Decentralized navigation functions for multiple agents with limited sensing capabilities

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Abstract

The decentralized navigation function methodology, established in previous work for navigation of multiple holonomic agents with global sensing capabilities is extended to the case of local sensing capabilities. Each agent plans its actions without knowing the destinations of the others and the positions of agents outside its sensing neighborhood. The overall system is modelled as a deterministic switched system and we use tools from nonsmooth analysis to check its stability properties. The collision avoidance and global convergence properties are verified through simulations.

1 Introduction

Navigation of mobile agents has been an area of significant interest in robotics and control communities. Most efforts have focused on the case of a single agent navigating in an environment with obstacles [8]. Recently, decentralized navigation for multiple agents has gained increasing attention. The basic motivation comes from two application domains: (i) decentralized conflict resolution in air traffic management(ATM) and (ii) the field of micro robotics, where a team of autonomous micro robots must cooperate to achieve manipulation precision in the sub micron level.

In both cases, the decentralized navigation procedure involves reassignment of the control tasks from the central authority, i.e. the Air Traffic Controllers, to the agents, i.e. the cockpit or the robots. The level of decentralization depends on the knowledge an agent has on the other agents' actions and objectives. In [14],[5] the decentralization factor lied in the fact that each agent had knowledge only of its own desired destination, but not of the desired destinations of the others. Clearly, this is a suitable model for a futuristic distributed ATM system, where each aircraft will have knowledge of the actions and positions of the other aircraft at each time instant, but not of their destinations.

Nevertheless, in practice, the sensing capabilities of each agent are limited. Consequently, each agent can not have knowledge of the positions and/or velocities of every agent in the workspace but only of the agents within its sensing zone at each time instant. The interpretation of the sensing zone of an agent that we use in this paper is a circle of constant radius around its center of mass. Taking those aspects into consideration, the multi agent navigation problem treated in this paper can be stated as follows: *derive a set of control laws (one for each agent) that drives a team of agents from any initial configuration to a desired goal configuration avoiding, at the same time, collisions. Each agent has no knowledge of the others desired destinations and has only local knowledge of their positions at each instant.* The same problem has been dealt in [1],[7] under a game theoretic perspective. In [13] a nonsmooth controller was designed to achieve flocking behavior in an environment with multiple agents with limited sensing capabilities. In this paper we use the navigation function method established in [8], [9],[14],[5] and solve the problem in a closed loop fashion.

The rest of the paper is organized as follows: in section 2 the system definition, the corresponding assumptions and the problem statement are presented. In section 3 we redefine the decentralized navigation functions introduced in [14],[5] to cope with the limited sensing capabilities of the agents. The stability analysis of the system is contained in sections 4,5. Simulation results are presented in section 6 while section 7 summarizes the conclusions and indicates our current research.

2 System and Problem Definition

Consider a system of N agents operating in the same workspace $W \subset \mathcal{R}^2$. Each agent i occupies a disc: $R = \{q \in \mathcal{R}^2 : \| q - q_i \| \leq r_i\}$ in the workspace where $q_i \in \mathcal{R}^2$ is the center of the disc and r_i is the radius of the agent. The configuration space is spanned by $q = [q_1, \ldots, q_N]^T$. The motion of each agent are described by the single integrator:

$$\dot{q}_i = u_i, i \in \mathcal{N} = [1, \ldots, N] \tag{1}$$

The desired destinations of the agents are denoted by the index d: $q_d = [q_{d1}, \ldots, q_{dN}]^T$. We make the following assumptions:

- Each agent has only knowledge of the position agents located in a cyclic neighborhood of specific radius d_C at each time instant, where $d_C > max_{i,j \in \mathcal{N}}(r_i + r_j)$.
- Each agent has knowledge only of its own desired destination but not of the others.
- Each agent knows the exact number of agents in the workspace.

- We consider spherical agents.
- The workspace is bounded and spherical.

Figure 1 shows a three-agent conflict situation. The multi agent navigation

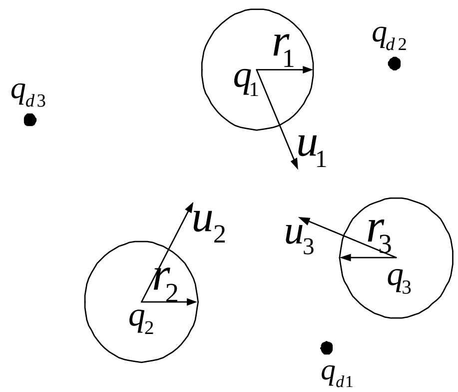

Figure 1: A conflict scenario with three agents.

problem treated in this paper can be stated as follows: "*derive a set of control laws (one for each agent) that drives a team of agents from any initial configuration to a desired goal configuration avoiding, at the same time, collisions. Each agent has no knowledge of the others desired destinations and has only local knowledge of their positions at each instant*".

3 Decentralized Navigation Functions for Agents with Limited Sensing Capabilities

In this section, we review the decentralized navigation function method used in [5],[14] for the case of multiple holonomic agents and modify it in order to cope with the limited sensing capabilities specification. Consider a system of n agents operating in the same workspace $W \subset R^2$. Each agent i occupies a disk: $R = \{q \in R^2 : \|q - q_i\| \leq r_i\}$ in the workspace where $q_i \in R^2$ is the center of the disk and r_i is the radius of the agent. The dynamics of each agent are given by $\dot{q}_i = u_i$ and the configuration space is spanned by $q = [q_1, \ldots, q_n]^T$. The proposed control law for each agent is given by

$$u_i = -K_i \cdot \frac{\partial \varphi_i}{\partial q_i} \tag{2}$$

where K_i is a positive gain and the *decentralized navigation function* φ_i is defined as

$$\varphi_i = \frac{\gamma_{di} + f_i}{((\gamma_{di} + f_i)^k + G_i)^{1/k}} \tag{3}$$

The term $\gamma_{di} = \|q_i - q_{di}\|^2$ in the potential function is the squared metric of the agent's i configuration from its desired destination q_{di}. The exponent k is a scalar positive parameter. The function G_i expresses all possible collisions of agent i with the others, while f_i guarantees that φ_i attains positive values whenever collisions with respect to i tend to occur even when i has already reached its destination.

3.1 Construction of the G_i function

We review now the construction of the "collision" function G_i for each agent i introduced in [14], [5]. In these papers, the decentralization feature of the whole scheme lied in the fact that each agent didn't have knowledge of the desired destinations of the rest of the team. On the other hand, each one had global knowledge of the positions of the others at each time instant. This is far from realistic in real world applications. The "Proximity Function" between agents i and j in [14], [5] was given by

$$\beta_{ij} = \|q_i - q_j\|^2 - (r_i + r_j)^2$$

In this work we take the limited sensing capabilities of each agent into account. Each agent has only local knowledge of the positions of the others at each time instant. Specifically, it only knows the position of agents which are in a cyclic neighborhood of specific radius d_C around its center. Therefore the Proximity Function between two agents has to be redefined in this case. We propose the following nonsmooth function:

$$\beta_{ij} = \begin{cases} \|q_i - q_j\|^2 - (r_i + r_j)^2, \text{for } \|q_i - q_j\| \leq d_C \\ d_C^2 - (r_i + r_j)^2, \text{for } \|q_i - q_j\| > d_C \end{cases} \tag{4}$$

Figure 2 represents a nonsmooth proximity function. Consider now the situation

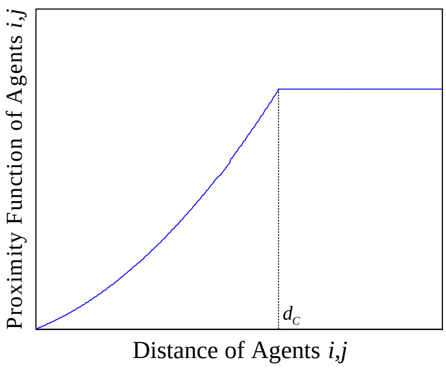

Figure 2: The function β_{ij} for $r_i + r_j = 1, d_C = 4$.

in figure 3. There are 5 agents and we proceed to define the function G_R for agent R.

Definition 3.1 *A* relation *with respect to agent R is every possible collision scheme that can occur in a multiple agents scene with respect R.*

Definition 3.2 *A* binary relation *with respect to agent R is a relation between agent R and another.*

Definition 3.3 *The* relation level *in the number of binary relations in a relation.*

We denote by $(R_j)_l$ the jth relation of level-l with respect to agent R. With this terminology in hand, the collision scheme of figure (3a) is a level-1 relation (one binary relation) and that of figure (3b) is a level-3 relation (three binary relations), always with respect to the specific agent R. We use the notation

$$(R_j)_l = \{\{R, A\}, \{R, B\}, \{R, C\}, \ldots\}$$

to denote the set of binary relations in a relation with respect to agent R, where $\{A, B, C, ...\}$ the set of agents that participate in the specific relation. For example, in figure 3b:

$$(R_1)_3 = \{\{R, O_1\}, \{R, O_2\}, \{R, O_3\}\}$$

where we have set arbitrarily $j = 1$.

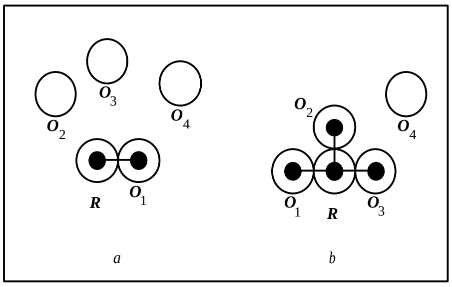

Figure 3: Part a represents a level-1 relation and part b a level-3 relation wrt agent R.

The complementary set $(R_j^C)_l$ of relation j is the set that contains all the relations of the same level apart from the specific relation j. For example in figure 3b:

$$\left(R_1^C\right)_3 = \{(R_2)_3, (R_3)_3, (R_4)_3\}$$

where

$$(R_2)_3 = \{\{R, O_1\}, \{R, O_2\}, \{R, O_4\}\}$$
$$(R_3)_3 = \{\{R, O_1\}, \{R, O_3\}, \{R, O_4\}\}$$
$$(R_4)_3 = \{\{R, O_2\}, \{R, O_3\}, \{R, O_4\}\}$$

A "Relation Proximity Function" (RPF) provides a measure of the distance between agent i and the other agents involved in the relation. Each relation has its own RPF. Let R_k denote the k^{th} relation of level l. The RPF of this relation is given by:

$$(b_{R_k})_l = \sum_{j \in (R_k)_l} \beta_{\{R,j\}} \tag{5}$$

where the notation $j \in (R_k)_l$ is used to denote the agents that participate in the specific relation of agent R. In the proofs, we also use the simplified notation $b_r = \sum_{j \in P_r} \beta_{ij}$ for simplicity, where r denotes a relation and P_r denotes the set of agents participating in the specific relation wrt agent i.

For example, in the relation of figure (2b) we have

$$(b_{R_1})_3 = \sum_{m \in (R_1)_3} \beta_{\{R,m\}} = \beta_{\{R,O_1\}} + \beta_{\{R,O_2\}} + \beta_{\{R,O_3\}}$$

A "Relation Verification Function" (RVF) is defined by:

$$(g_{R_k})_l = (b_{R_k})_l + \frac{\lambda (b_{R_k})_l}{(b_{R_k})_l + (B_{R_k^C})_l^{1/h}} \tag{6}$$

where λ, h are positive scalars and

$$(B_{R_k^C})_l = \prod_{m \in (R_k^C)_l} (b_m)_l$$

where as previously defined, $(R_k^C)_l$ is the complementary set of relations of level-l, i.e. all the other relations with respect to agent i that have the same number of binary relations with the relation R_k. Continuing with the previous example we could compute, for instance,

$$\left(B_{R_1^C}\right)_3 = (b_{R_2})_3 \cdot (b_{R_3})_3 \cdot (b_{R_4})_3$$

which refers to level-3 relations of agent R.

For simplicity we also use the notation $(B_{R_k^C})_l \equiv \tilde{b}_i = \prod_{m \in (R_k^C)_l} b_m$. Using the simplified notation $(b_{R_k})_l = b_i, (B_{R_k^C})_l = \tilde{b}_i$, the RVF can be written as $g_i = b_i + \frac{\lambda b_i}{b_i + \tilde{b}_i^{1/h}}$ It is obvious that for the highest level $l = n-1$ only one relation is possible so that $(R_k^C)_{n-1} = \emptyset$ and $(g_{R_k})_l = (b_{R_k})_l$ for $l = n-1$. The basic property that we demand from RVF is that it assumes the value of zero if a relation holds, while no other relations of the same or other levels hold. In other words it should indicate which of all possible relations holds. We have he following limits of RVF (using the simplified notation): (a) $\lim\limits_{b_i \to 0} \lim\limits_{\tilde{b}_i \to 0} g_i \left(b_i, \tilde{b}_i\right) = \lambda$ (b) $\lim\limits_{\substack{b_i \to 0 \\ \tilde{b}_i \neq 0}} g_i \left(b_i, \tilde{b}_i\right) = 0$. These limits guarantee that RVF will behave in the way we want it to, as an indicator of a specific collision.

The function G_i is now defined as

$$G_i = \prod_{l=1}^{n_L^i} \prod_{j=1}^{n_{R_l}^i} (g_{R_j})_l \tag{7}$$

where n_L^i the number of levels and $n_{R_l}^i$ the number of relations in level-l with respect to agent i.

3.2 An example

As an example, we will present steps to construct the function G with respect to a specific agent in a team of 4 agents indexed 1 through 4. We construct the function G_1 wrt agent 1. We begin by defining the Relation Proximity Functions (eq.(7)) in every level (Table 1):

Relation	Level 1	Level 2	Level 3
1	$(b_1)_1 = \beta_{12}$	$(b_1)_2 = \beta_{12} + \beta_{13}$	$(b_1)_3 = \beta_{12} +$ $+ \beta_{13} + \beta_{14}$
2	$(b_2)_1 = \beta_{13}$	$(b_2)_2 = \beta_{12} + \beta_{14}$	-
3	$(b_3)_1 = \beta_{14}$	$(b_3)_2 = \beta_{13} + \beta_{14}$	-

Table 1

It is now easy to calculate the Relation Verification Functions for each relation based on equation (8). For example, for the second relation of level 2, the complement (term $(B_{R_k^C})_l$ in eq.(8)) is given by $(B_{2^C})_2 = (b_1)_2 \cdot (b_3)_2$ and substituting in (8), we have

$$(g_2)_2 = (b_2)_2 + \frac{\lambda\,(b_2)_2}{(b_2)_2 + ((b_1)_2 \cdot (b_3)_2)^{1/h}}$$

The function G_1 is then calculated as the product of the Relation Verification Functions of all relations.

3.3 Construction of the f_i function

The key difference of the decentralized method with respect to the centralized case is that the control law of each agent ignores the destinations of the others. By using $\varphi_i = \frac{\gamma_{di}}{\left((\gamma_{di})^k + G_i\right)^{1/k}}$ as a navigation function for agent i, there is no potential for i to cooperate in a possible collision scheme when its initial condition coincides with its final destination. In order to overcome this limitation,we add a function f_i to γ_i so that the cost function φ_i attains positive values in proximity situations even when i has already reached its destination. We define the function f_i by:

$$f_i(G_i) = \begin{cases} a_0 + \sum\limits_{j=1}^{3} a_j G_i^j, \; G_i \le X \\ 0, \; G_i > X \end{cases}$$

where $X, Y = f_i(0) > 0$ are positive parameters the role of which will be made clear in the following. The parameters a_j are evaluated so that f_i is maximized when $G_i \to 0$ and minimized when $G_i = X$. We also require that f_i is continuously differentiable at X. Therefore we have:

$$a_0 = Y, a_1 = 0, a_2 = \frac{-3Y}{X^2}, a_3 = \frac{2Y}{X^3}$$

We require that $Y \leq \frac{\Theta_1}{k}$ where Θ_1 is an arbitrarily large positive gain. This will help in obtaining a lower bound of k analytically in the stability analysis that follows. The parameter X serves as a sensing parameter that activates the f_i function whenever possible collisions are bound to occur. The only requirement we have for X is that it must be small enough whenever the system has reached its equilibrium, i.e. when everyone has reached its destination. In mathematical terms:

$$X < G_i\left(q_{d1}, \ldots, q_{dN}\right) \forall i$$

That's the minimum requirement we have regarding knowledge of the destinations of the team. Intuitively, the destinations should be far enough from one another.

A key feature of navigation functions and in particular, Decentralized Navigation Functions, is that their gradient motion is repulsive with respect to the boundary of the free space. The free space for each agent is defined as the subset of W which is free of collisions with the other agents. Hence collision avoidance is reassured. For further information regarding terminology the reader is referred to [5], [4].

4 Elements from Nonsmooth Analysis

In this section, we review some elements from nonsmooth analysis and Lyapunov theory for nonsmooth systems that we use in the stability analysis of the next section.

We consider the vector differential equation with discontinuous right-hand side:

$$\dot{x} = f(x) \tag{8}$$

where $f : R^n \to R^n$ is measurable and essentially locally bounded.

Definition 4.1 [6]*: In the case when n is finite, the vector function $x(.)$ is called a solution of (8) in $[t_0, t_1]$ if it is absolutely continuous on $[t_0, t_1]$ and there exists $N_f \subset R^n, \mu(N_f) = 0$ such that for all $N \subset R^n, \mu(N) = 0$ and for almost all $t \in [t_0, t_1]$*

$$\dot{x} \in K[f](x) \equiv \overline{co}\{\lim_{x_i \to x} f(x_i) | x_i \notin N_f \cup N\}$$

Lyapunov stability theorems have been extended for nonsmooth systems in [12],[2]. The authors use the concept of *generalized gradient* which for the case of finite-dimensional spaces is given by the following definition:

Definition 4.2 [3]*: Let $V : R^n \to R$ be a locally Lipschitz function. The generalized gradient of V at x is given by*

$$\partial V(x) = \overline{co}\{\lim_{x_i \to x} \nabla V(x_i) | x_i \notin \Omega_V\}$$

where Ω_V is the set of points in R^n where V fails to be differentiable.

Lyapunov theorems for nonsmooth systems require the energy function to be *regular*. Regularity is based on the concept of *generalized derivative* which was defined by Clarke as follows:

Definition 4.3 [3]*: Let f be Lipschitz near x and v be a vector in R^n. The generalized directional derivative of f at x in the direction v is defined*

$$f^0(x; v) = \lim_{y \to x} \sup_{t \downarrow 0} \frac{f(y + tv) - f(y)}{t}$$

Definition 4.4 [3]*: The function $f : R^n \to R$ is called regular if*
1) $\forall v$, the usual one-sided directional derivative $f'(x; v)$ exists and
2) $\forall v, f'(x; v) = f^0(x; v)$

The following chain rule provides a calculus for the time derivative of the energy function in the nonsmooth case:

Theorem 4.5 [12]*: Let x be a Filippov solution to $\dot{x} = f(x)$ on an interval containing t and $V : R^n \to R$ be a Lipschitz and regular function. Then $V(x(t))$ is absolutely continuous, $(d/dt)V(x(t))$ exists almost everywhere and*

$$\frac{d}{dt}V(x(t)) \in^{a.e.} \dot{\tilde{V}}(x) := \bigcap_{\xi \in \partial V(x(t))} \xi^T K[f](x(t))$$

We shall use the following nonsmooth version of LaSalle's invariance principle to prove the convergence of the prescribed system:

Theorem 4.6 [12] *Let Ω be a compact set such that every Filippov solution to the autonomous system $\dot{x} = f(x), x(0) = x(t_0)$ starting in Ω is unique and remains in Ω for all $t \geq t_0$. Let $V : \Omega \to R$ be a time independent regular function such that $v \leq 0 \forall v \in \dot{\tilde{V}}$ (if $\dot{\tilde{V}}$ is the empty set then this is trivially satisfied). Define $S = \{x \in \Omega | 0 \in \dot{\tilde{V}}\}$. Then every trajectory in Ω converges to the largest invariant set,M, in the closure of S.*

Let $\dot{x} = f(x)$ be essentially locally bounded and $0 \in K[f](x)$ in $Q \supset \{x \in R^n | \|x\| \leq r\}$. Also, let $V : R^n \to R$ be a regular positive definite function in Q. Then
1) $\dot{\tilde{V}}(x) \leq 0$ in Q implies that the origin is stable.
2) If in addition, there exists a class K function $\omega(.)$ in Q with $\dot{\tilde{V}}(x) \leq \omega(x)$ $\forall x \in Q$ then the origin is asymptotically stable.

5 Stability Analysis

Following [5], we use the sum of the separate decentralized navigation functions $\varphi = \sum \varphi_i$ as a candidate Lyapunov function for the whole system. Specifically, the following holds:

Theorem 5.1 *The system is asymptotically stabilized to $q_d = [q_{d1}, \ldots, q_{dN}]^T$ up to a set of initial conditions of measure zero if the parameters k,h assume values bigger than a finite lower bound.*

We immediately note that the result of this theorem is existential rather than computational. We show that finite k, h that renders the system almost everywhere asymptotically stable *exist*, but we do not provide an analytical expression for this lower bound. However, practical values of k, h will be provided in the simulation section. In [4], we have used $\varphi = \sum_{i=1}^{n} \varphi_i$ as a Lyapunov function for the whole system. In this case this function is continuous everywhere, but nonsmooth whenever a switching occurs, i.e. whenever $\|q_i - q_j\| = d_c$ for some i, j. We define the *switching surface* as:

$$S = \{q : \exists i, j, i \neq j | \|q_i - q_j\| = d_c\} \tag{9}$$

We have proved that the system converges whenever $q \notin S$ (see [4]). On the switching surface the Lyapunov function is no longer smooth so we must use stability theory for nonsmooth systems. In the case when $q \in S$ we shall make use of theorem 4.6. First we must use the following lemma to ensure that φ is regular.

Lemma 5.2 *The function φ is regular $\forall q \in S$.*

Proof of Lemma 5.2: We show first that β_{ij} is regular whenever $\|q_i - q_j\| = d_C$. The directional derivative at d_C is

$$\beta'_{ij}(d_C; v) = \lim_{t \to 0} \frac{\beta_{ij}(d_C + tv) - \beta_{ij}(d_C)}{t} = \left\{ \begin{array}{l} 0, v \geq 0 \\ c < 0, v < 0 \end{array} \right.$$

The generalized directional derivative is

$$\beta^0_{ij}(d_C; v) = \limsup_{\substack{t \to 0 \\ y \to d_C}} \frac{\beta_{ij}(y + tv) - \beta_{ij}(y)}{t} = \left\{ \begin{array}{l} 0, v \geq 0 \\ c < 0, v < 0 \end{array} \right.$$

so that $\beta^0_{ij}(d_C; v) = \beta'_{ij}(d_C; v)\ \forall v$. It is easy to check that the terms $\frac{\partial b_i}{\partial \beta_{ij}}, \frac{\partial G_i}{\partial b_i}$ are nonnegative so by virtue of Theorem 2.3.9 (i), [3], the function G_i is regular at $q \in S$.

Function φ_i is continuously differentiable wrt G_i. In this case the term $\frac{\partial \varphi_i}{\partial G_i}$ is nonpositive but we are fortunate that G_i is 1-dimensional. Following the proof of theorem 2.3.9 (ii),[3] we can see that the generalized derivative of φ_i satisfies the following inequality: $\varphi^0_i(q; v) \leq \frac{\partial \varphi_i}{\partial G_i} G^0_i(q; v) = \frac{\partial \varphi_i}{\partial G_i} G'_i(q; v) = \varphi'_i(q; v)$. But we always have $\varphi'_i(q; v) \leq \varphi^0_i(q; v)$, so that $\varphi'_i(q; v) = \varphi^0_i(q; v)$, ensuring the

regularity of φ_i. The function φ is regular as the finite linear combination of regular functions.$\diamondsuit$
We now proceed with the proof of theorem 5.1. . We make use of the following matrix theorems in our analysis:

Theorem 5.3 *Given a matrix $A \in R^{n\times n}$ then all its eigenvalues lie in the union of n discs:*

$$\bigcup_{i=1}^{n} \left\{ z : |z - a_{ii}| \leq \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}| \right\} \triangleq \bigcup_{i=1}^{n} R_i(A) \triangleq R(A)$$

Each of these discs is called a Gersgorin disc of A.

Corollary 5.4 *Given a matrix $A \in R^{n\times n}$ and n positive real numbers $p_1, \ldots, p_n$ then all the eigenvalues of A lie in the union of n discs:*

$$\bigcup_{i=1}^{n} \left\{ z : |z - a_{ii}| \leq \frac{1}{p_i} \sum_{\substack{j=1 \\ j\neq i}}^{n} p_j \, |a_{ij}| \right\}$$

A key point of Corollary 5.4 is that if we bound the first $n/2$ Gersgorin discs of a matrix A sufficiently away from zero, then an appropriate choice of the numbers $p_1, \ldots, p_n$ renders the remaining $n/2$ discs sufficiently close to the corresponding diagonal elements. Hence, by ensuring the positive definiteness of the eigenvalues of the matrix M corresponding to the first $n/2$ rows, then we can render the remaining ones sufficiently close to the corresponding diagonal elements. This fact will be made clearer in the analysis that follows.

Proof of Theorem 5.1: In the global sensing case, the Proximity function between agents i and j is given by:

$$\beta_{ij}(q) = \|q_i - q_j\|^2 - (r_i + r_j)^2 = q^T D_{ij} q - (r_i + r_j)^2$$

where the $2N \times 2N$ matrix D_{ij} is defined in [9]. :

$$D_{ij} = \begin{bmatrix} & & O_{2(i-1)\times 2N} & & \\ O_{2\times 2(i-1)} & I_{2\times 2} & O_{2\times 2(j-i-1)} & -I_{2\times 2} & O_{2\times 2(N-j)} \\ & & O_{2(j-i-1)\times 2N} & & \\ O_{2\times 2(i-1)} & -I_{2\times 2} & O_{2\times 2(j-i-1)} & I_{2\times 2} & O_{2\times 2(N-j)} \\ & & O_{2(N-j)\times 2N} & & \end{bmatrix}$$

We can also write $b_r^i = q^T P_r^i q - \sum_{j\in P_r} (r_i + r_j)^2$,where $P_r^i = \sum_{j\in P_r} D_{ij}$, and P_r denotes the set of binary relations in relation r. It can easily be seen that

$\nabla b_r^i = 2P_r^i q, \nabla^2 b_r^i = 2P_r^i$. We also use the following notation for the r-th relation wrt agent i:

$$g_r^i = b_r^i + \frac{\lambda b_r^i}{b_r^i + \left(\tilde{b}_r^i\right)^{1/h}}, \tilde{b}_r^i = \prod_{\substack{s \in S_r \\ s \neq r}} b_s^i,$$
$$\nabla \tilde{b}_r^i = \sum_{\substack{s \in S_r \\ s \neq r}} \underbrace{\prod_{\substack{t \in S_r \\ t \neq s,r}} b_i^t}_{\tilde{b}_{s,r}^i} \cdot 2P_s^i q$$

where S_r denotes the set of relations in the same level with relation r. An easy calculation shows that

$$\nabla g_r^i = \ldots = 2\left[d_r^i P_r^i - w_r^i \tilde{P}_r^i\right] q \triangleq Q_r^i q, \tilde{P}_r^i \triangleq \sum_{\substack{s \in S_r \\ s \neq r}} \tilde{b}_{s,r}^i P_s^i$$

where $d_r^i = 1 + (1 - \frac{b_r^i}{b_r^i + (\tilde{b_r^i})^{1/h}})\frac{\lambda}{b_r^i + (\tilde{b_r^i})^{1/h}}$, $w_r^i = \frac{\lambda b_r^i (\tilde{b_r^i})^{\frac{1}{h}-1}}{h(b_r^i + (\tilde{b_r^i})^{1/h})^2}$. The gradient of the G_i function is given by:

$$G_i = \prod_{r=1}^{N_i} g_r^i \Rightarrow \nabla G_i = \sum_{r=1}^{N_i} \underbrace{\prod_{\substack{l=1 \\ l \neq r}}^{N_i} g_l^i}_{\tilde{g}_r^i} \nabla g_r^i = \sum_{r=1}^{N_i} \tilde{g}_r^i Q_r^i q \triangleq Q_i q$$

where N_i all the relations with respect to agent i. We define

$$\nabla G \triangleq \begin{bmatrix} \nabla G_1 \\ \vdots \\ \nabla G_N \end{bmatrix} = \begin{bmatrix} Q_1 \\ \vdots \\ Q_N \end{bmatrix} q \triangleq Qq$$

Remembering that $u_i = -K_i \frac{\partial \varphi_i}{\partial q_i}$ and that $\varphi_i = \frac{\gamma_{di} + f_i}{\left((\gamma_{di} + f_i)^k + G_i\right)^{1/k}}, f_i = \sum_{j=0}^{3} a_i G_i^j$ the closed loop dynamics of the system are given by:

$$\begin{aligned} \dot{q} &= \begin{bmatrix} -K_1 A_1^{-(1+1/k)} \left\{ G_1 \frac{\partial \gamma_{d1}}{\partial q_1} + \sigma_1 \frac{\partial G_1}{\partial q_1} \right\} \\ \vdots \\ -K_N A_N^{-(1+1/k)} \left\{ G_N \frac{\partial \gamma_{dN}}{\partial q_N} + \sigma_N \frac{\partial G_N}{\partial q_N} \right\} \end{bmatrix} = \ldots \\ &= -A_K G\left(\partial \gamma_d\right) - A_K \Sigma Q q \end{aligned}$$

where $\sigma_i = G_i \sigma(G_i) - \frac{\gamma_{di} + f_i}{k}, \sigma(G_i) = \sum_{j=1}^{3} j a_j G_i^{j-1}, A_i = (\gamma_{di} + f_i)^k + G_i$ and the matrices

$$A_K \triangleq \underbrace{diag\begin{pmatrix} K_1 A_1^{-(1+1/k)}, K_1 A_1^{-(1+1/k)}, \ldots \\ , K_N A_N^{-(1+1/k)}, K_N A_N^{-(1+1/k)} \end{pmatrix}}_{2N \times 2N}$$

$$G \triangleq \underbrace{diag\left(G_1, G_1, \ldots, G_N, G_N\right)}_{2N\times 2N}, (\partial\gamma_d) = \left[\frac{\partial\gamma_{d1}}{\partial q_1} \cdots \frac{\partial\gamma_{dN}}{\partial q_N}\right]$$

$$\Sigma \triangleq \underbrace{\left[\underbrace{\Sigma_1}_{2N\times 2N}, \ldots, \underbrace{\Sigma_N}_{2N\times 2N}\right]}_{2N\times 2N^2}, \Sigma_i = diag\begin{pmatrix} 0, 0, \ldots, \underbrace{\sigma_i, \sigma_i}_{2i-1,2i} \\ , \ldots, 0, 0 \end{pmatrix}$$

By using $\varphi = \sum\limits_i \varphi_i$ as a candidate Lyapunov function we have $\varphi = \sum\limits_i \varphi_i \Rightarrow \dot{\varphi} = \left(\sum\limits_i (\nabla\varphi_i)^T\right)\dot{q}, \nabla\varphi_i = A_i^{-(1+1/k)}\left\{G_i\nabla\gamma_{di} + \sigma_i\nabla G_i\right\}$ and after some trivial calculation

$$\sum_i (\nabla\varphi_i)^T = \ldots = (\partial\gamma_d)^T A_G + q^T Q^T A_\Sigma$$

where $A_G = \underbrace{diag\begin{pmatrix} G_1 A_1^{-(1+1/k)}, G_1 A_1^{-(1+1/k)}, \ldots, \\ G_N A_N^{-(1+1/k)}, G_N A_N^{-(1+1/k)} \end{pmatrix}}_{2N\times 2N}$ and

$$A_\Sigma = \underbrace{\begin{bmatrix} \underbrace{A_{\Sigma_1}}_{2N\times 2N} \\ \vdots \\ \underbrace{A_{\Sigma_N}}_{2N\times 2N} \end{bmatrix}}_{2N^2\times 2N}, A_{\Sigma_i} = \underbrace{diag\begin{pmatrix} A_i^{-(1+1/k)}\sigma_i, \ldots, \\ A_i^{-(1+1/k)}\sigma_i \end{pmatrix}}_{2N\times 2N}$$

The derivative of the candidate Lyapunov function is calculated as

$$\begin{aligned} \dot{\varphi} &= \left(\sum_i (\nabla\varphi_i)^T\right)\cdot\dot{q} = \ldots \\ &= -\begin{bmatrix} (\partial\gamma_d)^T & q^T \end{bmatrix} \underbrace{\begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix}}_{M} \begin{bmatrix} \partial\gamma_d \\ q \end{bmatrix} \end{aligned}$$

where $M_1 = A_G A_K G, M_2 = A_G A_K \Sigma Q, M_3 = Q^T A_\Sigma A_K G, M_4 = Q^T A_\Sigma A_K \Sigma Q$.

Let's return to the local sensing case.Let $S_1 = \{q : \exists i, j, i \neq j | (\|q_i - q_j\| = d_c) \bigwedge (\|q_k - q_l\| \neq d_c \forall k, l : k \neq i, j, l \neq i, j)\}$ denote the subset of S which corresponds to the simplest case of switching that involves only two agents. System dynamics are given by:

$$\dot{q} = f(q) = \left[-K_1\frac{\partial\varphi_1}{\partial q_1}, \ldots, -K_n\frac{\partial\varphi_n}{\partial q_n}\right]^T$$

The vector function $f(q)$ is nonsmooth at S_1 so that $\dot{q} \in K[f](q), q \in S_1$. We have $K[f](q \in S_1) = \overline{co}\{f_{S_1}^-, f_{S_1}^+\}$ where $S_1^{-(+)} = \{q : \|q_i - q_j\| < (>) d_C\}$ and

$$f_{S_1}^{-(+)}(q \in S_1) = \lim_{\substack{q^* \to q, \\ q^* \in S_1^{-(+)}}} f(q^*)$$

Likewise, the generalized gradient of the candidate Lyapunov function at the discontinuity surface is given by $\partial\varphi(q \in S_1) = \overline{co}\{\nabla\varphi_{S_1}^-, \nabla\varphi_{S_1}^+\}$ where

$$\nabla\varphi_{S_1}^{-(+)}(q \in S_1) = \lim_{\substack{q^* \to q, \\ q^* \in S_1^{-(+)}}} \nabla\varphi(q^*)$$

Each $\rho \in \partial\varphi(q \in S_1)$ is the convex combination of the limit points of the convex hull: $\rho = \mu\left(\nabla\varphi_{S_1}^-\right) + (1-\mu)\left(\nabla\varphi_{S_1}^+\right), \mu \in [0,1]$. Similarly, each $\eta \in K[f](q \in S_1)$ as $\eta = \lambda f_{S_1}^- + (1-\lambda) f_{S_1}^+, \lambda \in [0,1]$, so that $\rho^T\eta = \lambda\mu\left(\nabla\varphi_{S_1}^-\right)^T f_{s_1}^- + (1-\lambda)\mu\left(\nabla\varphi_{S_1}^-\right)^T f_{s_1}^+ + \lambda(1-\mu)\left(\nabla\varphi_{S_1}^+\right)^T f_{s_1}^- + (1-\lambda)(1-\mu)\left(\nabla\varphi_{S_1}^+\right)^T f_{s_1}^+$. By virtue of theorem ?? one has

$$\dot{\varphi}(q \in S_1) \in \bigcap_{\rho \in \partial\varphi(q \in S_1)} \rho^T\eta,\ \eta \in K[f](q \in S_1)$$

Going back to the previous analysis, it is easy to see that the matrices A_G, A_K, G, Σ, A_Σ are continuous in the discontinuity surface. The matrix Q is discontinuous at S_1 and that's due to the nonsmoothness of the functions G_i, G_j. By using the notation $Q^{-(+)}(q \in S_1) = \lim_{\substack{q^* \to q, \\ q^* \in S_1^{-(+)}}} Q(q^*)$ and noting that $\bigcap_{\rho \in \partial\varphi(q \in S_1)} \rho^T\eta = \bigcap_{\mu \in [0,1]} \left\{\rho^T\eta | \lambda \in [0,1]\right\}$ we conclude after some trivial calculation that

$$\dot{\varphi}\left(q \in S_1\right) \in \bigcap_{\mu \in [0,1]} \left\{ \begin{array}{l} -\left[\begin{array}{cc} (\partial\gamma_d)^T & q^T \end{array}\right] M \left[\begin{array}{c} \partial\gamma_d \\ q \end{array}\right] \\ |\lambda \in [0,1] \end{array} \right\}$$

with $M = \left[\begin{array}{cc} M_1 & M_2 \\ M_3 & M_4 \end{array}\right]$ where

$$M_1 = A_G A_K G, M_2 = A_G A_K \Sigma\left(\lambda Q^- + (1-\lambda) Q^+\right)$$

$$M_3 = \left(\mu\left(Q^-\right)^T + (1-\mu)\left(Q^+\right)^T\right) A_\Sigma A_K G$$

$$\begin{array}{l} M_4 = \lambda\mu\left(Q^-\right)^T A_\Sigma A_K \Sigma Q^- + (1-\lambda)\mu\left(Q^-\right)^T A_\Sigma A_K \Sigma Q^+ + \\ \lambda(1-\mu)\left(Q^+\right)^T A_\Sigma A_K \Sigma Q^- + (1-\lambda)(1-\mu)\left(Q^+\right)^T A_\Sigma A_K \Sigma Q^+ \end{array}$$

We first proceed by examining the Gersgorin discs of the first half rows of the matrix M. We denote this procedure as $M_1 - M_2$, as the main diagonal elements of M_1 are "compared" with the corresponding raw elements of M_2. Note that the submatrices M_1, M_2 are both diagonal, therefore the only nonzero elements of raw i of the $4N \times 4N$ matrix M are the elements $M_{ii}, M_{i,2N+i}$ where of course $1 \le i \le 2N$ as we calculate the Gersgorin discs of the first half rows of the matrix M. With respect to corollary 5.4, we have:

$$\begin{aligned} &|z - M_{ii}| \le \tfrac{1}{p_i} \sum_{j \ne i} p_j \, |M_{ij}|, 1 \le i \le 2N \Rightarrow \\ &\Rightarrow |z - (M_1)_{ii}| \le \tfrac{p_{2N+i}}{p_i} \, |(M_2)_{ii}| \end{aligned}$$

where

$$(M_1)_{ii} = A_i^{-2(1+1/k)} K_i G_i^2$$

and

$$|(M_2)_{ii}| = \left| A_i^{-2(1+1/k)} \sigma_i K_i G_i \cdot \left\{ \lambda \left(Q_{ii}^i\right)^+ + (1-\lambda)\left(Q_{ii}^i\right)^- \right\} \right|$$

Denote $\left| \lambda \left(Q_{ii}^i\right)^+ + (1-\lambda)\left(Q_{ii}^i\right)^- | \lambda \in [0,1] \right| \triangleq \left| \left(Q_{ii}^i\right)^\pm \right|$. It is then obvious that $\left| \left(Q_{ii}^i\right)^\pm \right|_{\max} = \max \left\{ \left| \left(Q_{ii}^i\right)^- \right|_{\max}, \left| \left(Q_{ii}^i\right)^+ \right|_{\max} \right\}$, which is always bounded in a bounded workspace. Therefore we have:

$$\begin{aligned} &\left| z - A_i^{-2(.)} K_i G_i^2 \right| \le \tfrac{p_{2N+i}}{p_i} \left| A_i^{-2(.)} \sigma_i K_i G_i \left(Q_{ii}^i\right)^\pm \right| \\ &\Rightarrow z \ge A_i^{-2(.)} K_i G_i^2 - \tfrac{p_{2N+i}}{p_i} \left| A_i^{-2(.)} \sigma_i K_i G_i \left(Q_{ii}^i\right)^\pm \right| \end{aligned}$$

We examine the following three cases:

- $G_i < \varepsilon$ At a critical point in this region, the corresponding eigenvalue tends to zero, so that the derivative of the Lyapunov function could achieve zero values. However, the result of Lemma 6 in [4] indicates that φ_i is a Morse function, hence its critical points are isolated[8]. Thus the set of initial conditions that lead to saddle points are sets of measure zero[11].

- $G_i > X$ The corresponding eigenvalue is guaranteed to be positive as long as:

$$\begin{aligned} &z > 0 \Leftarrow A_i^{-2(.)} K_i \left(G_i - \tfrac{p_{2N+i}}{p_i} |\sigma_i| \left| \left(Q_{ii}^i\right)^\pm \right| \right) > 0 \\ &\Leftarrow G_i \ge X > \tfrac{p_{2N+i}}{p_i} |\sigma_i| \left| \left(Q_{ii}^i\right)^\pm \right| = \tfrac{\gamma_{di}}{k} \tfrac{p_{2N+i}}{p_i} \left| \left(Q_{ii}^i\right)^\pm \right| \\ &\Leftarrow k > \tfrac{(\gamma_{di})_{\max}}{X} \tfrac{p_{2N+i}}{p_i} \left| \left(Q_{ii}^i\right)^\pm \right|_{\max} \end{aligned}$$

- $0 < \varepsilon \le G_i \le X$ In [4], we prove that $|\sigma_i(\varepsilon)| \le Y \left| \frac{1}{k} + \frac{8}{9} \right| + \left| \frac{\gamma_{di}}{k} \right|$ The corresponding eigenvalue is guaranteed to be positive as long as:

$$\begin{aligned} &z > 0 \Leftarrow \varepsilon > \left\{ Y \left| \tfrac{1}{k} + \tfrac{8}{9} \right| + \left| \tfrac{\gamma_{di}}{k} \right| \right\} \tfrac{p_{2N+i}}{p_i} \left| \left(Q_{ii}^i\right)^\pm \right|_{\max} \\ &\overset{Y \le \frac{\Theta_1}{k}}{\Longleftarrow} \end{aligned}$$

$$k > 2 \max \left\{ 2\sqrt{\frac{\Theta_1}{\varepsilon}}, \frac{16\Theta_1}{9\varepsilon}, \frac{(\gamma_{di})_{\max}}{\varepsilon} \right\} \frac{p_{2N+i}}{p_i} \left| \left(Q_{ii}^i\right)^\pm \right|_{\max}$$

A key point is that there is no restriction on how to select the terms $\frac{p_{2N+i}}{p_i}$. This will help us in deriving bounds that guarantee the positive definiteness of the matrix M.

We are now left to examine the Gersgorin discs of the second half rows of the matrix M. Likewise, we denote this procedure as $M_3 - M_4$. The discs of Corollary 5.4 are evaluated:

$$\begin{array}{l} |z - M_{ii}| \le \sum\limits_{j \ne i} \frac{p_j}{p_i} |M_{ij}|, 2N+1 \le i \le 4N, 1 \le j \le 4N \\ \Rightarrow |z - (M_4)_{ii}| \le R_i(M_3) + R_i(M_4) \end{array}$$

where $R_i(M_3) = \sum\limits_{j=1}^{2N} \frac{p_j}{p_i} \left|(M_3)_{ij}\right|$,$R_i(M_4) = \sum\limits_{\substack{j=2N+1 \\ j \ne i}}^{4N} \frac{p_j}{p_i} \left|(M_4)_{ij}\right|$ and

$$(M_4)_{ii} = \sum_j \left(\begin{array}{l} K_i A_i^{-(1+1/k)} A_j^{-(1+1/k)} \sigma_j \sigma_i \cdot \\ \cdot \left\{ \begin{array}{l} \lambda\mu \left(Q_{ii}^i\right)^- \left(Q_{ii}^j\right)^- + (1-\lambda)\mu \left(Q_{ii}^i\right)^- \left(Q_{ii}^j\right)^+ + \\ \lambda(1-\mu) \left(Q_{ii}^i\right)^+ \left(Q_{ii}^j\right)^- + \\ (1-\lambda)(1-\mu) \left(Q_{ii}^i\right)^+ \left(Q_{ii}^j\right)^+ | \lambda \in [0,1] \end{array} \right\} \end{array} \right)$$

Following the same procedure as in [4], it can easily be shown that $R_i(M_3) \ge R_i(M_4) \forall i$.

The corresponding eigenvalue is guaranteed to be positive as long as:

$$\begin{array}{l} z > 0 \Leftarrow (M_4)_{ii} > R_i(M_3) + R_i(M_4) \\ \Leftarrow (M_4)_{ii} > \max\left\{2R_i(M_3), 2R_i(M_4)\right\} = 2R_i(M_3) \end{array}$$

Choosing without loss of generality $p_i = p, 2N+1 \le i \le 4N$, we have after some non-trivial calculations:

$$R_i(M_3) = \sum_{j=1}^{2N} \frac{p_i}{p} \underbrace{\left| \begin{array}{l} A_j^{-2(1+1/k)} \sigma_j K_j G_j \left(\mu \left(Q_{ii}^j\right)^- + (1-\mu) \left(Q_{ii}^j\right)^+ \right) + \\ (A_j A_i)^{-(1+1/k)} \sigma_i K_j G_j \left(\mu \left(Q_{jj}^i\right)^- + (1-\mu) \left(Q_{jj}^i\right)^+ \right) \end{array} \right|}_{\left|(M_3)_{ij}\right|}$$

The fact that $(M_4)_{ii} > 0$ is guaranteed by Lemma 2.3 in [4]. This lemma also guarantees that there is always a finite upper bound on the terms We have

$$\begin{array}{l} (M_4)_{ii} > 2R_i(M_3) = 2 \sum\limits_{j=1}^{2N} \frac{p_j}{p} \left|(M_3)_{ij}\right| \Leftarrow \\ p > \frac{4N}{(M_4)_{ii}} \max\limits_j \left\{ p_j \left|(M_3)_{ij}\right| \right\}, \\ 2N+1 \le i \le 4N, 1 \le j \le 2N \end{array}$$

We can now directly apply theorem 4.6 to our case. We have proved that $v \le 0 \ \forall v \in \dot{\tilde{\varphi}}$ and that the only invariant subset of the set $S = \{q | 0 \in \dot{\tilde{\varphi}}(q)\}$ is $\left\{q_d = [q_{d1}, ..., q_{dn}]^T\right\}$. Hence the nonsmooth version of LaSalle's invariance principle guarantees convergence to the destination points. $\diamondsuit$

6 Simulations

To demonstrate the navigation properties of our decentralized approach, we present a simulation of four holonomic agents that have to navigate from an initial to a final configuration, avoiding collision with each other. Each agent has no knowledge of the positions of agents outside its sensing zone, which is the big circle around its center of mass in Fig.3, Pic.1. In this picture A-i,T-i denote the initial condition and desired destination of agent i respectively. The chosen configurations constitute non-trivial setups since the straight-line paths connecting initial and final positions of each agent are obstructed by other agents. The following have been chosen for the simulation of figure 3:
Initial Conditions:

$$q_1(0) = \begin{bmatrix} -.1732 & -.1 \end{bmatrix}^T, q_2(0) = \begin{bmatrix} .1732 & -.1 \end{bmatrix}^T,$$
$$q_3(0) = \begin{bmatrix} 0 & .2 \end{bmatrix}^T, q_4(0) = \begin{bmatrix} 0 & -.2 \end{bmatrix}^T$$

Final Conditions:

$$q_{d1} = \begin{bmatrix} .1732 & .1 \end{bmatrix}^T, q_{d2} = \begin{bmatrix} - & .1732 & .1 \end{bmatrix}^T,$$
$$q_{d3} = \begin{bmatrix} 0 & -.1 \end{bmatrix}^T, q_{d4} = \begin{bmatrix} 0 & .25 \end{bmatrix}^T$$

Parameters:

$$k = 110, r_1 = r_2 = r_3 = r_4 = .05, d_C = .11$$
$$\lambda = 1, h = 5, X = .001, Y = .01$$

Pictures 1-6 of Figure 3 show the evolution of the team configuration within a horizon of 6000 time units. One can observe that the collision avoidance as well as destination convergence properties are fulfilled.

In the next simulation (Fig.4) the sensing zone of the red agent A2 is shown in all the screenshots. The following have been chosen for the simulation:
Initial Conditions:

$$q_1(0) = \begin{bmatrix} -.1732 & -.1 \end{bmatrix}^T, q_2(0) = \begin{bmatrix} .1732 & -.1 \end{bmatrix}^T,$$
$$q_3(0) = \begin{bmatrix} 0 & .2 \end{bmatrix}^T, q_4(0) = \begin{bmatrix} 0 & -.2 \end{bmatrix}^T$$

Final Conditions:

$$q_{d1} = \begin{bmatrix} .15 & .05 \end{bmatrix}^T, q_{d2} = \begin{bmatrix} - & .1732 & .2 \end{bmatrix}^T,$$
$$q_{d3} = \begin{bmatrix} 0 & -.1 \end{bmatrix}^T, q_{d4} = \begin{bmatrix} 0 & .25 \end{bmatrix}^T$$

Parameters:

$$k = 100, r_1 = r_2 = r_3 = r_4 = .03, d_C = .08$$
$$\lambda = 1, h = 5, X = .001, Y = .01$$

The collision avoidance and destination requirements are met in this case as well. We point out that since the sensing zone of the red agent is always empty,i.e. it does not participate in a conflict situation, its trajectory is the straight line between its initial and final destination. This is due to the fact that the sensing parameter d_C is small in this case.

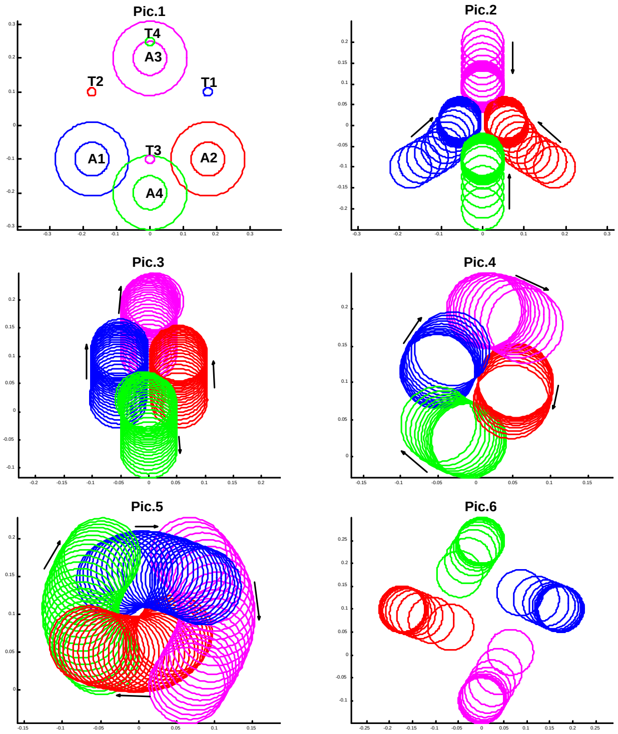

Figure 4: Simulation A

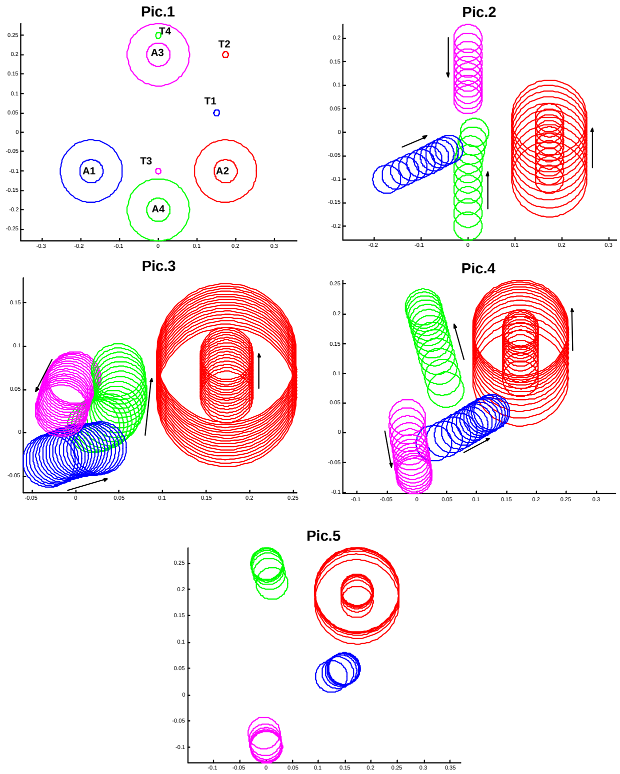

Figure 5: Simulation B

7 Conclusions

In this paper we extended the decentralized navigation method to the case of multiple holonomic agents with limited sensing capabilities. We proposed a nonsmooth extension of the navigation function of [5] and proved system convergence using tools from nonsmooth stability analysis. The effectiveness of the methodology is verified through computer simulations.

Current research includes applying this method to the case of distributed nonholonomic agents [10] as well as introducing new definitions of the sensing zone of an agent. Extensions of this method to 3-dimensional dynamics are also under investigation.

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