivative of f at x in the direction v is defined*

$$f^0(x;v) = \lim_{y\to x} \sup_{t\downarrow 0} \frac{f(y+tv)-f(y)}{t}$$

Regularity of a function is defined:

Definition 4.4 [4] *The function $f : R^n \to R$ is called regular if*

1) $\forall v$, the usual one-sided directional derivative $f'(x;v)$exists and

2) $\forall v, f'(x;v) = f^0(x;v)$

The following chain rule provides a calculus for the time derivative of the energy function in the nonsmooth case:

Theorem 4.5 [20] *Let x be a Filippov solution to $\dot{x} = f(x)$ on an interval containing t and $V : R^n \to R$ be a Lipschitz and regular function. Then $V(x(t))$ is absolutely continuous, $(d/dt)V(x(t))$ exists almost everywhere and*

$$\frac{d}{dt}V(x(t)) \in^{a.e.} \dot{\tilde{V}}(x) := \bigcap_{\xi\in\partial V(x(t))} \xi^T K[f](x(t))$$

We shall use the following nonsmooth version of LaSalle's invariance principle to prove the convergence of the prescribed system:

Theorem 4.6 [20] *Let Ω be a compact set such that every Filippov solution to the autonomous system $\dot{x} = f(x), x(0) = x(t_0)$ starting in Ω is unique and remains in Ω for all $t \geq t_0$. Let $V : \Omega \to R$ be a time independent regular function such that $v \leq 0 \forall v \in \dot{\tilde{V}}$(if $\dot{\tilde{V}}$ is the empty set then this is trivially satisfied). Define $S = \{x \in \Omega | 0 \in \dot{\tilde{V}}\}$. Then every trajectory in Ω converges to the largest invariant set,M, in the closure of S.*

V. Nonholonomic Control and Stability Analysis

We will show that the system is asymptotically stabilized under the control law

$$\begin{aligned} u_i &= -v_i\{|\nabla_i\varphi_i\cdot\eta_i| + M_i\} - g_i v_i - \tfrac{v_i}{\tanh(|v_i|)} K_{v_i} K_{z_i} \\ \omega_i &= -K_{\theta_i}(\theta_i - \theta_{di} - \theta_{nh_i}) + \dot{\theta}_{nh_i} \end{aligned} \tag{6}$$

where $K_{v_i}, K_{\theta_i}, g_i > 0$ are positive gains,

$$\theta_{nh_i} = \arg(\frac{\partial\varphi_i}{\partial x_i}\cdot s_i + \mathbf{i}\frac{\partial\varphi_i}{\partial y_i}\cdot s_i)$$

$$s_i = \mathrm{sgn}((q_i - q_{di})\cdot\eta_{di})$$

$$\eta_i = \begin{bmatrix} \cos\theta_i & \sin\theta_i \end{bmatrix}^T$$

$$\eta_{di} = \begin{bmatrix} \cos\theta_{di} & \sin\theta_{di} \end{bmatrix}^T$$

$$K_{zi} = \|\nabla_i\varphi_i\|^2 + \|q_i - q_{di}\|^2$$

$$M_i > |\sum_{j\neq i} \nabla_i\varphi_j\cdot\eta_i|_{max}$$

$$\nabla_i\varphi_j = \begin{bmatrix} \frac{\partial\varphi_j}{x_i} & \frac{\partial\varphi_j}{y_i} \end{bmatrix}$$

In particular, we prove the following theorem:

Theorem 5.1:*Under the control law (6), the system is asymptotically stabilized to $p_d = [p_{d1},\ldots,p_{dN}]^T$.*

Proof: Let us first consider the case $|v_i| > 0 \forall i$. We use

$$V = \sum V_i, V_i = \varphi_i + |v_i| + \frac{1}{2}(\theta_i - \theta_{di} - \theta_{nhi})^2$$

as a Lyapunov function candidate. For $|v_i| > 0$ we have

$$\dot{V} = \sum_i \dot{V}_i = \sum_i \left\{ \begin{array}{c} \sum_j v_j\left(\nabla_j\varphi_i\right)\cdot\eta_j + \mathrm{sgn}(v_i)\dot{v}_i + \\ +\left(\theta_i - \theta_{di} - \theta_{nhi}\right)\left(\dot{\theta}_i - \dot{\theta}_{nhi}\right) \end{array} \right\}$$

and substituting

$$\begin{aligned} \dot{V} = &\sum_i \left\{ \sum_j v_j\left(\nabla_j\varphi_i\right)\cdot\eta_j - |v_i|\left(|(\nabla_i\varphi_i)\cdot\eta_i| + M_i\right)\right\} \\ &-\sum_i \tfrac{|v_i|}{\tanh(|v_i|)} K_{v_i} K_{z_i} - \sum_i g_i|v_i| \\ &-\sum_i K_{\theta_i}\left(\theta_i - \theta_{di} - \theta_{nhi}\right)^2 \end{aligned}$$

The first term of the right hand side of the last equation can be rewritten as

$$\begin{aligned} &\sum_i \left\{ \sum_j v_j\left(\nabla_j\varphi_i\right)\cdot\eta_j - |v_i|\left(|(\nabla_i\varphi_i)\cdot\eta_i| + M_i\right)\right\} = \\ &= \sum_i \left\{ \begin{array}{c} v_i\left(\nabla_i\varphi_i\right)\cdot\eta_i + v_i\sum_{j\neq i}\left(\nabla_i\varphi_j\right)\cdot\eta_i - \\ -|v_i|\left(|(\nabla_i\varphi_i)\cdot\eta_i| + M_i\right) \end{array} \right\} \leq 0 \end{aligned}$$

so that

$$\dot{V} \leq -\sum_i K_{v_i}K_{z_i} - \sum_i g_i|v_i| - \sum_i K_{\theta_i}\left(\theta_i - \theta_{di} - \theta_{nhi}\right)^2$$

where the inequality $\frac{x}{\tanh x} \geq 1$ for $x \geq 0$.

The candidate Lyapunov function is nonsmooth whenever $v_i = 0$ for some i. The generalized gradient of V is given by

$$\partial V = \begin{bmatrix} \sum_i \nabla_1\varphi_i \\ \vdots \\ \sum_i \nabla_N\varphi_i \\ \partial|v_1| \\ \vdots \\ \partial|v_N| \\ 1/2\nabla_{\theta_1}\left(\theta_1 - \theta_{d1} - \theta_{nh1}\right)^2 \\ \vdots \\ 1/2\nabla_{\theta_N}\left(\theta_N - \theta_{dN} - \theta_{nhN}\right)^2 \\ 1/2\nabla_{\theta_{nh1}}\left(\theta_1 - \theta_{d1} - \theta_{nh1}\right)^2 \\ \vdots \\ 1/2\nabla^{\theta_{nhN}}\left(\theta_N - \theta_{dN} - \theta_{nhN}\right)^2 \end{bmatrix}$$

and the Filippov set(def.4.1) of the closed loop system by

$$K[f]=\begin{bmatrix} v_1\cos\theta_1 \\ v_1\sin\theta_1 \\ \vdots \\ v_N\cos\theta_N \\ v_N\sin\theta_N \\ u_1 \\ \vdots \\ u_N \\ \omega_1 \\ \vdots \\ \omega_N \\ \dot{\theta}_{nh1} \\ \vdots \\ \dot{\theta}_{nhN} \end{bmatrix}=\begin{bmatrix} v_1\cos\theta_1 \\ v_1\sin\theta_1 \\ \vdots \\ v_N\cos\theta_N \\ v_N\sin\theta_N \\ K[u_1] \\ \vdots \\ K[u_N] \\ \omega_1 \\ \vdots \\ \omega_N \\ \dot{\theta}_{nh1} \\ \vdots \\ \dot{\theta}_{nhN} \end{bmatrix}$$

We denote by

$$D \triangleq \{x : \exists i \in \{1,\ldots N\}\, \text{s.t.} v_i = 0\}$$

the "discontinuity surface" and

$$D_S \triangleq \{i \in \{1,\ldots N\}\, \text{s.t.} v_i = 0\}$$

the set of indices of agents that participate in D. We then have

$$\begin{aligned}
&\dot{\tilde{V}} = \bigcap_{\xi\in\partial V} \xi^T K[f] = \\
&v_1\left(\sum_i \nabla_1\varphi_i\right)\cdot\eta_1+\ldots+v_N\left(\sum_i \nabla_N\varphi_i\right)\cdot\eta_N \\
&+\bigcap_{\xi\in\partial|v_1|}\xi^T K[u_1]+\ldots+\bigcap_{\xi\in\partial|v_N|}\xi^T K[u_N] \\
&+\sum_i(\theta_i-\theta_{di}-\theta_{nhi})\left(\omega_i-\dot{\theta}_{nhi}\right)\Rightarrow
\end{aligned}$$

$$\begin{aligned}
\dot{\tilde{V}} &= \sum_{i\notin D_S}\left\{v_i\left(\sum_i \nabla_i\varphi_j\right)\cdot\eta_i+\operatorname{sgn}(v_i)\,u_i\right\} \\
&+\sum_{i\in D_S}\bigcap_{\xi\in\partial|v_i|}\xi^T K[u_i]-\sum_i K_{\theta_i}(\theta_i-\theta_{di}-\theta_{nhi})^2
\end{aligned}$$

For $i\in D_S$ we have $\partial\left|v_i\right|_{v_i=0}=[-1,1]$ and

$$K[u_i]|_{v_i=0}=[-|K_{vi}K_{zi}|,|K_{vi}K_{zi}|]$$

so that

$$\bigcap_{\xi\in\partial|v_i|}\xi^T K[u_i]=0$$

From the previous analysis we also derive that

$$\begin{aligned}
&\sum_{i\notin D_S}\left\{v_i\left(\sum_i \nabla_i\varphi_j\right)\cdot\eta_i+\operatorname{sgn}(v_i)\,u_i\right\}\le \\
&-\sum_{i\notin D_S}\{K_{vi}K_{zi}+g_i|v_i|\}
\end{aligned}$$

Going back to Theorem 4.6 it is easy to see that $v \le 0 \forall v \in \dot{\tilde{V}}$. Each function V_i is regular as the sum of regular functions ([20]) and V is regular for the same reason. The level sets of V are compact so we can apply this theorem. We have that $S=\{x|0\in\dot{\tilde{V}}\}=\{x:(v_i=0\forall i)\bigwedge(\theta_i-\theta_{di}=\theta_{nhi}\forall i)\}$. The trajectory of the system converges to the largest invariant subset of S. For this subset to be invariant we must have

$$\dot{v}_i=0\Rightarrow K_{vi}K_{zi}=0\Rightarrow(\nabla_i\varphi_i=0)\wedge(q_i=q_{di})\,\forall i$$

For $\nabla_i\varphi_i=0$ we have $\theta_{nhi}=0$ so that $\theta_i=\theta_{di}$. $\diamondsuit$

VI. Simulations

To demonstrate the navigation properties of our decentralized approach, we present a simulation of four nonholonomic agents that have to navigate from an initial to a final configuration, avoiding collision with each other. The chosen configurations constitute non-trivial setups since the straight-line paths connecting initial and final positions of each agent are obstructed by other agents.

Figures 3,4 involve four nonholonomic agents with dynamics described by (4) navigating under the control law (6). Figure 3 shows the initial positions and desired destinations of each agent. In this figure, $I-i, T-i$ denote the initial position and final desired destination of agent i respectively.

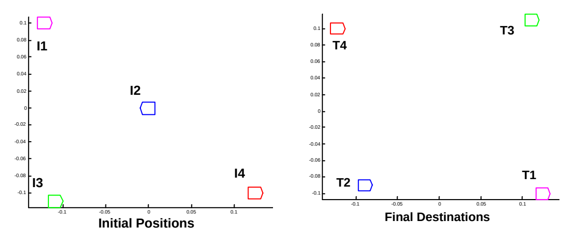

Fig. 3. Initial and Goal Configurations for 4 dynamic nonholonomic agents

In figure 4, screenshots A through G show the evolution of the four agent team. The conflict resolution procedure takes place in screenshots B through E, while in the last two screenshots the agents converge to their goal configuration free of collisions. Based on the control law (6), the parameters chosen for this simulation are $r_1=r_2=r_3=r_4=0.01$, $Y=0.001, X=0.0001$, $\varepsilon_{nh}=10^{-5}$, $k=90, \lambda=1, h=5$, $M=g=K_v=10^{-3}$, $K_\theta=1$.

The collision avoidance procedure shown in screenshots B through E reveals the cooperative nature of our approach. As seen in screenshots B,C agents 1,2 backtrack from their desired destination to create free space for agents 3,4 to navigate towards their desired destinations. The conflict resolution maneuvers of screenshots D,E allow agents 1,2 to find free space and converge to their goal configuration.

VII. Conclusions

The navigation functions' based methodology, established previously for decentralized navigation of multiple holonomic agents, has been extended to address the problem of decentralized navigation of multiple nonholonomic agents. In contrast to our previous work ([15]), each agent

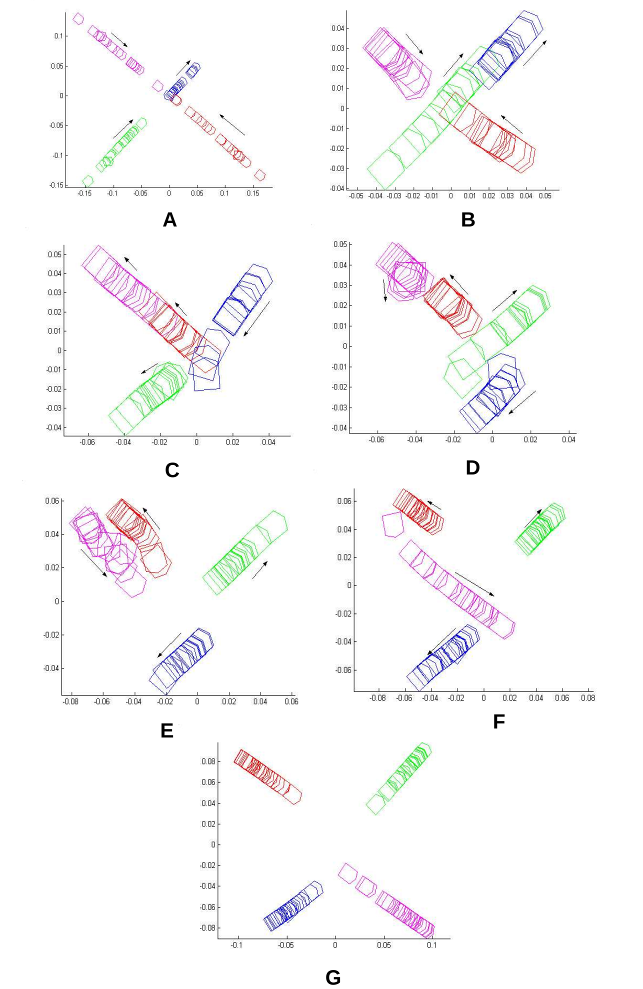

Fig. 4. Collision avoidance and convergence to the destination for 4 dynamic nonholonomic agents

does not require any knowledge about the velocities and the desired destinations of the other members of the team. Furthermore, the control inputs are the acceleration and rotational velocity of each vehicle, coping in this way with realistic dynamics of classes of mechanical systems. Asymptotic stability is guaranteed by LaSalle's Invariance Principle for nonsmooth systems. The collision avoidance and global convergence properties have been verified through simulations.

Current research directions are towards applying the methodology to the cases where each agent has limited knowledge of the positions of the others and where there is some form of uncertainty in the agent movement.

VIII. Acknowledgment

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