DECENTRALIZED MOTION CONTROL OF MULTIPLE AGENTS WITH DOUBLE INTEGRATOR DYNAMICS

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Abstract: The decentralized navigation function methodology, established in previous work for navigation of a multi-agent system with single integrator dynamics is extended to the case of double integrator. The main motivation of this work lies in the fact that acceleration control is more implementable in multiple vehicle systems than velocity control. Each agent plans its actions without knowing the destinations of the others. The stability of the overall system is guaranteed by LaSalle's Invariance Principle. The collision avoidance and global convergence properties are verified through simulations. *Copyright$^{©}$ 2005 IFAC.*

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1. INTRODUCTION

Navigation of mobile agents has been an area of significant interest in robotics and control communities. Most efforts have focused on the case of a single agent navigating in an environment with obstacles (Latombe (1991)). In the past few years, navigation for multiple agents has been considered by many researchers, mainly in the context of cooperative and formation control of multiple agents; see for example (Egerstedt and Hu (2001),Lin et al. (2003)). Recently, decentralized navigation for multiple agents has gained increasing attention (Feddema and Schoenwald (2002),Lawton et al. (2003),Saber and Murray (2003),Tanner et al. (2003)). The motivation comes from many application domains among which decentralized conflict resolution in Air Traffic Management(ATM) is one of the most important ones. Decentralized control in ATM has been considered in (Bicchi and Pallottino (2000),Inalhan et al. (2002)), under a game theoretic perspective.

Decentralized navigation approaches are more appealing to centralized ones, due to their reduced computational complexity and increased robustness with respect to agent failures. In (Dimarogonas et al. (2003)),the centralized navigation function methodology established in (Koditscheck and Rimon (1990)) for a single point agent navigating in a static environment and in (Loizou and Kyriakopoulos (2002)) for multiple holonomic agents, was extended to the case of decentralized navigation of a multi-agent team. The decentralization factor lied in the fact that each agent had no knowledge of the desired destinations of the others.

The dynamics of the agents in these papers were single integrator and the control inputs were the velocity of each agent. In practice however, multiagent systems and especially moving vehicles are controlled through their acceleration due to the fact that the real dynamics have to be taken into account. It is therefore both motivating and natural to extend this methodology to the case

of a multi-agent system with double integrator dynamics where the control input will be the acceleration of each agent.

Taking those aspects into consideration, we consider in this paper the decentralized conflict avoidance problem for the case of a multi-agent system with double integrator dynamics. The problem that we treat can be stated as follows: *"Derive a set of control laws (one for the acceleration of each agent) that drives a team of n agents from any initial configuration to a desired goal configuration avoiding at the same time collisions."* We make the following assumptions:

- Each agent has global knowledge of the position and velocity of the others at each time instant.
- Each agent has knowledge only of its own desired destination but not of the others.
- We consider spherical agents.
- The workspace is bounded and spherical.

Our assumption that we have spherical agents does not constrain the generality of this work since it has been proven that navigation properties are invariant under diffeomorphisms (Koditscheck and Rimon (1990)). Arbitrarily shaped agents diffeomorphic to spheres can be taken into account. Methods for constructing analytic diffeomorphisms are discussed in (Rimon and Koditscheck (1992)) for point agents and in (Tanner et al. (2001)) for rigid body agents.

The second assumption makes the problem decentralized. Clearly, in the centralized case a central authority has knowledge of everyones goals and positions at each time instant and it coordinates the whole team so that the desired specifications (destination convergence and collision avoidance) are fulfilled. In the current situation no such authority exists and we have to deal with the limited knowledge of each agent.

The rest of the paper is organized as follows: Section 2 provides a review of the concept of decentralized navigation functions and introduces the terminology and mathematical tools required for the analysis. Section 3 states the problem in hand and presents the proposed control scheme. The stability properties of the system are examined in Section 4. Section 5 presents simulation results for a number of non-trivial multi agent navigational tasks. Finally, section 6 summarizes the conclusions and indicates our current research.

2. DECENTRALIZED NAVIGATION FUNCTIONS

In this section, we review the decentralized navigation function method used in Dimarogonas et al. (2003) for the case of multiple holonomic agents with single integrator dynamics. A detailed analysis of the concepts described in the following analysis can be found in Dimarogonas et al. (2004).

Navigation functions are real valued maps realized through cost functions, whose negated gradient field is attractive towards the goal configuration and repulsive wrt obstacles. It has been shown by Koditschek and Rimon that almost global navigation is possible since a smooth vector field on any sphere world with a unique attractor, must have at least as many saddles as obstacles (Koditscheck and Rimon (1990)).

Consider a system of N agents operating in the same workspace $W \subset R^2$. Each agent i occupies a disk: $R = \{q \in R^2 : \|q - q_i\| \leq r_i\}$ in the workspace where $q_i \in R^2$ is the center of the disk and r_i is the radius of the agent. The motion of each agent is described by $\dot{q}_i = u_i$ and the configuration space is spanned by $q = [q_1, \ldots, q_n]^T$. The *decentralized navigation function* φ_i is defined as

$$\varphi_i = \frac{\gamma_{di} + f_i}{((\gamma_{di} + f_i)^k + G_i)^{1/k}} \tag{1}$$

The term $\gamma_{di} = \|q_i - q_{di}\|^2$ in the potential function is the squared metric of the agent's i configuration from its desired destination q_{di}. The exponent k is a scalar positive parameter. The function G_i expresses all possible collisions of agent i with the others, while f_i guarantees that the φ_i attains positive values whenever collisions with respect to i tend to occur, even when i has already reached its destination.

2.1 Construction of the G_i function

We review now the construction of the "collision" function G_i for each agent i. The "Proximity Function" between agents i and j is given by

$$\beta_{ij} = \|q_i - q_j\|^2 - (r_i + r_j)^2$$

We will use the term *relation* to describe the possible collision schemes that can occur in a multiple agents scene with respect to agent i. A *binary relation* is a relation between agent i and another. We will call the number of binary relations in a relation, the *relation level*. With this terminology in hand, the relation of figure (1a) is a level-1 relation (one binary relation) and that of figure (1b) is a level-3 relation (three binary relations), always with respect to the specific agent R. A "Relation Proximity Function" (RPF) provides a measure of the distance between agent i and the other agents involved in the relation. Each relation has its own RPF. Let R_k denote the
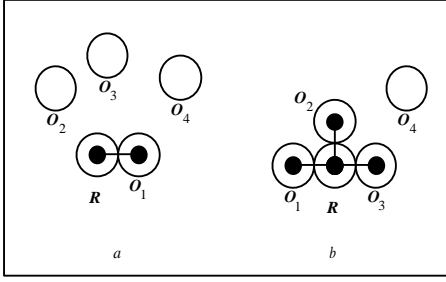
Fig. 1. Part *a* represents a level-1 relation and part *b* a level-3 relation wrt agent R.

k^{th} relation of level l. The RPF of this relation is given by:

$$(b_{R_k})_l = \sum_{j \in (R_k)_l} \beta_{ij}$$

A "Relation Verification Function" (RVF) is defined by:

$$(g_{R_k})_l = (b_{R_k})_l + \frac{\lambda (b_{R_k})_l}{(b_{R_k})_l + (B_{R_k^C})_l^{1/h}}$$

where λ, h are positive scalars and $(B_{R_k^C})_l = \prod_{m \in (R_k^C)_l} (b_m)_l$ where $(R_k^C)_l$ is the complementary set of relations of level-l, i.e. all the other relations with respect to agent i that have the same number of binary relations with the relation R_k. It is obvious that for the highest level $l = n-1$ only one relation is possible so that $(R_k^C)_{n-1} = \emptyset$ and $(g_{R_k})_l = (b_{R_k})_l$ for $l = n - 1$. The function G_i is now defined as

$$G_i = \prod_{l=1}^{n_L^i} \prod_{j=1}^{n_{R_l}^i} (g_{R_j})_l$$

where n_L^i the number of levels and $n_{R_l}^i$ the number of relations in level-l with respect to agent i.

The definition of the G function in the multiple moving agents situation is slightly different than the one introduced by the authors in (Koditscheck and Rimon (1990)). The collision scheme in that approach involved a single moving point agent in an environment with static obstacles. A collision with more than one obstacle was therefore impossible and the obstacle function was simply the product of the distances of the agent from each obstacle. In our case however, this is inappropriate, as can be seen in figure 2. The control law of agent A should distinguish when agent A is in conflict with B, C, or B and C simultaneously. Mathematically, the first two situations are level-1 relations and the third a level-2 relation with respect to A. Whenever the latter occurs, the RVF of the level-2 relation tends to zero while the RVFs of the two separate level-1 relations (A,B and A,C) are nonzero. The key property of an RVF is that it
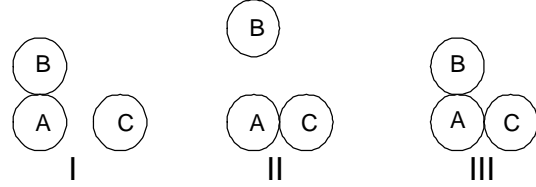

Fig. 2. I,II are level-1 relations with respect to A, while III is level-2. The RVFs of the level-1 relations are nonzero in situation III.

tends to zero only when the corresponding relation holds. Hence it serves as an analytic switch that is activated (tends to zero) only when the relation it represents is realized.

2.2 Construction of the f_i function

The key difference of the decentralized method with respect to the centralized case is that the control law of each agent ignores the destinations of the others. By using $\varphi_i = \frac{\gamma_{di}}{\left((\gamma_{di})^k + G_i \right)^{1/k}}$ as a navigation function for agent i, there is no potential for i to cooperate in a possible collision scheme when its initial condition coincides with its final destination. In order to overcome this limitation, we add a function f_i to γ_i so that the cost function φ_i attains positive values in proximity situations even when i has already reached its destination. A preliminary definition for this function was given in (Dimarogonas et al. (2003)). Here, we modify the previous definitions to ensure that the destination point is a non-degenerate local minimum of φ_i with minimum requirements on assumptions. We define the function f_i by:

$$f_i(G_i) = \begin{cases} a_0 + \sum_{j=1}^{3} a_j G_i^j, \ G_i \leq X \\ 0, \ G_i > X \end{cases}$$

where $X, Y = f_i(0) > 0$ are positive parameters the role of which will be made clear in the following. The parameters a_j are evaluated so that f_i is maximized when $G_i \to 0$ and minimized when $G_i = X$. We also require that f_i is continuously differentiable at X. Therefore we have:

$$a_0 = Y, a_1 = 0, a_2 = \frac{-3Y}{X^2}, a_3 = \frac{2Y}{X^3}$$

The parameter X serves as a sensing parameter that activates the f_i function whenever possible collisions are bound to occur. The only requirement we have for X is that it must be small enough whenever the system has reached its equilibrium so that f_i vanishes whenever all agents converge to their goal configuration. In mathematical terms:

$$X < G_i (q_{d1}, \ldots, q_{dN}) \ \forall i$$

That's the minimum requirement we have regarding knowledge of the destinations of the team. Intuitively, the destinations should be far enough from one another.

A key feature of navigation functions and in particular, DNF's, is that their gradient motion is repulsive with respect to the boundary of the free space. The free space for each agent is defined as the subset of W which is free of collisions with the other agents. Hence collision avoidance is reassured. For further information regarding terminology the reader is referred to (Dimarogonas et al. (2003), Dimarogonas et al. (2004)).

3. THE CASE OF DOUBLE INTEGRATOR DYNAMICS

In this section, we assume that the spherical agents' motion is described by the double integrator:

$$\begin{aligned} \dot{q}_i &= v_i \\ \dot{v}_i &= u_i \end{aligned}, i \in \{1, \ldots, N\} \qquad (2)$$

For the case of single integrator kinematics described in the previous section the control law had the simple form $v_i = -K_i \frac{\partial \varphi_i}{\partial q_i}$. In the present situation, agents are controlled through their acceleration and a different controller design is applied.

Specifically, we will show that the system is asymptotically stabilized under the control law

$$u_i = -K_i \frac{\partial \varphi_i}{\partial q_i} + \theta_i \left(v_i, \frac{\partial \varphi_i}{\partial t} \right) - g_i v_i \qquad (3)$$

where $K_i, g_i > 0$ are positive gains,

$$\theta_i \left(v_i, \frac{\partial \varphi_i}{\partial t} \right) \triangleq -\frac{c v_i}{\tanh \left(\|v_i\|^2 \right)} \left| \frac{\partial \varphi_i}{\partial t} \right|$$

and

$$\frac{\partial \varphi_i}{\partial t} = \sum_{j \neq i} \frac{\partial \varphi_i}{\partial q_j} \dot{q}_j$$

The first term of equation (3) corresponds to the potential field (decentralized navigation function) described in section 2. The second term exploits the knowledge each agent has of the velocities of the others, and is designed to guarantee convergence of the whole team to the desired configurations. The last term serves as a damping element that ensures convergence to the destination point by suppressing oscillatory motion around it.

By using the notation $x = \left[x_1^T, \ldots, x_N^T \right]^T$, $x_i^T = \left[q_i^T \ v_i^T \right]$ the closed loop dynamics of the system can be rewritten as

$$\dot{x} = \xi(x) = \left[\xi_1^T(x), \ldots, \xi_N^T(x) \right]^T \qquad (4)$$

with

$$\xi_i(x) = \begin{bmatrix} v_i \\ -K_i \frac{\partial \varphi_i}{\partial q_i} - \frac{c v_i}{\tanh \left(\|v_i\|^2 \right)} \left| \frac{\partial \varphi_i}{\partial t} \right| - g_i v_i \end{bmatrix}$$

We will use the function $V = \sum_i K_i \varphi_i + \frac{1}{2} \sum_i \|v_i\|^2$ as a candidate Lyapunov function to show that the agents converge to their destinations points . We will check the stability of the multi-agent system with LaSalle's Invariance Principle.

4. STABILITY ANALYSIS

In the following we prove the following theorem:
Theorem 4.1 *The system (4) is asymptotically stabilized to $\left[q_d^T \ 0 \right]$, $q_d = [q_{d1}, \ldots, q_{dN}]^T$ up to a set of initial conditions of measure zero if the exponent k assumes values bigger than a finite lower bound and $c > \max_i(K_i)$.*
Proof: The candidate Lyapunov Function we use is $V = \sum_i K_i \varphi_i + \frac{1}{2} \sum_i \|v_i\|^2$ and by taking its derivative we have

$$V = \sum_i K_i \varphi_i + \frac{1}{2} \sum_i \|v_i\|^2 \Rightarrow$$

$$\dot{V} = \sum_i K_i \dot{\varphi}_i + \sum v_i^T \dot{v}_i = \sum K_i \left(\frac{\partial \varphi_i}{\partial t} + v_i^T \frac{\partial \varphi_i}{\partial q_i} \right)$$

$$+ \sum v_i^T \left(-K_i \frac{\partial \varphi_i}{\partial q_i} + \theta_i \left(v_i, \frac{\partial \varphi_i}{\partial t} \right) - g_i v_i \right)$$

$$\Rightarrow \dot{V} = \sum \left(K_i \frac{\partial \varphi_i}{\partial t} + v_i^T \theta_i \left(v_i, \frac{\partial \varphi_i}{\partial t} \right) - g_i \|v_i\|^2 \right)$$

Using the notation $B_i \triangleq K_i \frac{\partial \varphi_i}{\partial t} + v_i^T \theta_i \left(v_i, \frac{\partial \varphi_i}{\partial t} \right)$ we first show that $\sum_i B_i \leq 0$ if $c > \max_i(K_i)$:

$$\frac{\partial \varphi_i}{\partial t} > 0 :$$

$$c > \max_i (K_i) \Rightarrow c > K_i \frac{\tanh \left(\|v_i\|^2 \right)}{\|v_i\|^2}$$

$$\Rightarrow K_i > \frac{c \|v_i\|^2}{\tanh \left(\|v_i\|^2 \right)} \operatorname{sgn} \left(\frac{\partial \varphi_i}{\partial t} \right)$$

$$\Rightarrow K_i \frac{\partial \varphi_i}{\partial t} + v_i^T \theta_i \left(v_i, \frac{\partial \varphi_i}{\partial t} \right) < 0 \forall i : \frac{\partial \varphi_i}{\partial t} > 0$$

$$\frac{\partial \varphi_i}{\partial t} < 0 :$$

$$c > 0 \Rightarrow c > -K_i \frac{\tanh \left(\|v_i\|^2 \right)}{\|v_i\|^2}$$

$$\Rightarrow K_i > \frac{c \|v_i\|^2}{\tanh \left(\|v_i\|^2 \right)} \operatorname{sgn} \left(\frac{\partial \varphi_i}{\partial t} \right)$$

$$\Rightarrow K_i \frac{\partial \varphi_i}{\partial t} + v_i^T \theta_i \left(v_i, \frac{\partial \varphi_i}{\partial t} \right) < 0 \forall i : \frac{\partial \varphi_i}{\partial t} < 0$$

Of course, $K_i \frac{\partial \varphi_i}{\partial t} + v_i^T \theta_i \left(v_i, \frac{\partial \varphi_i}{\partial t} \right) = 0$ for $\frac{\partial \varphi_i}{\partial t} = 0$. In the preceding equations we used the fact that

$0 \leq \frac{\tanh(x)}{x} \leq 1 \forall x \geq 0$. So we have $\sum_i B_i \leq 0$ with equality holding only when $\frac{\partial \varphi_i}{\partial t} = 0 \forall i$. We have

$$\dot{V} = \sum_i B_i - \sum_i g_i \left\| v_i \right\|^2 \leq 0$$

Hence, by LaSalle's Invariance Principle, the state of the system converges to the largest invariant set contained in the set

$$S = \left\{ q, v : \left(\frac{\partial \varphi_i}{\partial t} = 0 \right) \wedge (v_i = 0) \, \forall i \right\} =$$
$$= \{ q, v : (v_i = 0) \, \forall i \}$$

because by definition the set $\left\{ q, v : \left(\frac{\partial \varphi_i}{\partial t} = 0 \right) \forall i \right\}$ is contained in the set $\{ q, v : (v_i = 0) \, \forall i \}$. For this subset to be invariant we need

$$\dot{v}_i = 0 \Rightarrow \frac{\partial \varphi_i}{\partial q_i} = 0 \forall i$$

In (Dimarogonas et al. (2003)) we have proven that this situation occurs whenever the potential functions either reach the destination or a saddle point. By bounding the exponent k from below by a finite number, φ_i becomes a navigation function, hence its critical points are isolated (Koditscheck and Rimon (1990)). Thus the set of initial conditions that lead to saddle points are sets of measure zero (Milnor (1963)). Hence the largest invariant set contained in the set $\frac{\partial \varphi_i}{\partial q_i} = 0 \forall i$ is simply $q_d \diamond$

5. SIMULATIONS

To demonstrate the navigation properties of our decentralized approach, we present a simulation of four holonomic agents that have to navigate from an initial to a final configuration, avoiding collision with each other. Each agent has no knowledge of the desired destinations of the other agents. In this picture $I - i, T - i$ denote the initial condition and desired destination of agent i respectively. The chosen configurations constitute non-trivial setups since the straight-line paths connecting initial and final positions of each agent are obstructed by other agents. The following conditions have been chosen for the simulation of figure 3:
Initial Conditions:

$$q_1(0) = \begin{bmatrix} .1232 & -.1 \end{bmatrix}^T, q_2(0) = \begin{bmatrix} -.1 & -.1 \end{bmatrix}^T,$$
$$q_3(0) = \begin{bmatrix} -.1232 & .1 \end{bmatrix}^T, q_4(0) = \begin{bmatrix} .1 & .1 \end{bmatrix}^T$$

$$u_1(0) = u_2(0) = u_3(0) = u_4(0) = \begin{bmatrix} 10^{-3} & 0 \end{bmatrix}$$

Final Conditions:

$$q_{d1} = \begin{bmatrix} -.1232 & .1 \end{bmatrix}^T, q_{d2} = \begin{bmatrix} .1 & .1 \end{bmatrix}^T,$$
$$q_{d3} = \begin{bmatrix} .1732 & -.1 \end{bmatrix}^T, q_{d4} = \begin{bmatrix} -.1 & -.1 \end{bmatrix}^T$$
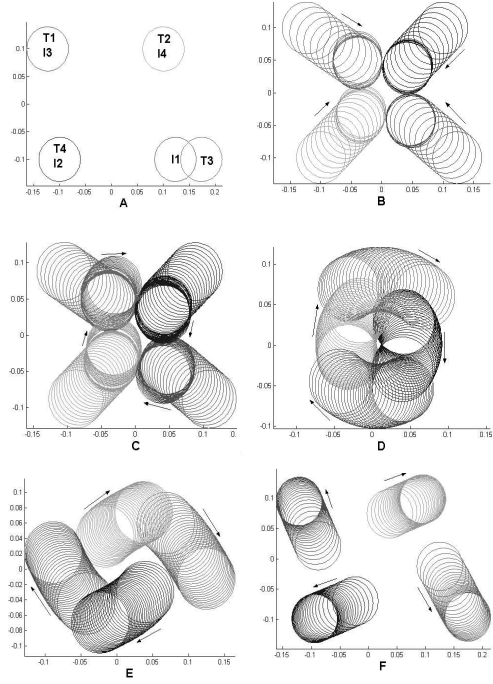

Fig. 3. Simulation 1

Screenshots A-F of Figure 3 show the evolution of the team configuration in time. Screenshot A shows the initial and final destination of each agent, whilst B-E show the conflict resolution procedure. The agents converge to their targets in screenshot F.

In the second simulation we show the influence of the f function in the collision avoidance procedure. The initial and final condition for agent 4 are the same in this case:
Initial Conditions:

$$q_1(0) = \begin{bmatrix} .1732 & -.1 \end{bmatrix}^T, q_2(0) = \begin{bmatrix} -.15 & -.15 \end{bmatrix}^T,$$
$$q_3(0) = \begin{bmatrix} -.1232 & .1 \end{bmatrix}^T, q_4(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$$

$$u_1(0) = u_2(0) = u_3(0) = u_4(0) = \begin{bmatrix} 10^{-3} & -10^{-3} \end{bmatrix}$$

Final Conditions:

$$q_{d1} = \begin{bmatrix} -.1732 & .1 \end{bmatrix}^T, q_{d2} = \begin{bmatrix} .15 & .15 \end{bmatrix}^T,$$
$$q_{d3} = \begin{bmatrix} .1732 & -.1 \end{bmatrix}^T, q_{d4} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$$

Screenshots A-F of Figure 4 show the evolution of the team configuration in time. One can see that agent 4 cooperates with the rest of the team in the conflict resolution procedure.

6. CONCLUSIONS

The decentralized navigation function methodology, established in previous work for navigation of a multi-agent system with single integrator dynamics has successfully been extended to the case of double integrator. The main motivation
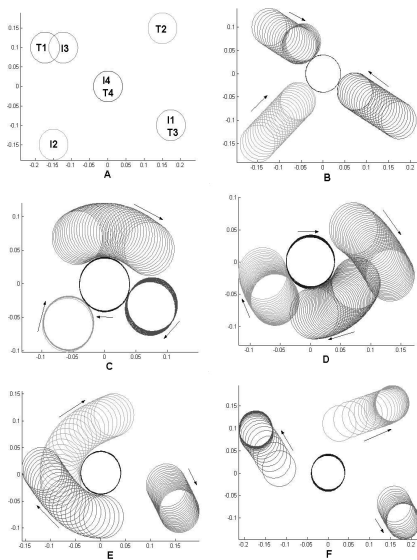
Fig. 4. Simulation 2

of this work lies in the fact that acceleration control is more implementable in multiple vehicle systems than velocity control. The stability of the overall system has been proven by LaSalle's Invariance Principle. The collision avoidance and global convergence properties have been verified through computer simulations.

Current research aims at limiting the sensing capabilities each agent has for the rest of the team,i.e. increasing the decentralization of the whole scheme. This has been accomplished for the case of single integrator dynamics in a recent paper (Dimarogonas and Kyriakopoulos (2005)). Another interesting issue that has to be dealt with is acceleration control for the case of nonholonomic dynamics.

7. ACKNOWLEDGMENT

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