A connection between formation control and flocking behavior in nonholonomic multiagent systems

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***Abstract*— This paper contains two main features: a provably correct distributed control strategy for convergence of multiple nonholonomic agents to a desired feasible formation configuration and a connection between formation infeasibility and flocking behavior in nonholonomic kinematic multi-agent systems. In particular, it is shown that when inter-agent formation objectives cannot occur simultaneously in the state-space then, under certain assumptions, the agents velocity vectors and orientations converge to a common value at steady state, under the same control strategy that would lead to a feasible formation. Convergence guarantees are provided in both cases using tools form algebraic graph theory and Lyapunov analysis. The results are verified through computer simulations. This is an extension of a result established in our previous work for multiple holonomic kinematic agents.**

I. INTRODUCTION

Multi-agent Navigation is a field that has recently gained increasing attention both in the robotics and the control communities, due to the need for autonomous control of more than one mobile robotic agents in the same workspace. While most efforts in the past had focused on centralized planning, specific real-world applications have lead researchers throughout the globe to turn their attention to decentralized concepts. The motivation for this work comes from many application domains one of the most important of which is the field of micro robotics ([7],[15]), where a team of a potentially large number of autonomous micro robots must cooperate in the sub micron level.

Among the various specifications that the control design aims to impose on the multi-agent team, formation convergence and achievement of flocking behavior are two objectives that have been pursued extensively in the last few years. The main feature of formation control is the cooperative nature of the equilibria of the system. Agents must converge to a desired configuration encoded by the inter-agent relative positions. Many feedback control schemes that achieve formation stabilization to a desire formation in a distributed manner have been proposed in literature, see for example [23],[12],[10],[3],[5] for some recent efforts. Of particular interest is also the so-called agreement problem, in which agents must converge to the same point in the state space ([16], [13], [19],[2],[9],[20],[11]). On the other hand, flocking behavior involves convergence of the velocity vectors and orientations of the agents to a common value at steady state; contributions include [8], [22],[18],[21],[14].

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A common mathematical tool that is used to analyze such multi-agent systems is algebraic graph theory ([1], [6]).

In most cases, formation convergence involves kinematic models of the agents' motion, while flocking behavior dynamic ones. Hence the problem of flocking motion has rarely been examined in the context of kinematic models of motion. In this paper, algebraic graph theory and Lyapunov stability analysis are used to establish a connection between formation control and flocking behavior for multiple nonholonomic *kinematic* agents. A discontinuous time invariant feedback control strategy is proposed that drives the multi-agent system to the desired formation configuration in the case of formation feasibility. The case of formation infeasibility is examined and an interesting result is derived. In particular, it is shown that when inter-agent formation objectives cannot occur simultaneously in the state-space then, under certain assumptions, the agents velocity vectors and orientations converge to a common value at steady state, under the same control strategy that would lead to a feasible formation. Hence a connection between formation infeasibility and flocking behavior is obtained. To the best of the authors' knowledge, the current paper contains the first concrete results regarding the connection of formation infeasibility and flocking behavior, for the case of nonholonomic multiple kinematic agents.

In our earlier work ([4]), a similar result was established for the case of multiple holonomic kinematic agents, i.e. agents whose motion is described by the single integrator. To the best of the authors' knowledge, the current paper and the previous one ([4]) contain the first concrete results regarding the connection of formation infeasibility and flocking behavior, for both nonholonomic and holonomic multiple kinematic agents.

The rest of the paper is organized as follows: section II describes the system and the problem that is treated in this paper. Assumptions regarding the communication topology between the agents are presented and the desired formation specification is modelled in terms of an undirected graph. The notion of formation feasibility and infeasibility is also introduced. Section III begins with some background on algebraic graph theory that is used in the sequel and proceeds with the introduction of the distributed feedback control strategy that drives the multi-agent team to the desired formation configuration in the case of formation feasibility as well as the corresponding stability analysis. The fact that formation infeasibility results in flocking behavior is discussed and proved in section IV. Some computer simulation results are included in section V while section VI summarizes the results of this paper and indicates current research efforts.

II. System and Problem Definition

Consider a system of N nonholonomic point agents operating in the same workspace $W \subset \mathcal{R}^2$. Let $q_i = [x_i, y_i]^T \in \mathcal{R}^2$ denote the position of agent i (see figure 1). The configuration space is spanned by $q = [q_1, \ldots, q_N]^T$. Each of the N mobile agents has a specific orientation θ_i with respect to the global coordinate frame. The orientation vector of the agents is represented by $\theta = [\theta_1 \ldots \theta_N]$. The configuration of each agent is represented by $p_i = \left[\begin{array}{cc} q_i & \theta_i \end{array}\right] \in R^2 \times (-\pi, \pi]$. Agent motion is described by the following nonholonomic kinematics:

$$\begin{array}{l} \dot{x}_i = u_i \cos\theta_i \\ \dot{y}_i = u_i \sin\theta_i \\ \dot{\theta}_i = \omega_i \end{array}, i \in N = [1, \ldots, N] \tag{1}$$

where u_i, ω_i denote the translational and rotational velocity of agent i, respectively. These are considered as the control inputs of the multi-agent system.

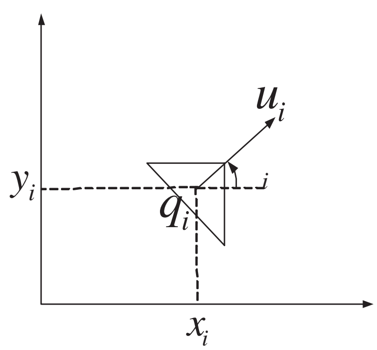

Fig. 1. Nonholonomic agent

Each agent's objective is to converge to a desired relative configuration with respect to a certain subset of the rest of the team, in a manner that will lead the whole team to a desired formation. Specifically, each agent is assigned with a specific subset N_i of the rest of the team, called agent i's *communication set* with which it can communicate in order to achieve the desired formation. Following the literature on formation control [17],[22], the desired formation can be encoded in terms of a *formation graph*:

Definition 1: The *formation graph* $G = \{V, E, C\}$ is an undirected graph that consists of (i) a set of vertices $V = \{1, ..., N\}$ indexed by the team members, (ii) a set of edges, $E = \{(i, j) \in V \times V | i \in N_j\}$ containing pairs of nodes that represent inter-agent formation specifications and (iii) a set of labels $C = \{c_{ij}\}$, where $(i, j) \in E$, that specify the desired inter-agent relative positions in the formation configuration.

The objective of each agent i is to be stabilized in a desired relative position c_{ij} with respect to each member j of N_i. Each agent has only knowledge of the state of agents that belong to its communication set. This fact highlights the distributed nature of the approach.

We assume that the formation graph is undirected, in the sense that

$$i \in N_j \Leftrightarrow j \in N_i, \forall i, j \in \mathcal{N}, i \neq j$$

It is obvious that $(i, j) \in E$ iff $i \in N_j \Leftrightarrow j \in N_i$. The formation configuration is called *feasible* if there are no conflicting interagent objectives, in the sense that

$$c_{ij} = -c_{ji}, \forall i, j \in \mathcal{N}, i \neq j$$

Whenever the latter does not hold, the formation configuration is called *infeasible*.

As an example, the next figure represents the communication graph of a team of seven agents with corresponding communication sets:

$$N_1 = \{2, 6\}, N_2 = \{1, 5\}, N_3 = \{6, 7\}$$

$$N_4 = \{5\}, N_5 = \{2, 4, 7\}, N_6 = \{1, 3, 7\}, N_7 = \{3, 5, 6\}$$

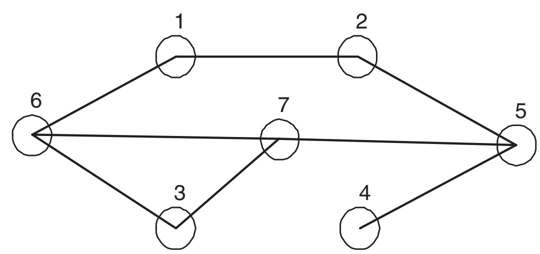

Fig. 2. The communication graph of a seven-agent team

In conclusion, the problem treated in this paper can be stated as follows: "*under the preceding assumptions, derive a set of control laws (one for each agent) that drives the team of agents from any initial configuration to the desired formation configuration.*".

III. Control Strategy and Stability Analysis

A. Tools from Algebraic Graph Theory

In this subsection we review some tools from algebraic graph theory that we shall use in the stability analysis of the next sections. The following can be found in any standard textbook on algebraic graph theory(e.g. [1], [6]).

For an undirected graph G with n vertices the *adjacency matrix* $A = A(G) = (a_{ij})$ is the $n \times n$ matrix given by

$$a_{ij} = \left\{ \begin{array}{l} 1, \text{if } (i, j) \in E \\ 0, \text{otherwise} \end{array} \right.$$

If there is an edge connecting two vertices i, j, i.e. $(i, j) \in E$, then i, j are called *adjacent*. A *path* of length r from a vertex i to a vertex j is a sequence of $r+1$ distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. If there is a path between any two vertices of the graph G, then G is called *connected* (otherwise it is called *disconnected*). The *degree* d_i of vertex i is defined as the number of its neighboring vertices, i.e. $d_i = \{\#j : (i, j) \in E\}$. Let Δ be the $n \times n$ diagonal matrix of d_i's. The (combinatorial) *Laplacian* of G is the symmetric positive semidefinite matrix $\mathcal{L}=\Delta-A$. The Laplacian captures many interesting topological properties of the graph. Of particular interest in our case is the fact that for a connected graph, the Laplacian has a single zero eigenvalue and the corresponding eigenvector is the vector of ones, $\overrightarrow{\mathbf{1}}$.

The last property has lead to the interesting result regarding the connection between formation non-feasibility and flocking behavior discussed in section IV. The next paragraphs of this section contain the stability analysis of the formation scheme.

As an example, the Laplacian matrix of the formation graph in figure 2 is given by:

$$\mathcal{L}=\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 & 0 \\ -1 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 3 & 0 & -1 \\ -1 & 0 & -1 & 0 & 0 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & -1 & 3 \end{bmatrix}$$

B. Stability of a feasible formation

Denote $c_{ii}=-\sum\limits_{j\in N_i} c_{ij}$ and $c_l=[c_{11},\ldots,c_{NN}]^T$. In the analysis that follows, we use the decoupling of the stack vector $q=[x,y]^T$ and the vector $c_l=[c_x,c_y]^T$ into the coefficients that correspond to the x,y directions of the agents respectively.We also use the function

$$\operatorname{sgn}(x)=\begin{cases} 1, x\geq 0 \\ -1, x<0 \end{cases}$$

The function $\arctan 2(x,y)$ that is also used in the sequel is the same as arc tangent of the two variables x and y with the distinction that the signs of both arguments are used to determine the quadrant of the result. It should also be noted that $\arctan 2(0,0)=0$ by definition. Finally, the notation $(a)_i$ denotes the i-th element of a vector a.

Convergence of the agents to the desired formation configuration in the case of formation feasibility is guaranteed by the following theorem:

Theorem 1: Assume that the formation configuration is feasible and that the formation graph is connected. Then the feedback control strategy:

$$u_i=-\operatorname{sgn}\left\{\gamma_{xi}\cos\theta_i+\gamma_{yi}\sin\theta_i\right\}\cdot\left(\gamma_{xi}^2+\gamma_{yi}^2\right)^{1/2} \quad (2)$$

$$\omega_i=\dot{\theta}_{nh_i}-\left(\theta_i-\theta_{nh_i}\right) \quad (3)$$

where

$$\gamma_{xi}=\left(\mathcal{L}x+c_x\right)_i, \gamma_{yi}=\left(\mathcal{L}y+c_y\right)_i$$

and the "nonholonomic angle"

$$\theta_{nh_i}=\arctan 2\left(\gamma_{yi},\gamma_{xi}\right)$$

drives the agents to the desired formation configuration with zero orientation.

Proof: We use the positive semidefinite function

$$V=\sum_i\gamma_i+\sum_i\left(\theta_i-\theta_{nh_i}\right)^2$$

as a candidate Lyapunov function, where

$$\gamma_i=\frac{1}{2}\sum_{j\in N_i}\left\|q_i-q_j-c_{ij}\right\|^2$$

First note that

$$\sum_i\nabla\gamma_i=\sum_i\begin{bmatrix}\frac{\partial\gamma_i}{\partial q_1}\\ \vdots \\ \frac{\partial\gamma_i}{\partial q_N}\end{bmatrix}$$

and

$$\frac{\partial\gamma_i}{\partial q_j}=\begin{cases}\sum\limits_{j\in N_i}\left(q_i-q_j\right)+c_{ii}, i=j\\ -\left(q_i-q_j-c_{ij}\right), j\in N_i, j\neq i\\ 0, j\notin N_i\end{cases}$$

where $c_{ii}=-\sum\limits_{j\in N_i}c_{ij}$, so that

$$\sum_i\frac{\partial\gamma_i}{\partial q_j}=\frac{\partial\gamma_j}{\partial q_j}+\sum_{i\in N_j}\frac{\partial\gamma_i}{\partial q_j}=$$

$$\sum_{i\in N_j}\left(q_j-q_i\right)+c_{jj}+\sum_{i\in N_j}\left(-\left(q_i-q_j-c_{ij}\right)\right)=$$

$$=2\cdot\sum_{i\in N_j}q_j-2\cdot\sum_{i\in N_j}q_i+2\cdot c_{jj}$$

$$=2\cdot d_jq_j-2\cdot\sum_{i\in N_j}q_i+2\cdot c_{jj}$$

Therefore

$$\sum_i\nabla\gamma_i=\sum_i\begin{bmatrix}\frac{\partial\gamma_i}{\partial q_1}\\ \vdots\\ \frac{\partial\gamma_i}{\partial q_N}\end{bmatrix}=2\begin{bmatrix}d_1\cdot q_1\\ \vdots\\ d_N\cdot q_N\end{bmatrix}-$$

$$2\begin{bmatrix}\sum\limits_{j\in N_1}q_j\\ \vdots\\ \sum\limits_{j\in N_N}q_j\end{bmatrix}+2\begin{bmatrix}c_{11}\\ \vdots\\ c_{NN}\end{bmatrix}=2\left(Lq+c_l\right)$$

where $L=\mathcal{L}\otimes I_2$. The last equation is a direct consequence of the fact that the formation configuration has been assumed to be feasible.

Differentiating the candidate Lyapunov function wrt time we get

$$V=\sum_i\gamma_i+\sum_i\left(\theta_i-\theta_{nh_i}\right)^2\Rightarrow$$

$$\Rightarrow\dot{V}=\left\{\sum_i\left(\nabla\gamma_i\right)^T\right\}\cdot\dot{q}+2\sum_i\left(\theta_i-\theta_{nh_i}\right)\cdot\left(\dot{\theta}_i-\dot{\theta}_{nh_i}\right)=$$

$$=2\left(Lq+c_l\right)^T\begin{bmatrix}u_1\cos\theta_1\\ u_1\sin\theta_1\\ \vdots\\ u_N\cos\theta_N\\ u_N\sin\theta_N\end{bmatrix}+$$

$$2\sum_i\left(\theta_i-\theta_{nh_i}\right)\cdot\left(\dot{\theta}_i-\dot{\theta}_{nh_i}\right)=$$

$$= 2\left(\mathcal{L}x + c_x\right)^T \begin{bmatrix} u_1 \cos\theta_1 \\ \vdots \\ u_N \cos\theta_N \end{bmatrix} +$$

$$+2\left(\mathcal{L}y + c_y\right)^T \begin{bmatrix} u_1 \sin\theta_1 \\ \vdots \\ u_N \sin\theta_N \end{bmatrix} +$$

$$+2\sum_i \left(\theta_i - \theta_{nh_i}\right) \cdot \left(\omega_i - \dot{\theta}_{nh_i}\right) =$$

$$= \sum_i \left\{2u_i \left(\left(\mathcal{L}x + c_x\right)_i \cos\theta_i + \left(\mathcal{L}y + c_y\right)_i \sin\theta_i\right)\right\}$$

$$+2\sum_i \left(\theta_i - \theta_{nh_i}\right) \cdot \left(\omega_i - \dot{\theta}_{nh_i}\right)$$

With the choice of control laws (2),(3) we get

$$\dot{V} = 2\sum_i \left\{-\left|\gamma_{xi}\cos\theta_i + \gamma_{yi}\sin\theta_i\right| \left(\gamma_{xi}^2 + \gamma_{yi}^2\right)^{1/2}\right\}$$

$$-2\sum_i \left(\theta_i - \theta_{nh_i}\right)^2 \le 0$$

It is easy to see that this definition of θ_{nh_i} implies that

$$\left|\gamma_{xi}\cos\theta_i + \gamma_{yi}\sin\theta_i\right| = \sqrt{\gamma_{xi}^2 + \gamma_{yi}^2} \cdot \left|\cos\left(\theta_i - \theta_{nh_i}\right)\right|$$

so that

$$\dot{V} = 2\sum_i \left\{-\left(\gamma_{xi}^2 + \gamma_{yi}^2\right)\left|\cos\left(\theta_i - \theta_{nh_i}\right)\right|\right\}$$

$$-2\sum_i \left(\theta_i - \theta_{nh_i}\right)^2 \le 0$$

By LaSalle's invariance principle, the trajectories of the system converge to the largest invariant set contained in the set

$$\left\{ \begin{array}{c} \left(\gamma_{xi} = \gamma_{yi} = 0\right) \vee \\ \left(\left|\theta_i - \theta_{nh_i}\right| = \frac{\pi}{2}\right) \end{array} \right\} \wedge \left(\theta_i = \theta_{nh_i}\right) \equiv$$

$$\equiv \left(\gamma_{xi} = \gamma_{yi} = 0\right) \wedge \left(\theta_i = \theta_{nh_i}\right)$$

For $\left(\gamma_{xi} = \gamma_{yi} = 0\right)$ we have $\theta_{nhi} = 0$ by definition of θ_{nhi}, so that $\theta_i = 0 \; \forall \; i$. In addition $\left(\gamma_{xi} = \gamma_{yi} = 0\right)\forall i$ guarantees that the agents converge to the desired formation configuration. This is easily derived by the fact that

$$\left(\gamma_{xi} = \gamma_{yi} = 0\right)\forall i \Rightarrow Lq + c_l = 0$$

For all $i \in N$, let c_i denote the configuration of agent i in a desired formation configuration with respect to the global coordinate frame. It is then obvious that $c_{ij} = c_i - c_j \forall (i,j) \in E$ for all possible desired final formations. Define $q_i - q_j - c_{ij} = q_i - q_j - (c_i - c_j) = \tilde{q}_i - \tilde{q}_j$. Then the feasibility assumption implies that

$$Lq + c_l = 0 \Rightarrow L\tilde{q} = 0 \Rightarrow \mathcal{L}\tilde{x} = \mathcal{L}\tilde{y} = 0$$

where $\tilde{x}, \tilde{y}$ the stack vectors of $\tilde{q}$ in the x, y directions. The fact that the formation graph is connected implies that the Laplacian has a simple zero eigenvalue with corresponding eigenvector the vector of ones, $\overrightarrow{\mathbf{1}}$. This guarantees that both $\tilde{x}, \tilde{y}$ are eigenvectors of $\mathcal{L}$ belonging to span$\{\overrightarrow{\mathbf{1}}\}$. Hence

$$\tilde{q}_i = c \forall i \Rightarrow q_i - q_j = c_{ij} \forall i, j, j \in N_i$$

We conclude that the agents converge to the desired relative configuration. $\diamondsuit$

It must be stressed out that the proposed feedback control strategy (2),(3) is purely *decentralized*, since each agent requires information only of the states of agents within each neighboring set at each time instant. This is a consequence of the definitions of the terms $\gamma_{xi}, \gamma_{yi}, \theta_{nh_i}$ and the form of the Laplacian matrix $\mathcal{L}$ of the communication graph.

IV. FORMATION NON-FEASIBILITY RESULTS IN FLOCKING BEHAVIOR

The key assumption behind the stability analysis of the previous section is *formation feasibility*, in the sense discussed in section II. But what happens when the formation configuration is infeasible, i.e. there does *not* exist such a configuration in the state space? In this case, equation $\sum_i \nabla\gamma_i = 2\left(Lq + c_l\right)$ is no longer valid and the deductions of the previous section do not hold. The next theorem shows what happens in the case of formation infeasibility:

Theorem 2: If the formation graph is connected, the non-holonomic multi-agent system reaches a configuration in which all agents have the same velocities and orientations even if the formation configuration is infeasible.

Proof: Using the notation $\theta_i - \theta_{nh_i} \equiv \varphi_i$, equation (3) implies that

$$\dot{\varphi}_i = -\varphi_i \Rightarrow \varphi_i \stackrel{t\to\infty}{\to} 0$$

Hence $\theta_i = \theta_{nh_i} \forall i$ at steady state. The closed loop kinematics for the x, y-cofficients then become

$$\begin{array}{l} \dot{x}_i = u_i \cos\theta_{nh_i} = -\mathrm{sgn}\left\{\gamma_{xi}\cos\theta_{nh_i} + \gamma_{yi}\sin\theta_{nh_i}\right\}\gamma_{xi} \\ \dot{y}_i = u_i \sin\theta_{nh_i} = -\mathrm{sgn}\left\{\gamma_{xi}\cos\theta_{nh_i} + \gamma_{yi}\sin\theta_{nh_i}\right\}\gamma_{yi} \end{array}$$

But since by definition of θ_{nh_i} we have $\gamma_{xi}\cos\theta_{nh_i} + \gamma_{yi}\sin\theta_{nh_i} > 0$, then at steady state the previous equations reduce to:

$$\begin{array}{l} \dot{x}_i = -\gamma_{xi} \\ \dot{y}_i = -\gamma_{yi} \end{array}, i \in N = \{1, ..., N\} \tag{4}$$

Using

$$W = \frac{1}{2}\sum_i \left(\dot{x}_i^2 + \dot{y}_i^2\right)$$

as a candidate Lyapunov function for the system (4) and differentiating wrt time we get:

$$\dot{W} = \sum_i \left(\dot{x}_i\ddot{x}_i + \dot{y}_i\ddot{y}_i\right) = -\sum_i \left(\dot{x}_i\dot{\gamma}_{xi} + \dot{y}_i\dot{\gamma}_{yi}\right)$$

$$= -\sum_i \left(\dot{x}_i\left(\mathcal{L}\dot{x}\right)_i + \dot{y}_i\left(\mathcal{L}\dot{y}\right)_i\right) \Rightarrow$$

$$\Rightarrow \dot{W} = -\dot{x}^T\mathcal{L}\dot{x} - \dot{y}^T\mathcal{L}\dot{y} \le 0$$

LaSalle's Invariance Principle guarantees that the state of the system (4) converges to the largest invariant subset of the set $S = \left\{\dot{q} | \dot{W} = 0\right\}$. Using the same arguments as in the proof of Theorem 1, we deduce that at steady state both $\dot{x} = \left[\dot{x}_1, ..., \dot{x}_N\right]^T, \dot{y} = \left[\dot{y}_1, ..., \dot{y}_N\right]^T$ are eigenvectors of $\mathcal{L}$ corresponding to the zero eigenvalue, meaning that $\dot{x}, \dot{y}$ belong to span$\{\overrightarrow{\mathbf{1}}\}$, which ensures that all agent velocity vectors will have the same components at steady state, and will therefore be equal. It is obvious then that the nonholonomic angles θ_{nh_i} of all agents are equal (since all γ_{xi}, γ_{yi} are equal) and the fact that $\theta_i = \theta_{nh_i} \forall i$ guarantees that at steady state all agents will have a common orientation. $\diamondsuit$

This simple result shows that formation non-feasibility is directly related to a phenomenon with many similarities to what is known as flocking behavior in multi-agent systems. Agents converge to a configuration in which all have the same velocities and orientations even if the formation configuration is infeasible. A similar result holds in the case of holonomic kinematic agents as well ([4]).

V. Simulations

To verify the result of the previous paragraphs we provide some computer simulations of the proposed control framework (2),(3).

In the first simulation, four nonholonomic agents starting from arbitrary initial position, navigate under the proposed control scheme. The communication sets in this simulation have been chosen as

$$N_1 = \{2\}, N_2 = \{1,3\}, N_3 = \{2,4\}, N_4 = \{3\}$$

It is easily verified that the corresponding communication graph is connected. The four agents aim to converge to a line formation and the desired inter-agent relative positions are chosen accordingly. Specifically we have set $c_{12} = c_{23} = c_{34} = [-0.01, 0]$. Screenshots I-VI show the evolution in time of the multi-agent team. In screenshot I, A-i denotes the initial position of agent i. In the last screenshot, the agents converge to the desired line formation configuration.

The second simulation involves four agents and a non-feasible formation configuration. The values of the parameters in this simulation are the same as previously while the desired inter-agent distances have been slightly perturbed in order to achieve formation infeasibility. Specifically we have set $c_{12} = -c_{21} + [.002, .002]$ and $c_{34} = -c_{43} + [.001, .001]$. The formation configuration is rendered infeasible in this way. Screenshots I-IV of figure 4 show the evolution in time and achievement of flocking motion for the multi-agent system. In screenshot IV the velocities and orientations of the four agents converge to a common value, something that is also witnessed in the velocity diagram of the last screenshot.

VI. Conclusions

In this paper, two main results were presented: a provably correct distributed control strategy for convergence of multiple nonholonomic agents to a desired feasible formation and a connection between formation infeasibility and flocking behavior in nonholonomic kinematic multi-agent systems. In particular, it has been shown that when inter-agent formation objectives cannot occur simultaneously in the state-space then, under certain assumptions, the agents velocity vectors and orientations converge to a common value at steady state, under the same control strategy that would lead to a feasible formation. Convergence guarantees are provided in both

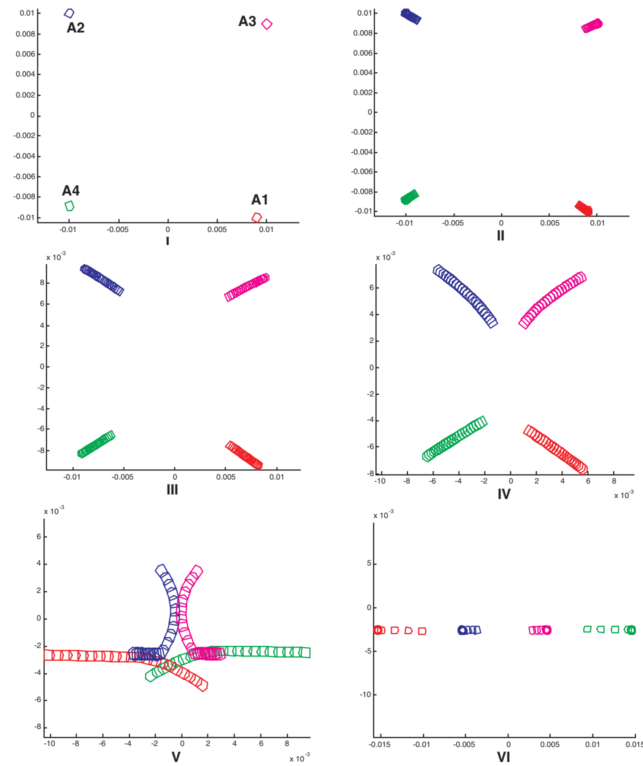

Fig. 3. Four nonholonomic agents converge to a line formation

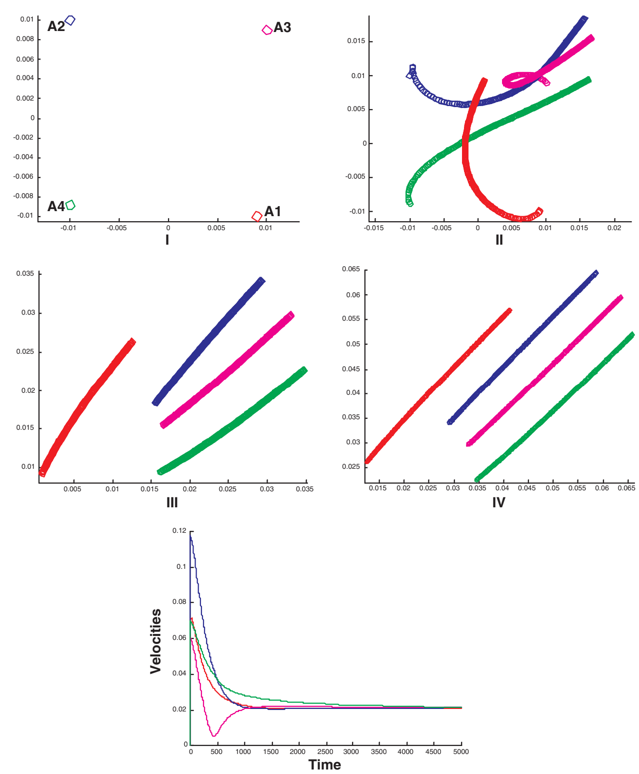

Fig. 4. Formation infeasibility results in flocking behavior for the multi-agent system.

cases using tools form algebraic graph theory and Lyapunov analysis. This is an extension of a result established in our previous work ([4]) for multiple holonomic kinematic agents. To the best of the authors' knowledge, these two papers contain the first concrete results regarding the connection of formation infeasibility and flocking behavior, for both nonholonomic and holonomic multiple kinematic agents.

Current research involves extending the current results to more general motion models, including three-dimensional models and general nonlinear dynamics. Collision avoidance among the members of the multi-agent team is a specification that is crucial for the control design in practical situations and is currently pursued. Another direction of research is to take into account directed graphs and switching communication topology. As a parallel result of this work, the state agreement or rendezvous problem for multiple unicycles is also under investigation.

VII. Acknowledgements

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