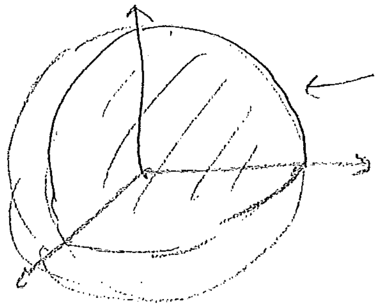
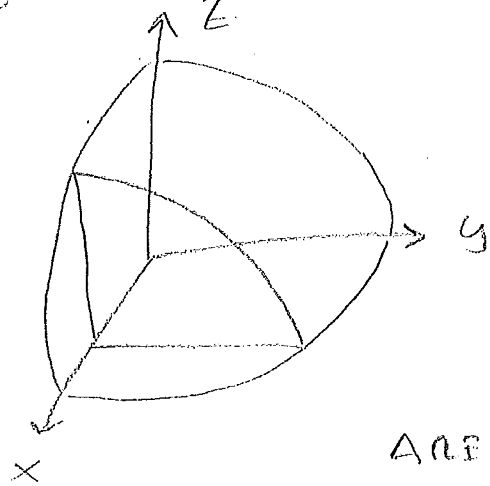
EXAMPLE: SPHERE OF RADIUS r



WE'LL COMPUTE THIS AND MULTIPLY BY 8



FIX A VALUE OF x

$$x^2+y^2+z^2=r^2$$

$$\underbrace{y^2+z^2}_{\text{CIRCLE!}} = \underbrace{r^2-x^2}_{\text{CONSTANT}}$$

RADIUS $= \sqrt{r^2-x^2}$

AREA $= (r^2-x^2)\cdot\pi$ $\left(\cdot\frac{1}{4}\right.$ AS WE ARE ONLY TAKING  $\left.\right)$

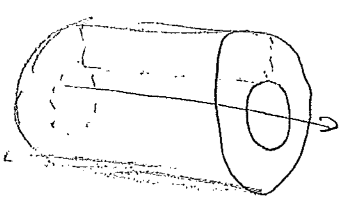
SO VOL OF OCTANT =

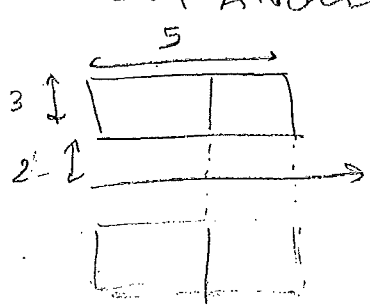
$$\int_0^r \frac{(r^2-x^2)\pi}{4}\,dx = \frac{\pi}{4}\left(xr^2-\frac{x^3}{3}\right)\Big|_0^r = \frac{\pi}{4}\cdot\frac{2}{3}r^3 = \frac{\pi}{6}r^3$$

VOL OF SPHERE $8\cdot\frac{\pi}{6}r^3 = \frac{4}{3}\pi r^3$

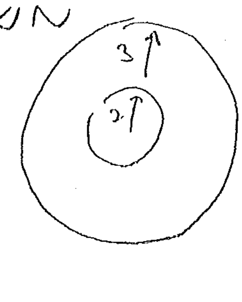
EXAMPLE: REVOLVING A REGION

WE CONSTRUCT AN HOLLOW CYLINDER BY REVOLVING A RECTANGLE AROUND THE X-AXIS





SLICE: A CIRCULAR CROWN

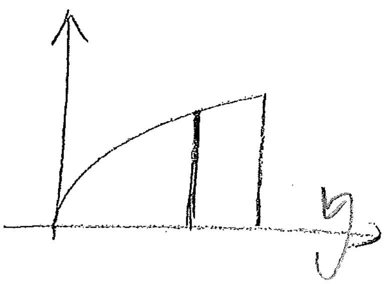


AREA: $5^2\cdot\pi - 2^2\cdot\pi = 21\pi$

So VOLUME $\displaystyle\int_0^5 21\pi \, dx = 21\pi x \Big|_0^5 = 105\pi$

EXAMPLE: MORE REVOLVING

WE REVOLVE THE REGION BETWEEN $y = \sqrt{x}$, $y = 0$ AND $x = 4$ AROUND $y = 0$



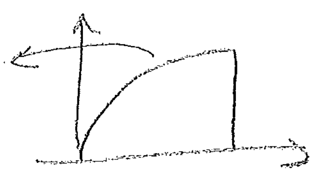
SLICE = CIRCLE OF RADIUS $\sqrt{x}$

AREA OF SLICE $= \pi(\sqrt{x})^2 = \pi x$

VOLUME $\displaystyle\int_0^4 \pi x \, dx = \frac{\pi x^2}{2}\Big|_0^4 = 8\pi$

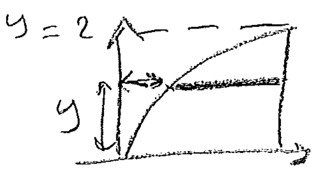
EXAMPLE: EVEN MORE REVOLVING

SAME REGION IS ROTATED AROUND $x = 0$



WE PICK <u>HORIZONTAL</u> SLICES

(WE ALWAYS WANT FLAT PANCAKES)



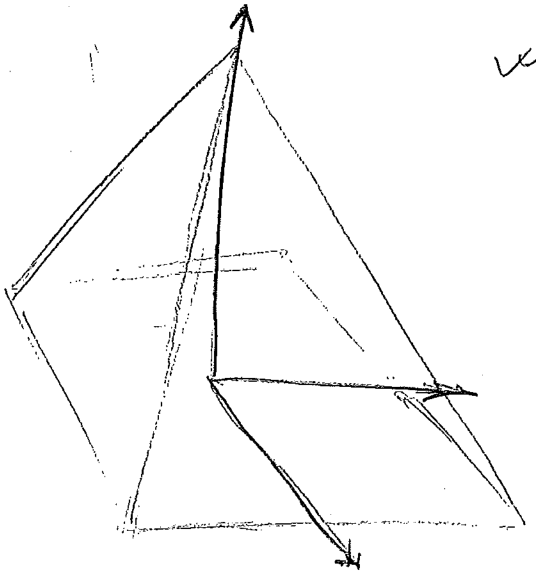
SLICE = CIRCULAR CROWN,

$R = 4$, $r = y^2$ $\quad (\sqrt{x} = y \Rightarrow y^2 = x)$

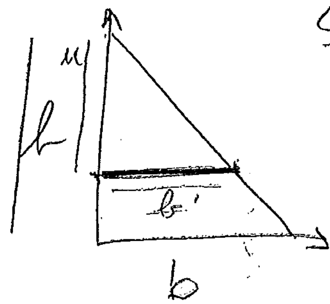
AREA OF SLICE IS $\pi R^2 - \pi r^2 = (16 - y^4)\pi$

VOL: $\displaystyle\pi\int_0^2 16 - y^4 \, dy = \pi\left(16y - \frac{y^5}{5}\right)\Big|_0^2 = \frac{128\pi}{5}$

EXAMPLE: SQUARE PYRAMID BASE 2b, HEIGHT h



WE PICK ONE OCTANT



SLICE: SQUARE

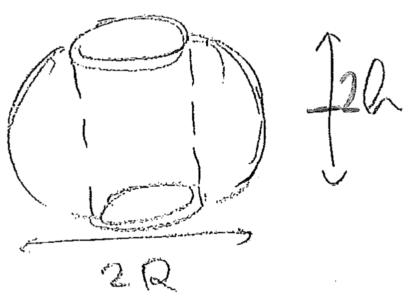
$$\frac{b}{h} = \frac{b'}{u} \qquad b' = \frac{u \cdot b}{h}$$

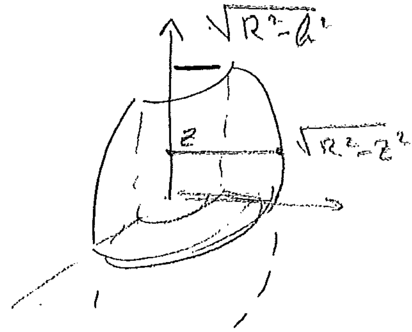
$$(u = z - h)$$

AREA OF SLICE $(b')^2 = u^2 \cdot \frac{b^2}{h^2}$

$$\frac{1}{4}\text{VOL} = \int_0^h u^2 \cdot \frac{b^2}{h^2}\,du = \frac{u^3}{3} \cdot \frac{b^2}{h^2}\Big|_0^h = \frac{h \cdot b^2}{3} \qquad \boxed{\text{VOL} = \frac{4}{3} h b^2}$$

EXAMPLE: NAPKIN RING





SLICE: CIRCULAR CROWN

$$R = \sqrt{R^2 - z^2}$$

$$r = \sqrt{R^2 - h}$$

AREA OF SLICE $= \pi(R^2 - R^2 - z^2 + h^2) =$
$\pi(h^2 - z^2)$

$$\text{VOL} : 2\int_0^h \pi(h^2 - z^2)\,dz = 2\pi\left(h^2 z - \frac{z^3}{3}\right)\Big|_0^h = \pi \cdot \frac{4}{3} h^3$$

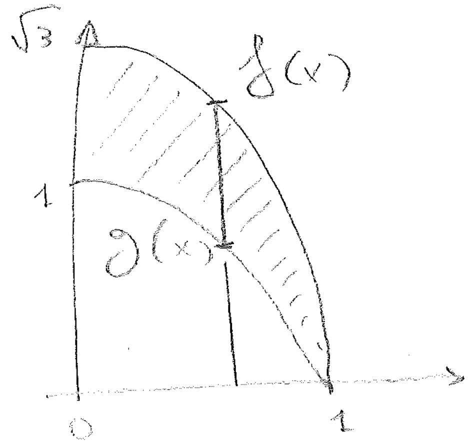
EXAMPLE: "BOWL"

SOLID OBTAINED BY ROTATING AROUND X-AXIS SHAPE BETWEEN

$y = \sqrt{3-3x}$ $\qquad$ $y = \sqrt{1-x^2}$

$\quad$ " $\qquad\qquad\qquad$ "

$\quad f \qquad\qquad\qquad g$



SLICE ALONG X-AXIS; SLICE IS A CIRCULAR CROWN OF AREA $\pi f(x)^2 - \pi g(x)^2$

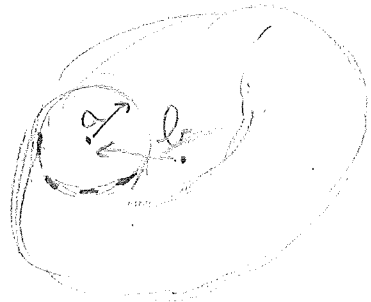
(NOTE: <u>NOT</u> $\pi(f(x)-g(x))^2$!)

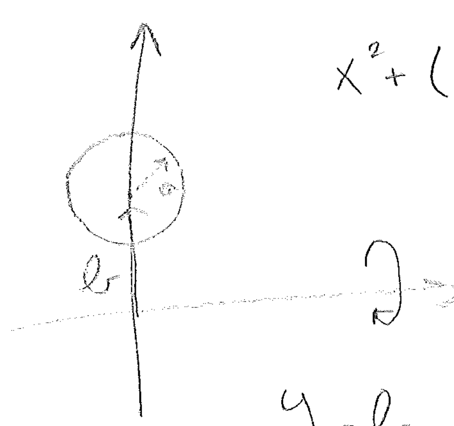
SO VOLUME $= \displaystyle\int_0^1 \pi\left(f(x)^2 - g(x)^2\right) dx =$

$\pi \displaystyle\int_0^1 \left(3-3x-(1-x^2)\right) dx = \pi \int_0^1 x^2-3x+2 \, dx =$

$\pi \left[\frac{x^3}{3} - \frac{3}{2}x^2 + 2x\right]_0^1 = \pi\left(\frac{1}{3} - \frac{3}{2} + 2\right) = \frac{5}{6}\pi$

EXAMPLE: TORUS





$x^2 + (y-b)^2 = a^2$

$(b > a)$

$y - b = \pm\sqrt{a^2 - x^2}$