A Self-triggered Model Predictive Control Framework for the Cooperation of Distributed Nonholonomic Agents

Alina Eqtami, Shahab Heshmati-alamdari, Dimos V. Dimarogonas and Kostas J. Kyriakopoulos

Abstract— **In this paper we propose a decentralized Model Predictive Control (MPC) framework with a self-triggering mechanism, for a team of cooperating agents. The nonholonomic agents are controlled locally and exchange information with their neighbors. The aim at scheduling the control updates based on a self-triggering criterion is twofold: to reduce the updates of the control law for each agent and to reduce the communication effort between the agents. The input-to-state (abbr. ISS) stability of the agents is proven, the condition for triggering is provided and the theoretic results are then depicted by a simulated example.**

I. Introduction

The formulation of control schemes in event-based frameworks has gained a lot of research interest in the recent years, [1], [2], [20], [21]. The decision for sampling in event-based schemes takes into account state or output feedback in order to sample as infrequently as possible while guaranteeing to preserve the stability of the system, [8], [9], [13]. Thus, it can be proven to be less conservative with respect to the constant sampling where the worst case scenario is considered.

The event-based approaches, either it is event-triggered control or self-triggered control has a particular relevance in network systems and to distributed / decentralized frameworks. Both approaches are comprised, inter alia, by triggering mechanisms that determine when the new control update should be. Nevertheless, the event-triggered techniques require a constant measurement of the state of the plant, or in the case of distributed schemes, it requires the continuous monitoring of the state of the neighbors in order to decide when the control update must be triggered. In the case of self-triggered control only the latest state measurement needs to be known for determining the next triggering instant, which in fact can reduce the communication effort between the distributed agents. Related works on event/ self-triggered control in the distributed frameworks can be found in [4], [11], [15], [16], [17] and the references quoted therein.

Nonlinear Model Predictive controllers have the capability to deal with nonlinearities and constraints. This is particularly desired when a large-scale problem must be formulated. There are many approaches on formulating these kind of problems under the MPC framework, namely the hierarchical, the decentralized or the distributed architectures. Related results on NMPC for large-scale systems can be found in [7], [12], [18] and in the review paper [19] and the number of papers quoted therein.

In this paper, a distributed framework is considered for a team of cooperating agents governed by nonholonomic kinematic models. The agents run local predictive controllers and they are exchanging information with a set of neighboring agents only on their own triggering instants. The contribution of this paper relies in finding sufficient conditions for triggering in the self-triggered control context: each one of the agents monitors its own triggering condition and between the intersampling periods applies in open-loop, the previously computed control sequence. This cooperative scenario has been introduced in the classic constant sampling framework, [7] and in an event-based framework, [5] for general nonlinear systems. However, with the self-triggered approach, the updates of the local control laws are reduced and additionally the communication effort between the agents is mitigated.

The event-based set-up for MPC controllers has just started to gain attention whereas few results have been presented for the self-triggered MPC. For the self-triggered MPC the reader is referred to [10], [3], [14] and [6]. The organization of the paper is the following: the mathematical modeling of the agents as well as the design of the local controllers are given in Section II. Section III accommodates the ISS stability analysis of the local MPC schemes which leads to Section IV where the conditions for the self-triggered framework are provided. A simulated example and the conclusions are reported in Sections V and VI, respectively.

II. Problem Formulation

In this section, the cooperative scenario of multiple agents that work in the same environment is formulated. We consider a distributed framework, and for this reason, the model, the constraints and the design of the controllers for each of the agents, are given. In the subsequent sections the overall problem is stated rigorously.

A. Mathematical Modeling

Consider a general system which is composed by M local subsystems. The subsystems are all described by the same form of nonholonomic kinematic equations of the following

Alina Eqtami, Shahab Heshmati-alamdari and Kostas J. Kyriakopoulos are with the Control Systems Lab, Department of Mechanical Engineering, National Technical University of Athens, 9 Heroon Polytechniou Street, Zografou 15780, Greece {alina, shahab, kkyria@mail.ntua.gr}. Dimos V. Dimarogonas is with the ACCESS Linnaeus Centre, School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden. He is also affiliated with the KTH Centre for Autonomous Systems and is supported by the Swedish Research Council (VR) through contract 2009-3948. {dimos@ee.kth.se}.

form:

$$x^i(k+1) = f(x^i(k), u^i(k)) \Rightarrow \tag{1a}$$

$$\begin{bmatrix}\chi^i(k+1)\\ y^i(k+1)\\ \theta^i(k+1)\end{bmatrix} = \begin{bmatrix}\chi^i(k) + dt\cos\theta^i(k)v^i(k)\\ y^i(k) + dt\sin\theta^i(k)v^i(k)\\ \theta^i(k) + dt\omega^i(k)\end{bmatrix} \tag{1b}$$

with $k \in \mathbb{Z}_{\geq 0}$ and $i = 1, \ldots, M$. The state of the subsystem i is denoted by $x^i(k) \triangleq [\chi^i(k), y^i(k), \theta^i(k)]^\top$, while $u^i(k) \triangleq [v^i(k), \omega^i(k)]^\top$ denotes the control variable. Suppose that the agents evolve on the same discrete-time space, i.e., they are synchronized. The state and the control vectors are required to fulfill the following constraints

$$x^i(k) \in X^i \qquad u^i(k) \in U \tag{2}$$

where $X^i \subseteq \mathbb{R}^3$ and $U^i \subseteq \mathbb{R}^2$ are compact sets containing the origin as an interior point. In particular, the constraints of the input are of the form $|v^i| \leq \bar{v}$ and $|\omega^i| \leq \bar{\omega}$.

The distributed system comprised of the M subsystems is decoupled, but in order to achieve some degree of cooperation it is assumed that each agent $\mathcal{A}^i$ for all $i = 1, \ldots, M$ exchanges information with a set of neighboring agents $\mathcal{G}^i \triangleq \{\mathcal{A}^j, j \in G^i\}$, where G^i denotes the set of indexes identifying the agents belonging to the set $\mathcal{G}^i$. Consider, now, a generic time-step k, then, for each $i = 1, \ldots, M$ the agent $\mathcal{A}^i$ receives from all neighboring agents $\mathcal{A}^j \in \mathcal{G}^i$ their state vectors $x^j(k)$ and their velocity vectors $u^j(k)$. More precisely, the information received by an agent $\mathcal{A}^i$ at time step k, can be written as

$$w^i(k) \triangleq col[x^j(k)]\, \forall j \in G^i \tag{3a}$$

$$w_u^i(k) \triangleq col[u^j(k)]\, \forall j \in G^i \tag{3b}$$

with $w^i(k) \in W^i \triangleq \Pi_{j\in G^i} X^j$ and $w_u^i(k) \in \Pi_{j \in G^i} U$. It is assumed that (i) this information is always available and accurate and (ii) can be exchanged without a delay. Notice however that we consider a self-triggering framework, so this exchange of information is not taking place at each time-step, but whenever it is *necessary* as it will be explained later on.

B. Control Design and Objective

The goal for each generic agent $\mathcal{A}^i$, described by (1a) and is subject to (2), is to be driven to a desired state which is included in X^i. In order to achieve this task local NMPC controllers, for each of the agents, are employed. For all the subsystems, it can be proven that the closed-loop systems are ISS with respect to the information vectors received by their neighbors and more specifically that the state of each subsystem is converging to a desired terminal set. Inside this set, an auxiliary terminal controller is employed to drive the system to the desired point. The design of the local NMPC controllers for a generic subsystem (1a) is presented next.

For each agent $\mathcal{A}^i$ and at a time-step k, the local NMPC control law is computed by solving a finite-horizon, open-loop optimal control problem (OCP), based on its state measurement $x^i(k)$ and based on the information received from the neighbors; the state and the velocity vectors $w^i(k)$ and $w_u^i(k)$, respectively. The optimal problem consists in minimizing, with respect to a control sequence $\{u^i(k|k), u^i(k+1|k), \ldots, u^i(k+N^i-1|k)\}$, a cost function $J_N^i(x^i(k), w^i(k), w_u^i(k), N^i)$. The cost function for the OCP, is given by

$$J_N^i(x^i(k), w^i(k), w_u^i(k), N^i) = \tag{4a}$$
$$\sum_{t=0}^{N^i-1} \{L^i(\hat{x}^i(k+t|k), u^i(k+t|k))$$
$$+ Q^i(\hat{x}^i(k+t|k), \hat{w}^i(k+t|k))\} + V^i(\hat{x}^i(k+N^i|k))$$

subject to

$$\hat{x}^i(k+t|k) \in X^i \qquad \forall t = 1, \ldots, N^i - 1 \tag{4b}$$
$$u^i(k+t|k) \in U \qquad \forall t = 0, \ldots, N^i - 1 \tag{4c}$$
$$\hat{w}^i(k+t|k) \in W^i \qquad \forall t = 0, \ldots, N^i - 1 \tag{4d}$$
$$\hat{x}^i(k+N^i|k) \in X_f^i \tag{4e}$$

where X_f^i denotes the terminal constraint set and $\hat{x}^i(k|k) = x^i(k)$. The positive integer $N^i \in \mathbb{Z}_{\geq 0}$, denotes the prediction horizon. The notation $\hat{x}^i(\cdot|\cdot)$ used in (4a), (4b) and (4e), denotes the predicted state of the agent $\mathcal{A}^i$ and is given as

$$\hat{x}^i(k+t+1|k) = f(\hat{x}^i(k+t|k), u^i(k+t|k)) \tag{5}$$

which accounts for the predicted state at time $k+t+1$ with $t \in \mathbb{Z}_{\geq 0}$, based on the measurement of the state at time k while using a control input u_{k+t}^i and the model of the system from (1a). In the same manner, the predicted states of the neighbors of the agent $\mathcal{A}^i$ are given as

$$\hat{w}^i(k+t+1|k) = f(\hat{w}^i(k+t|k), w_u^i(k+t|k)) \tag{6}$$

which is equivalent to

$$col[\hat{x}^j(k+t+1|k)] = col[f(\hat{x}^j(k+t|k), u^j(k+t|k))],\; j \in G^i$$

The vector $\hat{w}^i(k+t+1|k)$ for $t = 0, \ldots, N^i - 1$ denotes the prediction of the neighbors' states. However, at a generic time instant k, only $\hat{w}^i(k|k) \triangleq w^i(k)$ as well as $w_u^i(k)$ are known to the agent $\mathcal{A}^i$. In order to solve the OCP (4a)-(4e), the controller of the agent $\mathcal{A}^i$, assumes the following for the prediction horizon: the agents $\mathcal{A}^j$ for all $j \in G^i$, maintain the same velocity during the whole prediction horizon N^i, i.e., $w_u^i(k+t|k) = w_u^i(k)$, $\forall t \in [0, N^i - 1]$. This assumption is utilized only for the prediction of the controller, and it is clear that the trajectories of the neighboring agents will diverge from the predicted ones due to individual dynamics. However, the closed-loop nature of the overall framework is able to overcome this limitation, as it will be shown in the stability analysis.

In order to proceed to the subsequent analysis, some standard stability conditions for the design of the local predictive controllers are introduced, in order to assert that the MPC strategy results in stabilizing local controllers for each of the subsystems.

Assumption 1: The stage cost $L^i(x^i, u^i)$ is Lipschitz continuous in $X^i \times U$ and it holds that $L^i(0,0) = 0$. Moreover, there is a $\mathcal{K}_\infty$-function r^i, such that $L^i(x_k^i, u_k^i) \geq r^i(||x_k^i||)$.

Assumption 2: The running cost $Q^i(x^i, w^i)$ is such that $Q^i(x^i, w^i) \geq 0$. Moreover, Q^i is Lipschitz continuous in $X^i \times W^i$, with Lipschitz constants L^i_{qx} and L^i_{qw}, respectively.

Assumption 3: Let the terminal set X^i_f be such that $X^i_f \subset X^i$, X^i_f to be closed, and $0 \in X^i_f$. Assume that there is a locally stabilizing controller $h^i(x_k)$ for the terminal set. The associated Lyapunov function $V^i(\cdot)$ has the following property, for all $x^i \in X^i_f$ and for all $w^i \in W^i$,

$$\begin{aligned} & V^i(f^i(x^i(k), h^i(x^i(k)))) - V^i(x^i(k)) \leq \\ & - L^i(x^i(k), h^i(x^i(k))) - Q^i(x^i(k), w^i(k)) \end{aligned}$$

C. Problem Preliminaries

The solution of the OCP (4a)-(4e) at a time-step k provides an optimal control sequence. The classic framework of the MPC consists in applying to the system only the first control vector, i.e., $u^{*i}(k|k)$ and to discard all the remaining elements of the sequence. At the next time-step $k+1$, new state measurements are received and the whole procedure is repeated again. This is iteratively repeated until the system has reached to the desired terminal set. However, the self-triggering framework suggests that a portion of the computed control sequence may be applied to the system and not only the first vector. Suppose a triggering instant k_i. The control sequence that is applied to the plant is of the form

$$\{u^*(k_i|k_i), u^*(k_i+1|k_i), \ldots u^*(k_i+t|k_i)\} \tag{7}$$

for all $t \in [0, k_{i+1} - k_i - 1]$, where k_{i+1} is the next triggering instant. During the time interval $[k_i, k_{i+1})$ the control law is applied to the plant in an open-loop fashion, while no measurements from the neighboring agents are received. A question that naturally arises is how large this time interval can be? Notice, though, the smallest time interval is obviously 1, that is if $k_{i+1} = k_i + 1$. The self-triggered strategy that will be presented later in this paper, answers to this question and provides sufficient conditions for finding the recalculation periods, or in other words sufficient conditions for triggering the computation of the NMPC law. This leads us to the statement of the problem treated in this paper:

Problem Statement 1: Consider a generic subsystem (1a) that is subject to constraints (2), while measuring (3a)-(3b) from the neighboring agents. The objective is (i) to design a feedback control law provided by (4a)-(4e) such that the subsystem (1a) converges to its terminal constraint set and (ii) to find a mechanism to decide when the recalculation instants of the local control law should be.

III. Stability Analysis of NMPC

Consider a time-step k when an event is triggered, then a new OCP (4a)-(4e) is solved which provides an optimal control sequence $\{u^{i*}(k|k), \ldots, u^{i*}(k+N^i-1|k)\}$. The optimal cost $J^{i*}_N(k)$, is the cost (4a) under this optimal control sequence.

Consider now, control sequences $\bar{u}^i(\cdot)$ for time-steps $m = 1, \ldots, N^i - 1$, based on the optimal solution at the triggering instant k, given as

$$\bar{u}^i(k+t|k+m) = \begin{cases} u^{i*}(k+t|k) & \text{for } t = m, \ldots, N^i - 1 \\ h^i(\hat{x}^i(k+t|k+m)) & \text{for } t = N^i, \ldots, N^i + m - 1 \end{cases} \tag{8}$$

These control sequences are admissible and in general sub-optimal. From the feasibility of the optimal control trajectory at time-step k it follows that for all $t, m = 1, \ldots, N^i - 1$ we have $\bar{u}^i(k+t|k+m) \in U$ and $\hat{x}^i(k+N^i|k+m) \in X^i_f$. Now, let $\bar{J}^i_N(k+m)$ to be the "feasible" cost at a time step $k+m$, $\forall m \in [1, N^i - 1]$. This cost is derived from (4a) for a control sequence (8). This "feasible" cost will help us to obtain the difference $J^{i*}_N(k+m) - J^{i*}_N(k)$. First we are going to evaluate this difference for $m = 1$, then for $m = 2$ and finally invoke the general formulation.

For $m = 1$ we have

$$\begin{aligned} \bar{J}^i_N(k+1) &= J^{i*}_N(k) - L^i(x^i(k), u^i(k)) - Q^i(x^i(k), w^i(k)) \\ &+ \sum_{t=1}^{N^i-1} \{L^i(\bar{x}^i(k+t|k+1), \bar{u}^i(k+t|k+1)) \\ &+ Q^i(\bar{x}^i(k+t|k+1), \hat{w}^i(k+t|k+1)) \\ &- L^i(\hat{x}^i(k+t|k), u^{i*}(k+t|k)) \\ &- Q^i(\hat{x}^i(k+t|k), \hat{w}^i(k+t|k))\} \\ &+ L^i(\bar{x}^i(k+N^i|k+1), h^i(\bar{x}^i(k+N^i|k+1)) \\ &+ Q^i(\bar{x}^i(k+N^i|k+1), \hat{w}^i(k+N^i|k+1)) \\ &+ V^i(\bar{x}^i(k+N^i+1|k+1)) - V^i(\hat{x}^i(k+N^i|k)) \end{aligned} \tag{9}$$

where $\bar{x}^i(\cdot)$ is the state of the agent $\mathcal{A}^i$ while a feasible control input from (8) is being applied. Notice that we consider nominal stability of the agents, thus, the predicted state $\hat{x}(\cdot)$ and the "feasible" state $\bar{x}(\cdot)$, computed at the same time-step are coinciding.

Using Assumption 2, the following result can be obtained

$$\begin{aligned} & Q^i(\bar{x}^i(k+t|k+1), \hat{w}^i(k+t|k+1)) \\ & - Q^i(\hat{x}^i(k+t|k), \hat{w}^i(k+t|k)) \leq \\ & ||Q^i(\cdot, \hat{w}^i(k+t|k+1)) - Q^i(\cdot, \hat{w}^i(k+t|k))|| \leq \\ & L^i_{qw}||\hat{w}^i(k+t|k+1) - \hat{w}^i(k+t|k)|| \end{aligned} \tag{10}$$

From the Appendix and in particular from (21), it yields

$$\begin{aligned} & L^i_{qw}||\hat{w}^i(k+t|k+1) - \hat{w}^i(k+t|k)|| \leq \\ & L^i_{qw} dt(m-1) \sum_{j \in G^i} \{(2(\bar{v} + v^j_k)^2 + \bar{\omega}^2)^{1/2}\} \end{aligned} \tag{11}$$

Using the inequality from Assumption 3, and substituting (11) to (9), we obtain

$$\begin{aligned} \bar{J}^i_N(k+1) &\leq J^{i*}_N(k) - L^i(x^i(k), u^i(k)) - Q^i(x^i(k), w^i(k)) \\ &+ L^i_{qw} dt(m-1) \sum_{j \in G^i} \{(2(\bar{v} + |v^j_k|)^2 + \bar{\omega}^2)^{1/2}\} \end{aligned} \tag{12}$$

From the optimality of the solution that yields $J^{i*}_N(k+1) \leq \bar{J}^i_N(k+1)$ and with the help of the Assumption 1, the following is derived

$$J_N^{i*}(k+1) - J_N^{i*}(k) \leq -r^i(||x^i(k)||) + L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{v} + |v_k^j|)^2 + \bar{\omega}^2)^{1/2}\} \tag{13}$$

For $m = 2$ we get

$$\begin{aligned} \bar{J}_N^i(k+2) &\leq J_N^{i*}(k) - L^i(x^i(k), u^i(k)) \\ &- Q^i(x^i(k), w^i(k)) - L^i(\hat{x}^i(k+1|k), u^i(k+1|k)) \\ &- Q^i(\hat{x}^i(k+1|k), \hat{w}^i(k+1|k)) \\ &+ \sum_{t=1}^{N^i-2} \{Q^i(\hat{x}(k+t+1|k+2), \hat{w}(k+t+1|k+2)) \\ &- Q^i(\hat{x}(k+t+1|k), \hat{w}(k+t+1|k))\} \end{aligned} \tag{14}$$

Using similar arguments as before, we obtain the following

$$J_N^{i*}(k+2) - J_N^{i*}(k) \leq -r^i(||x^i(k)||) - r^i(||\hat{x}^i(k+1|k)||) + L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{v} + |v_k^j|)^2 + \bar{\omega}^2)^{1/2}\} \tag{15}$$

From the above it can be concluded using the same procedure that for random $m \in [1, N^i - 1]$, we get

$$\begin{aligned} &J_N^{i*}(k+m) - J_N^{i*}(k) \leq \\ &- r^i(||x^i(k)||) - \sum_{\rho=1}^{m-1} \{r^i(||\hat{x}^i(k+\rho|k)||)\} \\ &+ L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{v} + |v_k^j|)^2 + \bar{\omega}^2)^{1/2}\} \end{aligned} \tag{16}$$

In (16), it is shown that the difference $J_N^{i*}(k+m) - J_N^{i*}(k)$ is bounded.

IV. The self-triggered Framework

In this section the self-triggering mechanism is going to be presented. Consider that at time k_i, an event is triggered. Then (16) becomes

$$\begin{aligned} &J_N^{i*}(k_i+m) - J_N^{i*}(k_i) \leq \\ &- r^i(||x^i(k)||) - \sum_{\rho=1}^{m-1} \{r^i(||\hat{x}^i(k_i+\rho|k_i)||)\} \\ &+ L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{v} + |v^j(k_i)|)^2 + \bar{\omega}^2)^{1/2}\} \end{aligned} \tag{17}$$

For $m = [1, N^i - 1]$, and if the following is valid

$$\begin{aligned} &L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{v} + |v^j(k_i)|)^2 + \bar{\omega}^2)^{1/2}\} \\ &\leq \sigma(r^i(||x^i(k)||) + \sum_{\rho=1}^{m-1} \{r^i(||\hat{x}^i(k_i+\rho|k_i)||)\}) \end{aligned} \tag{18}$$

for $0 < \sigma < 1$, then the Lyapunov function $J_N^{i*}(k)$ is decreasing and the ISS property of the system is guaranteed.

Next we describe the self-triggering mechanism for a generic agent $\mathcal{A}^i$. At time k_i a control update is triggered, the controller reads the local state measurement and receives the information from the neighboring agents and finally it provides a control sequence for $[k_i, k_i + N^i - 1]$. The controller checks for how many steps inequality (18) is valid, and applies the optimal trajectory that was computed at time-step k_i for all those steps, in an open-loop fashion, until the next triggering instant k_{i+1}. The aforementioned procedure is repeated until the subsystem converges to the terminal constraint set.

We are now ready to state the stability result for this self-triggered MPC framework:

Theorem 1: Consider the subsystem (1a) that is subject to constraints (2) under the NMPC strategy and assume that *Assumptions* 1-3 hold. The control update times that are provided by (18) and the NMPC law provided by (4a)-(4e) which is applied to the system in an open-loop fashion during the inter-sampling periods, drive the closed-loop system towards a compact set X_f^i where it is ultimately bounded.

V. Simulation results

In this section, a simulated example of the proposed framework for a team of three nonholonomic agents moving in $\mathbb{R}^2$ is presented. The objective is to control each agent through a local NMPC law of the form (4a)-(4e) to reach the desired position, without colliding. The models of the subsystems are of the form (1b). The discretization time is $dt = 0.1$ and the cost functions are of quadratic form, i.e., $(x^i)^\top S^i x^i$, $(u^i)^\top R^i u^i$ and $(w^i - x^i + d^i)^\top Q^i (w^i - x^i + d^i)$, with $S^1 = S^2 = S^3 = diag[3, 5, 0.1]$, $R^1 = R^2 = R^3 = diag[1, 1]$ and $Q^1 = diag[8, 8, 0.1]$, $Q^2 = diag[6, 6, 0.1]$, $Q^3 = diag[5, 14, 0.1]$. The term $d^1 = d^2 = d^3 = [3, 3, 0]$ is the minimum desired distance between the agents. The initial and the desired position of agent $\mathcal{A}^1$ is $x^1_{initial} = [-20, 7, \pi/4]$, $x^1_{desired} = [6, -9, 0]$. For the agent $\mathcal{A}^2$ is $x^2_{initial} = [-10, -7, \pi/3]$, $x^2_{desired} = [14, 18, 0]$ and for the agent $\mathcal{A}^3$ is $x^3_{initial} = [10, -7, \pi - \pi/3]$, $x^3_{desired} = [-14, 18, \pi]$. Finally, the input is bounded by $\bar{u} = [10, 0.1]$ and the term σ is taken equal to 0.8.

In Fig.1, the trajectories of the agents are depicted. All three of them converge to a terminal constraint set that includes their desired states. It should be noted that the collision between the agents is avoided with the proposed framework. This is more apparent in Fig.2, where the χ^i and y^i positions are depicted. The agents are not coinciding at any sampling time. The coloring follows the same rule as in Fig.1, where the the red lines represent the agent $\mathcal{A}^1$, the blue lines represent the agent $\mathcal{A}^2$ and the magenta lines represent agent $\mathcal{A}^3$.

In the following the sampling times are depicted. Notice that when diagram has 1 value, there is a triggering instant, while when it has the value 0, the agents are controlled open-loop. Fig.3 depicts the triggering instants for agent $\mathcal{A}^1$ and Fig.4, Fig.5 depict the triggering instants for agents $\mathcal{A}^2$ and $\mathcal{A}^3$, respectively.

It is apparent from the figures, that the updates of the control laws, as well as, the communication load between the agents is significantly reduced, while the systems have

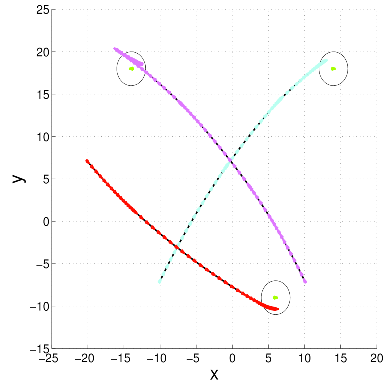

Fig. 1. Trajectories of the team of agents. The red triangles represent the agent $\mathcal{A}^1$. The blue triangles represent the agent $\mathcal{A}^2$ and the magenta triangles represent agent $\mathcal{A}^3$.

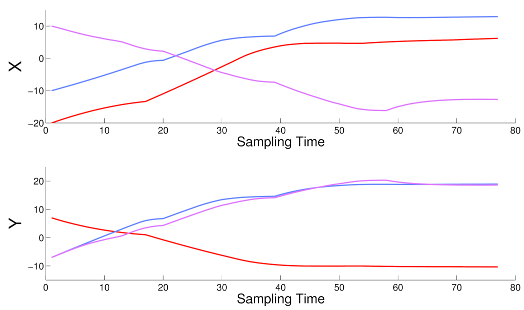

Fig. 2. The χ^i and y^i positions of the agents with respect to sampling times, for $i = 1, 2, 3$.

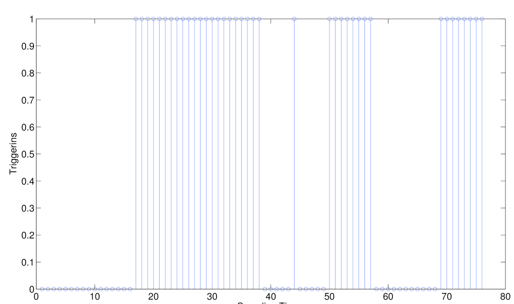

Fig. 3. Triggering instants for agent $\mathcal{A}^1$.

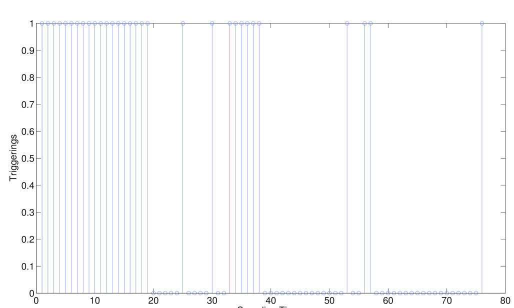

Fig. 4. Triggering instants for agent $\mathcal{A}^2$.

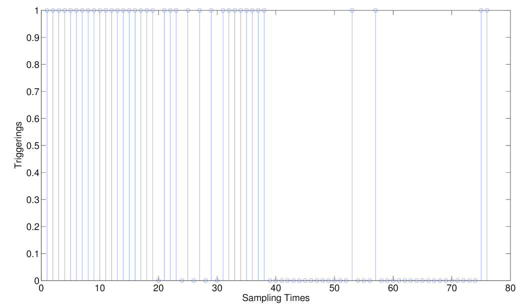

Fig. 5. Triggering instants for agent $\mathcal{A}^3$.

succeeded to converge to their desired states and to avoid collision.

VI. Summary and Future Work

In this paper, a cooperative framework for distributed non-holonomic agents under local model predictive controllers was considered. Also, for each subsystem a self-triggering condition was proposed. The main idea is to trigger the solution of the optimal control problem of the predictive controllers only when it is needed and not periodically as in the case of classic MPC schemes. During the inter-sampling times the control trajectory from the NMPC is applied to the system in an open-loop fashion. With the self-triggered approach both the control input and the next control update time are evaluated in order to avoid continuous supervision of the states of the neighboring agents. Thus, this approach results to a reduction of the updates of the control laws for each subsystem, as well to a reduction of the communication effort between the subsystems.

Future work involves an extension of the proposed distributed framework using less abstractions and having more realistic formulation. Namely, finding triggering conditions under the presence of disturbances and in the case where the information received by the neighbors is either delayed or not accurately known.

Appendix

In this section we are going to evaluate the inequality (10), which is crucial in order to reach to the triggering mechanism. First, the expression for the predicted states at a time-step $k+t$, with $t, m \in [1, N^i - 1]$, of the neighbors of the agent $\mathcal{A}^i$ measured from the the generic triggering instant k, i.e., $\hat{w}^i(k+t|k)$, is going to be given in Lemma 1 and then the predicted states $\hat{w}^i(k+t|k+m)$, measured from the time-step $k+m$, i.e., $\hat{w}^i(k+t|k+m)$ are going to be given in Lemma 2. Finally the expression for (10) will be provided.

Lemma 1: The predicted states $\hat{w}^i(k+t|k)$ for $t \in$

$[1, N^i - 1]$, are given as

$$\hat{w}^i(k+t|k) \triangleq col[x^j(k+t|k)] = \quad (19)$$

$$col[\hat{\chi}^j(k+t|k), \hat{y}^j(k+t|k), \hat{\theta}^j(k+t|k)]^\top =$$

$$col \begin{cases} \chi_k^j + dt\cos\theta_k^j \upsilon_k^j + dt\upsilon_k^j \sum_{l=1}^{t-1} \cos(\theta_k^j + ldt\omega_k^j) \\ y_k^j + dt\sin\theta_k^j \upsilon_k^j + dt\upsilon_k^j \sum_{l=1}^{t-1} \sin(\theta_k^j + ldt\omega_k^j) \\ \theta_k^j + tdt\omega_k^j \end{cases}$$

with $j \in G^i$.

Proof: At a triggering time-step k the vector $u_k = [\upsilon_k, \omega_k]^\top$ is measured and for the prediction horizon we assume that $[u_{k+1}, \ldots, u_{k+N-1}] = [u_k, \ldots, u_k]$. Having that, we get for $t = 2$,

$$\begin{cases} \hat{\chi}^j(k+2|k) \\ \hat{y}^j(k+2|k) \\ \hat{\theta}^j(k+2|k) \end{cases} = \begin{cases} \hat{\chi}^j(k+1|k) + dt\cos(\hat{\theta}^j(k+1|k))\upsilon_k^j \\ \hat{y}^j(k+1|k) + dt\sin(\hat{\theta}^j(k+1|k))\upsilon_k^j \\ \hat{\theta}^j(k+1|k) + dt\omega_k^j \end{cases}$$

Moving forward and by recursion we reach to the general rule (19). ■

Also we have that,

Lemma 2: The predicted states $\hat{w}^i(k+t|k+m)$ for $t \in [1, N^i - 1]$ and for $m = [1, N^i - 1]$, are given as

$$\hat{w}^i(k+t|k+m) \triangleq col[x^j(k+t|k+m)] = \quad (20)$$

$$col[\hat{\chi}^j(k+t|k+m), \hat{y}^j(k+t|k+m), \hat{\theta}^j(k+t|k+m)]^\top =$$

$$col \begin{cases} \chi_k^j + dt\cos\theta_k^j \upsilon_k^j \ldots \\ +dt\bar{\upsilon} \sum_{l=1}^{t-1} \cos(\theta_k^j + dt\omega_k^j + (l-1)dt\bar{\omega}) \\ y_k^j + dt\sin\theta_k^j \upsilon_k^j \ldots \\ +dt\bar{\upsilon} \sum_{l=1}^{t-1} \sin(\theta_k^j + dt\omega_k^j + (l-1)dt\bar{\omega}) \\ \theta_k^j + tdt\omega_k^j + dt(t-1)\bar{\omega} \end{cases}$$

with $j \in G^i$.

Proof:

Assume that $m = 1$, therefor, at time-step $k+1$ which follows the generic triggering time-step k, it is assumed that $[u_{k+1}, \ldots, u_{k+N-1}] = [\bar{u}, \ldots, \bar{u}]$ with $\bar{u} = [\bar{\upsilon}, \bar{\omega}]^\top$. The states of the neighbors of the agent $\mathcal{A}^i$, for a time-step $k+t$, with $t = 2$, are

From (1b) we get

$$[\chi^j(k+2|k+1), y^j(k+2|k+1), \theta^j(k+2|k+1)]^\top =$$

$$= \begin{cases} \chi^j(k+1|k) + dt\cos(\theta^j(k+1|k))\bar{\upsilon} \\ y^j(k+1|k) + dt\sin(\theta^j(k+1|k))\bar{\upsilon} \\ \theta^j(k+1|k) + dt\bar{\omega} \end{cases}$$

$$= \begin{cases} \chi_k^i + dt\cos\theta_k^i \upsilon_k^i + dt\cos(\theta_k^i + dt\omega_k^i)\bar{\upsilon} \\ y_k^i + dt\sin\theta_k^i \upsilon_k^i + dt\sin(\theta_k^i + dt\omega_k^i)\bar{\upsilon} \\ \theta_k^i + dt\omega_k^i + dt\bar{\omega} \end{cases}$$

which yields, by recursion for a $t \in [1, N^i - 1]$, the general form (20). The same applies for all $m \in [1, N^i - 1]$ as we consider nominal stability. ■

It should be noted that we used the abstraction $\sum_1^0 \equiv 0$.

From Lemma 1 and Lemma 2, while making some easy manipulations that is omitted due to space limitations, it can be concluded that for an agent $\mathcal{A}^i$, the predicted states of its neighbors at a time step $k+t$ are bounded by

$$||\hat{w}^i(k+t|k+m) - w^i(k+t|k)|| \le \sum_{j \in G^i} \{dt(m-1)(2(\bar{\upsilon} + |\upsilon_k^j|)^2 + \bar{\omega}^2)^{1/2}\} \quad (21)$$

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