On the State Agreement Problem for Multiple Unicycles with Varying Communication Links

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Abstract— **This paper presents a decentralized feedback control strategy that drives a system of multiple nonholonomic kinematic unicycles to agreement. The communication links between the members of the team may change over time and hence the communication topology is dynamic. The case of static communication topology was dealt in [7]. The proposed nonholonomic control law is discontinuous and time-invariant and tools from nonsmooth stability theory and matrix theory are used to check the stability of the overall system. Similarly to the linear case, the convergence of the multi-agent system relies on the connectivity of the communication graph that represents the inter-agent communication topology. The convergence properties are verified through computer simulations.**

I. Introduction

Navigation of multi-agent systems is a topic that has recently attracted researchers from both the robotics and the control communities, due to the need for autonomous control of more than one mobile robotic agents in the same workspace. While most approaches in the past focused at centralized planning ([17]), specific real-world applications have lead researchers throughout the globe to turn their attention to decentralized concepts. One such important application is the field of micro robotics ([12]), where a team of a potentially large number of autonomous micro robots must cooperate in the sub micron level.

Among the various objectives that the control design aims to impose on the multi-agent system, convergence of the multi-agent team to a desired formation is a design specification that has been extensively pursued during the last few years. The main feature of formation control is the cooperative nature of the equilibria of the system. Agents must converge to a desired configuration encoded by the inter-agent relative positions. Many feedback control schemes that achieve formation stabilization to a desire formation in a distributed manner have been proposed in literature, see for example [29],[18],[16],[10] for some recent efforts. The so-called agreement or rendezvous problem, in which agents must converge to the same point in the state space ([1],[25],[6],[14], [22],[13],[24]), is also an issue of particular interest.

There have been many approaches to the state agreement problem addressing the control design issue for several vehicle models. In most cases, single integrator kinematic models of motion are taken into account, while the information exchange topology has been considered both static and dynamic, as well as bidirectional or unidirectional. A recent review of the various approaches of the state agreement problem for linear models of motion is [27]. The agreement problem for general nonlinear models has been considered in [19].

In this contribution, a feedback control strategy that drives a system of multiple nonholonomic unicycles to agreement is introduced. Inspired by our previous work on decentralized navigation of multiple nonholonomic agents [20],[8], we propose in this paper a distributed nonholonomic feedback control strategy that is discontinuous and time invariant, something expected, as nonholonomic systems do not satisfy Brockets necessary smooth feedback stabilization condition ([3]). This type of controllers have in general better convergence properties than time-varying ones. An experimental comparison between these two types of nonholonomic controllers that supports our preference to time-invariant strategies has appeared in [15]. In that reference, it was deduced that time varying controllers were too slow and oscillatory for most practical situations. On the other hand, time-invariant controllers achieved a significantly better behavior.

The current paper is a follow up to our previous work in [7], where the agreement problem for multiple unicycles with *static* communication topology was taken into account. The main contribution of the current paper is that it takes into account the case of *switching* communication topology, which is a novelty with respect to previous work on the state agreement problem for nonholonomic unicycles. Our treatment is similar to that of [18], where merely static interconnection topology is considered. Furthermore, the authors of that work use a similar control strategy to that of [30], which is time varying periodic and smooth. Hence it provides in general worse convergence results with respect to the time invariant case encountered in our approach.

The rest of the paper is organized as follows: section II describes the system and the problem that is treated in this paper. Assumptions regarding the communication topology between the agents are presented and modelled in terms of an undirected graph. Section III begins with some background on matrix and graph theory and nonsmooth analysis that is used in the sequel and proceeds with the introduction of the distributed nonsmooth time invariant feedback control strategy that drives the multi-agent team to a common configuration in the state space as well as the corresponding stability analysis. Computer simulation results are included in section IV while section V summarizes the results of this paper and indicates current research efforts.

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II. System and Problem Definition

We consider a system of N nonholonomic kinematic point agents operating in the same workspace $W \subset \mathbb{R}^2$. Let $q_i = [x_i, y_i]^T \in \mathbb{R}^2$ denote the position of agent i. The configuration space is spanned by $q = [q_1, \ldots, q_N]^T$. Each of the N mobile agents has a specific orientation θ_i with respect to the global coordinate frame. The orientation vector of the agents is represented by $\theta = [\theta_1 \ldots \theta_N]$. The configuration of each agent is represented by $p_i = \left[\begin{array}{cc} q_i & \theta_i \end{array}\right] \in \mathbb{R}^2 \times (-\pi, \pi]$. Agent motion is described by the following nonholonomic kinematics:

$$\begin{array}{l} \dot{x}_i = u_i \cos\theta_i \\ \dot{y}_i = u_i \sin\theta_i \\ \dot{\theta}_i = \omega_i \end{array}, i \in \mathcal{N} = [1, \ldots, N] \tag{1}$$

where u_i, ω_i denote the translational and rotational velocity of agent i, respectively. These are considered as the control inputs of the multi-agent system.

The design objective is to construct feedback controllers that lead the multi-agent system to agreement, i.e. all agents should converge to a common point in the state space. Each agent is assigned with a specific subset N_i of the rest of the team, called agent i's *communication set*, that includes the agents with which it can communicate in order to achieve the desired agreement objective. Following the literature on cooperative control [23],[29], inter-agent communication can be encoded in terms of a *communication graph*:

Definition 1: The *communication graph* $\mathcal{G} = \{V, E\}$ is an undirected graph that consists of a set of vertices $V = \{1, ..., N\}$ indexed by the team members and (ii) a set of edges, $E = \{(i,j) \in V \times V | i \in N_j\}$ containing pairs of nodes that represent inter-agent communication specifications.

Figure 1 presents a three agent scenario and the corresponding communication graph. The neighboring sets are given by $N_1 = \{2,3\}, N_2 = N_3 = \{1\}$.

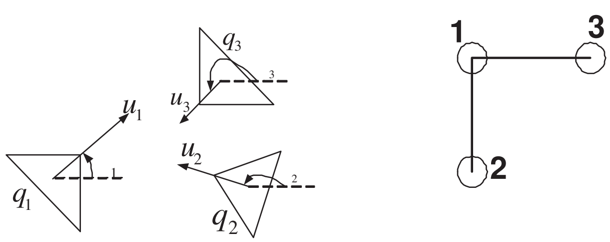

Fig. 1. A three agent scenario with communication sets $N_1 = \{2,3\}, N_2 = N_3 = \{1\}$.

In [7], we considered the case of static interconnection topology between the members of the nonholonomic multi-agent team. In this paper, we extend the results presented previously to the case of switching topology. In this case, the communication set of each agent is time-varying. In this paper, we assume that inter-agent communication is created/lost each time an agent enters/leaves a cyclic area of specific radius $d > 0$ around another agent. This cyclic area is called the *sensing zone* of the agent and the parameter d, which is assumed common for all agents, the *sensing radius*. Hence for agent i, the set N_i is defined as

$$N_i = \{j : \|q_i - q_j\| \leq d\} \tag{2}$$

Hence an edge is created in the communication graph each time an agent enters the sensing zone of another agent.

Each agent has only knowledge of the state of agents that belong to its communication set at each time instant. This fact highlights the distributed nature of the approach. We also assume that the communication graph is undirected, in the sense that $i \in N_j \Leftrightarrow j \in N_i, \forall i,j \in \mathcal{N}, i \neq j$. It is obvious that $(i,j) \in E$ iff $i \in N_j \Leftrightarrow j \in N_i$.

The control design is of the form

$$\begin{array}{l} u_i = u_i(p_i, p_j) \\ \omega_i = \omega_i(p_i, p_j) \end{array}, j \in N_i, i \in \mathcal{N} \tag{3}$$

copying in this way with the limited communication capabilities of each agent. The problem treated in this paper can now be stated as follows: "*under the preceding assumptions, derive a set of distributed control laws of the form (3) that drives the team of agents from any initial configuration to a common configuration in the state space*".

III. Control Design and Stability Analysis

A. Tools from Matrix Theory

In this subsection we review some tools from graph theory and matrix analysis that we shall use in the stability analysis of the proposed control framework. The following analysis on graph theory can be found in [2], while the elements from matrix analysis in [11],[21].

For an undirected graph $\mathcal{G} = (V, E)$ with n vertices we denote by V its set of vertices and by E its set of edges. If there is an edge connecting two vertices i, j, i.e. $(i,j) \in E$, then i, j are called *adjacent*. A *path* of length r from a vertex i to a vertex j is a sequence of $r+1$ distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. If there is a path between any two vertices of the graph $\mathcal{G}$, then $\mathcal{G}$ is called *connected* (otherwise it is called *disconnected*).

The *undirected graph $\mathcal{G} = (V, E)$ corresponding to a real symmetric $n \times n$ matrix M* is a graph with n vertices indexed by $1, \ldots, n$ such that there is an edge between vertices $i, j \in V$ if and only if $M_{ij} \neq 0$, i.e. $(i,j) \in E \Leftrightarrow M_{ij} \neq 0$.

A $n \times n$ real symmetric matrix with non-positive off-diagonal elements and zero row sums is called a *symmetric Metzler* matrix. It is shown in [21] that all the eigenvalues of a symmetric Metzler matrix are non-negative and zero is a trivial eigenvalue. The multiplicity of zero as an eigenvalue of a symmetric Metzler matrix is one (i.e. it is a simple eigenvalue) if and only if the corresponding undirected graph is connected. The trivial corresponding eigenvector is the vector of ones, $\overrightarrow{\mathbf{1}}$. This result has been used in the proof of the consensus algorithm for single integrator kinematic agents presented in [22]. Its usefulness in the present framework is verified in the sequel.

B. Tools from Nonsmooth Analysis

In this subsection, we review some elements from nonsmooth analysis and Lyapunov theory for nonsmooth systems that we use in the stability analysis of the next section.

For a differential equation with discontinuous right-hand side we have the following definition:

Definition 2: [9] In the case when the state-space is finite dimensional, the vector function $x(.)$ is called a *Filippov solution* of $\dot{x} = f(x)$ if it is absolutely continuous and $\dot{x} \in K[f](x)$ almost everywhere where

$$K[f](x) \equiv \overline{co}\{\lim_{x_i \to x} f(x_i) | x_i \notin N\}$$

where N is a set of measure zero.

Lyapunov stability theorems have been extended for nonsmooth systems in [28],[4]. The following chain rule provides a calculus for the time derivative of the energy function in the nonsmooth case:

Theorem 1: [28] Let x be a Filippov solution to $\dot{x} = f(x)$ on an interval containing t and $V : \mathbb{R}^n \to \mathbb{R}$ be a Lipschitz and regular function. Then $V(x(t))$ is absolutely continuous, $(d/dt)V(x(t))$ exists almost everywhere and

$$\frac{d}{dt}V(x(t)) \in^{a.e.} \dot{\tilde{V}}(x) := \bigcap_{\xi \in \partial V(x(t))} \xi^T K[f](x(t))$$

where "a.e." stands for "almost everywhere".

In this theorem, ∂V is *Clarke's generalized gradient*. The definition of the generalized gradient and of the *regularity* of a function can be found in [5]. In the case we encounter in this paper, the candidate Lyapunov function V we use is smooth and hence regular, while its generalized gradient is a singleton which is equal to its usual gradient everywhere in the state space: $\partial V(x) = \{\nabla V(x)\} \forall x \in \mathbb{R}^n$.

We shall use the following nonsmooth version of LaSalle's invariance principle to prove the convergence of the prescribed system:

Theorem 2: [28] Let Ω be a compact set such that every Filippov solution to the autonomous system $\dot{x} = f(x), x(0) = x(t_0)$ starting in Ω is unique and remains in Ω for all $t \geq t_0$. Let $V : \Omega \to \mathbb{R}$ be a time independent regular function such that $v \leq 0 \forall v \in \dot{\tilde{V}}$(if $\dot{\tilde{V}}$ is the empty set then this is trivially satisfied). Define $S = \{x \in \Omega | 0 \in \dot{\tilde{V}}\}$. Then every trajectory in Ω converges to the largest invariant set, M, in the closure of S.

C. Control Law and Stability Analysis

In order to cope with the limited sensing capabilities of each agent, we define a "cost" function γ_i for each agent i as follows:

$$\gamma_i = \sum_{j \in N_i} \gamma_{ij} \left(\underbrace{\|q_i - q_j\|^2}_{\beta_{ij}} \right) = \sum_{j \in N_i} \gamma_{ij}\left(\beta_{ij}\right)$$

where

$$\gamma_{ij} = \begin{cases} \frac{1}{2}\beta_{ij}, \ 0 \leq \beta_{ij} \leq c^2 \\ \phi(\beta_{ij}), \ c^2 \leq \beta_{ij} \leq d^2 \\ h, \ d^2 \leq \beta_{ij} \end{cases}$$

The positive constant scalar parameters c, d, h and the function ϕ are chosen in such a way so that γ_{ij} is everywhere continuously differentiable. In this paper, we choose the following polynomial function:

$$\phi(x) = a_2 x^2 + a_1 x + a_0$$

The parameters of this function satisfy the differentiability requirement for γ_{ij}, provided that they fulfil the following relations:

$$a_2 = \frac{1}{4\left(c^2 - d^2\right)}, a_1 = \frac{d^2}{2\left(d^2 - c^2\right)},$$

$$a_0 = \frac{c^4}{4\left(c^2 - d^2\right)}, h = \frac{d^2 + c^2}{4}$$

Figure 2 shows a plot of the function γ_{ij} with respect to β_{ij} for $c^2 = 0.56$ and $d^2 = 0.96$.

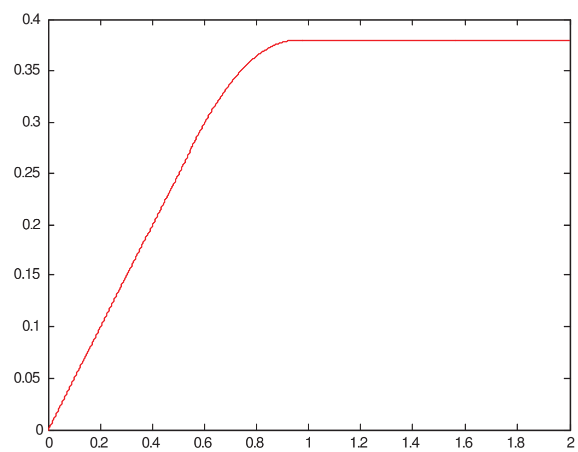

Fig. 2. The function γ_{ij} for $c^2 = 0.56$ and $d^2 = 0.96$.

The gradient and the partial derivative of γ_{ij} are computed by $\nabla \gamma_{ij} = 2\rho_{ij} D_{ij} q$ and $\frac{\partial \gamma_{ij}}{\partial q_i} = 2\rho_{ij}\left(D_{ij}\right)_i q$ where

$$\rho_{ij} \triangleq \frac{\partial \gamma_{ij}}{\partial \beta_{ij}}$$

and the matrices D_{ij},$(D_{ij})_i$, for $i < j$, can be shown to be given by

$$D_{ij} = \left[\begin{array}{ccccc} \multicolumn{5}{c}{O_{(i-1)\times N}} \\ O_{1\times(i-1)} & 1 & O_{1\times(j-i-1)} & -1 & O_{1\times(N-j)} \\ \multicolumn{5}{c}{O_{(j-i-1)\times N}} \\ O_{1\times(i-1)} & -1 & O_{1\times(j-i-1)} & 1 & O_{1\times(N-j)} \\ \multicolumn{5}{c}{O_{(N-j)\times N}} \end{array} \right] \otimes I_2$$

and

$$\left(D_{ij}\right)_i = \left[\begin{array}{ccccc} O_{1\times(i-1)} & 1 & O_{1\times(j-i-1)} & -1 & O_{1\times(N-j)} \end{array} \right] \otimes I_2$$

The definition of the matrices D_{ij},$(D_{ij})_i$, for $i > j$ is straightforward.

This choice of ϕ guarantees that $\rho_{ij} > 0$ for $0 < \beta_{ij} < d^2$. Define now

$$\gamma = \sum_i \sum_{j \neq i} \gamma_{ij}$$

Taking the gradient of γ we get

$$\nabla\gamma = \sum_i \sum_{j \neq i} \nabla\gamma_{ij} = 2\left(\sum_i \sum_{j \neq i} \rho_{ij} D_{ij}\right) q = 2\left(R_1 \otimes I_2\right) q$$

where the matrix R_1 is computed by

$$\left(R_1\right)_{ij} = \begin{cases} \sum\limits_{j \neq i} \rho_{ij} + \sum\limits_{j \neq i} \rho_{ji}, i = j \\ -\rho_{ij} - \rho_{ji}, i \neq j \end{cases}$$

Using the fact that $\rho_{ij} = 0$ for $\beta_{ij} > d^2$, the following equation is straightforward:

$$\frac{\partial \gamma_i}{\partial q_i} = \sum_{j \in N_i} \frac{\partial \gamma_{ij}}{\partial q_i} = \sum_{j \neq i} \frac{\partial \gamma_{ij}}{\partial q_i}$$

so that

$$\begin{bmatrix} \frac{\partial \gamma_1}{\partial q_1} \\ \vdots \\ \frac{\partial \gamma_N}{\partial q_N} \end{bmatrix} = \begin{bmatrix} \sum\limits_{j \in N_1} \frac{\partial \gamma_{1j}}{\partial q_1} \\ \vdots \\ \sum\limits_{j \in N_N} \frac{\partial \gamma_{Nj}}{\partial q_N} \end{bmatrix} = \begin{bmatrix} \sum\limits_{j \neq 1} \frac{\partial \gamma_{1j}}{\partial q_1} \\ \vdots \\ \sum\limits_{j \neq N} \frac{\partial \gamma_{Nj}}{\partial q_N} \end{bmatrix} =$$
$$= -2\left(R_2 \otimes I_2\right) q$$

where

$$\left(R_2\right)_{ij} = \begin{cases} \sum\limits_{j \neq i} \rho_{ij}, i = j \\ -\rho_{ij}, i \neq j \end{cases}$$

Using now the symmetry of the potential functions we get

$$\rho_{ij} = \rho_{ji} \Rightarrow R_1 = 2R_2$$

In the analysis that follows, we use the decoupling of the stack vector $q = [x, y]^T$ into the coefficients that correspond to the x, y directions of the agents respectively. Furthermore, the notation $(a)_i$ for a vector a, denotes its i-th element. We are now ready to state the main result of this paper:

Theorem 3: Assume that the communication graph remains connected for every topology induced by the switching communication law (2). Then the discontinuous time-invariant feedback control strategy:

$$u_i = -\mathrm{sgn}\left\{\gamma_{xi} \cos\theta_i + \gamma_{yi} \sin\theta_i\right\} \cdot \left(\gamma_{xi}^2 + \gamma_{yi}^2\right)^{1/2} \tag{4}$$

$$\omega_i = -\left(\theta_i - \theta_{nh_i}\right) \tag{5}$$

where

$$\gamma_{xi} = \frac{\partial \gamma_i}{\partial x_i} = \left(2R_2 x\right)_i, \gamma_{yi} = \frac{\partial \gamma_i}{\partial y_i} = \left(2R_2 y\right)_i$$

and the "nonholonomic angle"

$$\theta_{nh_i} = \arctan 2\left(\gamma_{yi}, \gamma_{xi}\right)$$

drives the agents to a common configuration in the state space.

Proof: We use the continuously differentiable positive definite function $W = \gamma$ as a candidate Lyapunov function.

Since the proposed control law is discontinuous we use the concept of Theorem 1 for the time derivative of the candidate Lyapunov function. Since W is smooth we have

$$\partial W = \{\nabla W\} = \{\nabla\gamma\}$$

so that

$$W = \gamma \Rightarrow \dot{\widetilde{W}} = \left\{\sum_i \left(\nabla\gamma_i\right)^T\right\} \cdot K \begin{bmatrix} u_1 \cos\theta_1 \\ u_1 \sin\theta_1 \\ \vdots \\ u_N \cos\theta_N \\ u_N \sin\theta_N \end{bmatrix} \subset$$
$$2q^T\left(R_1 \otimes I_2\right) \begin{bmatrix} K\left[u_1\right] \cos\theta_1 \\ K\left[u_1\right] \sin\theta_1 \\ \vdots \\ K\left[u_N\right] \cos\theta_N \\ K\left[u_N\right] \sin\theta_N \end{bmatrix} \subset$$
$$\subset 2\left(R_1 x\right)^T \begin{bmatrix} K\left[u_1\right] \cos\theta_1 \\ \vdots \\ K\left[u_N\right] \cos\theta_N \end{bmatrix} +$$
$$+2\left(R_1 y\right)^T \begin{bmatrix} K\left[u_1\right] \sin\theta_1 \\ \vdots \\ K\left[u_N\right] \sin\theta_N \end{bmatrix} \subset$$
$$\subset \sum_i \left\{2K\left[u_i\right]\left(\left(R_1 x\right)_i \cos\theta_i + \left(R_1 y\right)_i \sin\theta_i\right)\right\}$$

where we used Theorem 1.3 in [26] to calculate the inclusions of the Filippov set in the previous analysis. Since $K\left[\mathrm{sgn}(x)\right] x = \{|x|\}$([26],Theorem 1.7), the choice of control laws (4),(5) results in

$$\dot{\widetilde{W}} = 4\sum_i \left\{-\left|\gamma_{xi} \cos\theta_i + \gamma_{yi} \sin\theta_i\right| \left(\gamma_{xi}^2 + \gamma_{yi}^2\right)^{1/2}\right\} \leq 0$$

The assumption that the communication graph remains connected guarantees that the set

$$\Omega = \left\{q : \left\|q_i - q_j\right\| \leq (N-1)\, d, \forall i, j \in \mathcal{N}\right\}$$

is compact and invariant for the closed loop system with respect to the relative positions of all agents belonging to $\mathcal{N}$.

By the nonsmooth version of LaSalle's invariance principle (Theorem 2), the trajectories of the system converge to the largest invariant set contained in the set

$$S = \left\{ \begin{array}{r} \left(\gamma_{xi} = \gamma_{yi} = 0\right) \vee \left(\gamma_{xi} \cos\theta_i + \gamma_{yi} \sin\theta_i = 0\right), \\ \forall i \in N \end{array} \right\}$$

However, for each $i \in \mathcal{N}$, we have $\left|\omega_i\right| = \frac{\pi}{2}$ whenever $\gamma_{xi} \cos\theta_i + \gamma_{yi} \sin\theta_i = 0$, due to the proposed angular velocity control law. In particular, this choice of angular velocity renders the surface $\gamma_{xi} \cos\theta_i + \gamma_{yi} \sin\theta_i = 0$ repulsive for agent i, whenever i is not located at the desired equilibrium, namely when $\gamma_{xi} = \gamma_{yi} = 0$. Hence the largest invariant set E contained in S is

$$S \supset E = \left\{\gamma_{xi} = \gamma_{yi} = 0, \forall i \in \mathcal{N}\right\}$$

In addition $(\gamma_{xi} = \gamma_{yi} = 0)\,\forall i$ guarantees that the agents converge to a common configuration. This is easily derived by the fact that

$$(\gamma_{xi} = \gamma_{yi} = 0)\,\forall i \Rightarrow (R_2 \otimes I_2)\,q = 0 \Rightarrow R_2 x = R_2 y = 0$$

where x, y the stack vectors of q in the x, y directions. The symmetric matrix R_2 has zero row sums and non-positive off-diagonal elements. Using the same arguments and terminology as in [22], the matrix R_2 is a Metzler matrix. As mentioned in Section IIIA, the eigenvalues of R_2 are nonnegative and zero is the smallest eigenvalue. Following [22], we deduce that since the communication graph remains connected, zero is a simple eigenvalue of R_2 with trivial corresponding eigenvector the vector of ones, $\overrightarrow{\mathbf{1}}$. Hence equations $R_2 x = R_2 y = 0$ guarantee that both x, y are eigenvectors of R_2 belonging to span$\{\overrightarrow{\mathbf{1}}\}$. Hence all q_i tend to the same value, implying that all agents converge to a common configuration at steady state. $\diamondsuit$

Remark: It must be stressed out that the proposed feedback control strategy (4),(5) is purely *decentralized*, since each agent requires information only of the states of agents within its neighboring set at each time instant. This can also easily be realized by expressing the terms γ_{xi}, γ_{yi} in the proposed control law as

$$\gamma_{xi} = \frac{\partial \gamma_i}{\partial x_i} = 2 \sum_{j \in N_i} \rho_{ij}\,(x_i - x_j)$$

$$\gamma_{yi} = \frac{\partial \gamma_i}{\partial y_i} = 2 \sum_{j \in N_i} \rho_{ij}\,(y_i - y_j)$$

Hence each agent i must be aware of the relative positions only of those agents belonging to N_i at each time instant.

IV. Simulations

To support the results of the previous paragraphs we provide some computer simulations of the proposed control framework (4),(5).

In the next simulations, we have six nonholonomic agents that start from arbitrary initial conditions and navigate under the switching communication control scheme (equations (4),(5)). In the framework of this paper, an edge is created in the communication graph once an agent enters/leaves the communication set of another. The parameters c, d play a crucial role in the convergence properties of the system, since they affect the connectivity properties of the communication graph. This is demonstrated in the following two simulations. Denote by q_{min} the minimum distance between two agents in the initial conditions of the multiagent team.

In the first simulation we choose $d = 2q_{min}, c = 0.7d$. This choice of d successfully drives the system to the desired configuration, as witnessed in Figure 3. In this figure, screenshots I-V show the evolution in time of the of the six unicycles under the proposed control strategy (4),(5). In the first screenshot, agents are located at their initial condition. Due to the choice of d, the communication graph remains connected and the agents converge to a common configuration. This is depicted in the last screenshot V.

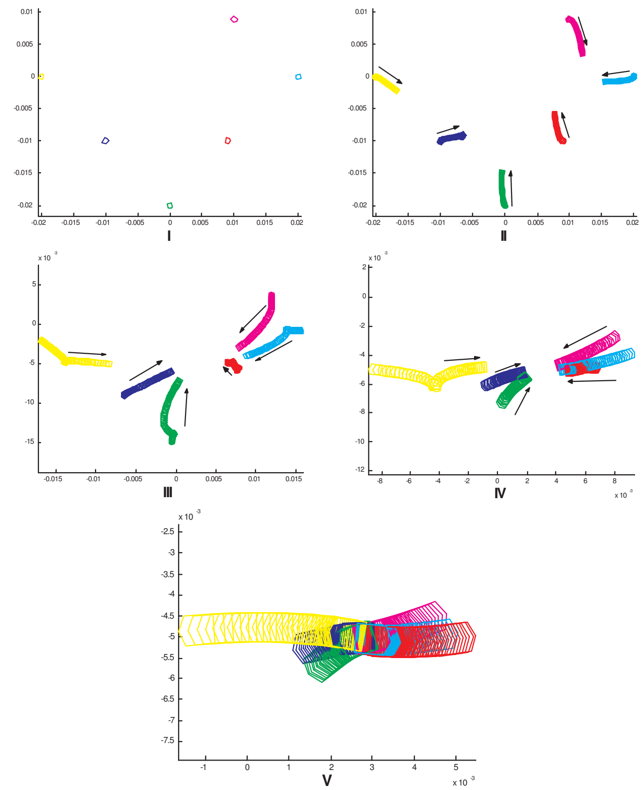

Fig. 3. Convergence to a common configuration for six unicycles under switching communication topology

In the next simulation we choose $d = 1.5q_{min}, c = 0.6d$. The initial conditions remain the same as in the first simulation. As can be witnessed in Figure 4, this choice does not guarantee that the communication graph remains connected. Hence the agents eventually split into two separate groups that correspond to the connected components of the communication graph. In the last screenshot of Figure 4, the two components converge to a different agreement point.

V. Conclusions

In this paper, we presented a decentralized feedback control strategy that drives a system of multiple nonholonomic kinematic unicycles to agreement. The communication links between the members of the team may change over time and hence the communication topology is dynamic. The case of static communication topology was dealt in [7]. The proposed nonholonomic control law is discontinuous and time-invariant and tools from nonsmooth stability theory and matrix theory were used to check the stability of the overall system. Time invariant controllers have been shown to produce better convergence properties for nonholonomic systems than time varying ones ([15]). Similarly to the linear case, the convergence of the multi-agent system was shown to rely on the connectivity of the communication graph that represents the inter-agent communication topology. The convergence properties were verified through computer simulations.

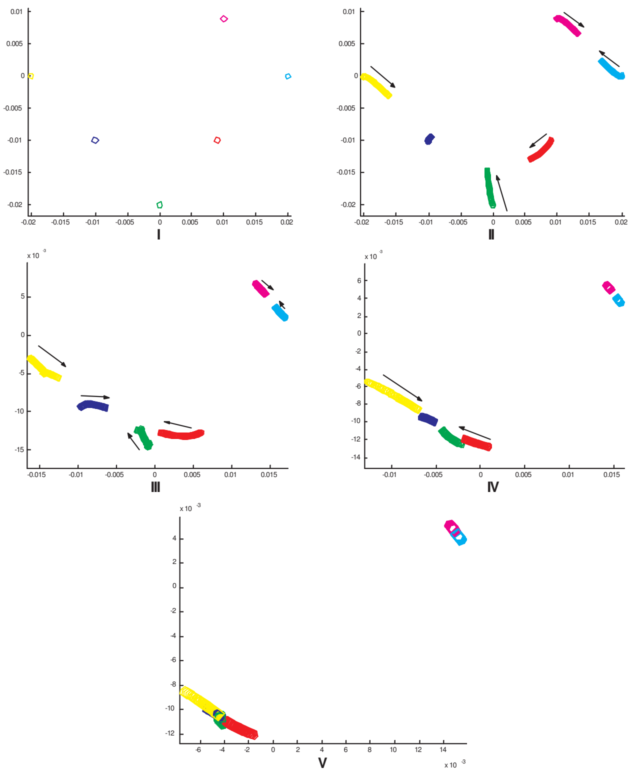

Fig. 4. Loss of connectivity decouples the system into the connected components of the communication graph

Current research involves extending the proposed framework to directed graphs. More general motion models such as three-dimensional kinematics are also currently pursued. As a parallel result of this work, formation convergence to arbitrary feasible formation configurations for multiple unicycles is also under investigation.

VI. Acknowledgements

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