THE ERROR IS THEN

$$\frac{36}{180}\frac{(1-0)^5}{m^4} = \frac{1}{5m^4}$$

SO $\quad 5m^4 \geq 10^6 \qquad m^4 \geq 2\cdot 10^5 = 200{,}000$

$m \geq 22$.

IMPROPER INTEGRALS

IN PHISICS AND ENGINEERING WE OFTEN HAVE TO COMPUTE INTEGRALS ON INFINITE DOMAINS, OR LIMITS OF UNBOUNDED FUNCTIONS.

SOME EXAMPLES ARE

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx \qquad \int_{-1}^{1} \frac{1}{x^2}\,dx \qquad \int_0^1 \log(x)$$

UNBOUNDED FUNCTION

$$\int_0^\infty e^{-x}\,dx \qquad \int_1^\infty \frac{1}{\sqrt{x}}\,dx \qquad \int_0^\infty \frac{1}{1+x^2}\,dx$$

INFINITE DOMAIN

HOW DO WE DEAL WITH THESE?

DIVIDE AND CONQUER PLUS LIMITS, LIMITS

LIMITS.

IDEA / DEFINITION (INFINITE DOMAIN)

i) IF $\int_a^R f(x)\,dx$ EXIST FOR ALL $R > a$

THEN $$\int_a^\infty f(x)\,dx = \lim_{R\to\infty} \int_a^R f(x)\,dx$$

(EXISTS IF AND ONLY IF LIMIT EXISTS, AND IS FINITE)

ii) IF $\int_r^a f(x)\,dx$ EXISTS FOR ALL $r < a$

THEN $$\int_{-\infty}^a f(x)\,dx = \lim_{r\to-\infty} \int_r^R f(x)\,dx$$

(EXISTS IF AND ONLY IF LIMIT EXISTS AND IS FINITE)

iii) IF BOTH $\int_a^{+\infty} f(x)\,dx$ AND $\int_{-\infty}^a f(x)\,dx$ EXIST (ANY a CAN BE USED) THEN

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_a^{+\infty} f(x)\,dx + \int_{-\infty}^a f(x)\,dx$$

IF THIS HAPPENS, THE INTEGRAL IS SAID TO BE CONVERGENT, DIVERGENT OTHERWISE.

EXAMPLE:

- $$\int_1^{\infty} \frac{1}{x^2+1}\,dx = \lim_{R\to\infty} \int_1^{R} \frac{1}{x^2+1}\,dy = \lim_{R\to\infty} \arctan(R) - \arctan(1)$$

$$= \lim_{R\to\infty} \arctan(R) - \frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

- $$\int_{-\infty}^{1} \frac{1}{x^2+1}\,dx = \lim_{r\to-\infty} \int_r^{1} \frac{1}{x^2+1}\,dx = \lim_{r\to-\infty} +\frac{\pi}{4} + \arctan(r)$$

$$= \frac{3\pi}{4}$$ so

- $$\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx = \pi$$

EXAMPLE: $\int_1^{\infty} \frac{1}{x^p}\,dx \qquad p > 0$

$p>1$) $\int \frac{1}{x^p}\,dx = \frac{x^{1-p}}{1-p} + C$ so $\lim_{R\to\infty} \int_1^{R} \frac{1}{x^p}\,dx =$

$$\lim_{R\to\infty} \frac{R^{1-p}}{1-p} - \frac{1}{1-p} \underset{\substack{\uparrow \\ 1-p<0}}{=} -\frac{1}{1-p} = \boxed{\frac{1}{p-1} = \int_1^{\infty} \frac{1}{x^p}\,dx}$$

$p<1$) $\int \frac{1}{x^p}\,dx = \frac{x^{1-p}}{1-p} + C$ so $\lim \int_1^{R} \frac{1}{x^p}\,dx =$

$$\lim_{R\to\infty} \frac{R^{1-p}}{1-p} - \frac{1}{1-p} \underset{\substack{\uparrow \\ 1-p>0}}{=} \infty$$ <u>DIVERGENT</u>

$p=1$) $\lim \int_1^{R} \frac{1}{x}\,dx = \lim_{R\to\infty} \log(|R|) = \infty$ <u>DIVERG.</u>

EXAMPLE: $p \geq 1$,

$$\int_2^\infty \frac{1}{x \log(x)^p}\, dx = \lim_{R \to +\infty} \int_2^R \frac{1}{x \log(x)^p}\, dx$$

$$\underset{\substack{\uparrow \\ u = \log x}}{=} \lim_{R \to \infty} \int_{\log 2}^{\log R} \frac{1}{u^p}\, du = \lim_{R \to \infty} \begin{cases} \left[\log|u| \right]_{\log 2}^{\log R} & p = 1 \\ \left[\dfrac{u^{-p+1}}{-p+1} \right]_{\log 2}^{\log R} & p \neq 1 \end{cases}$$

$$= \begin{cases} \lim\limits_{R \to \infty} \log(\log R) - \log(\log 2) = +\infty \\ \lim\limits_{R \to \infty} \dfrac{-\log 2^{-p+1}}{-p+1} + \dfrac{\log R^{-p+1}}{-p+1} = \dfrac{\log 2^{-p+1}}{p-1} \end{cases}$$