So $\frac{d}{dx} F(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$

$$= \lim_{h \to 0} \frac{\int_a^{x+h} f(x)\,dx - \int_a^{x} f(x)\,dx}{h} =$$

SHOULD BE MADE MORE PRECISE

$$\lim_{h \to 0} \frac{\int_x^{x+h} f(x)\,dx}{h} = \lim_{h \to 0} \frac{h f(x)}{h} = f(x)!$$

SO ROUGHLY STATING,

"DIFFERENTIATING UNDOES INTEGRATING"

EXAMPLE:

- $F(x) = \int_0^x \cos T\,dT$, THEN $F'(x) = \cos(x)$
- $F(x) = \int_1^x (T^2 + \sqrt{T+1})\,dT$, THEN $F'(x) = x^2 + \sqrt{x+1}$
- $F(x) = \int_1^{x^2} \cos T\,dT$, FIND $F'(x)$

CHAIN RULE: $F(x) = G(x^2)$, $G(x) = \int_1^x \cos T\,dT$

$$\frac{d}{dx} G(x^2) = \frac{d}{dx} x^2 \cdot \frac{d}{dx} G(x^2) = 2x \cos x^2$$

THIS IS STILL NOT SOLVING OUR MAIN PROBLEM OF COMPUTING INTEGRALS, THOUGH!

OR IS IT?

DEF: GIVEN A CONTINUOUS FUNCTION $f(x)$ WE SAY A FUNCTION $F(x)$ S.T. $\frac{d}{dx}F(x) = f(x)$ IS AN <u>ANTIDERIVATIVE</u> OF $f(x)$.

EXAMPLE: x^2 IS AN ANTIDERIVATIVE OF $2x$!

BUT SO ARE x^2+3, $x^2+\pi$, x^2-10...

HOW DO DIFFERENT ANTIDERIVATIVES RELATE?

THEOREM: IF $F(x)$, $G(x)$ ARE ANTIDERIVATIVES OF $f(x)$ THEN $F(x)-G(x)$ IS A CONSTANT, I.E. $G(x) = F(x) + c$.

<u>WHY?</u> $\frac{d}{dx}\left(F(x) - G(x)\right) = f(x) - f(x) = 0$;

AND A DIFFERENTIABLE FUNCTION WHOSE DERIVATIVE IS ALWAYS 0 IS <u>CONSTANT</u>.

SOME KNOWN DERIVATIVES:

$x^m \to mx^{m-1}$ $\quad \sin(x) \to \cos(x)$ $\quad \cos(x) \to -\sin(x)$

$e^x \to e^x$ $\quad \log(x) \to \frac{1}{x}$

SO HOW DO WE COMPUTE INTEGRALS?

- WE WANT $\int_a^b f(x)\,dx$
- FIND ANTIDERIVATIVE $F(x)$, SO THAT

$$\int_a^b f(x)\,dx = F(b) + c \quad \text{FOR SOME } c$$

- WHAT IS c? $\int_a^a f(x)\,dx = 0$, SO

$$F(a) + c = 0 \Rightarrow c = -F(a)$$

- $\int_a^b f(x)\,dx = F(b) - F(a)$!!

THEOREM (FTC, PART 2): $f(x)$ CONTINUOUS ON $[a, b]$, $F(x)$ ANTIDERIVATIVE OF $f(x)$.

THEN $\int_a^b f(x)\,dx = F(b) - F(a)$.

EXAMPLE: $f(x) = 2x$, $a = 0$, $b = 1$ $\boxed{F(x) = x^2}$

$$\int_0^1 2x\,dx = 1^2 - 0^2 = 1$$

$\boxed{F(x) = x^2 + 3}$ $\int_0^1 2x\,dx = (1^2 + 3) - (0^2 + 3) = 1 + 3 - 3 = 1$

THIS ALSO SHOWS THAT

"INTEGRATION UNDOES DIFFERENTIATION, UP TO A CONSTANT"

AS $$\int_a^x f'(\tau)\,d\tau = f(x) - f(a)$$

ALSO: "THE INTEGRAL OF RATE OF CHANGE IS NET CHANGE"

NOTATION: • TO SIMPLIFY OUR FORMULAS, WE WILL WRITE

$$F(x)\Big|_a^b \overset{\text{def}}{=} \Big[F(x)\Big]_a^b = F(b) - F(a)$$

SO THAT $$\int_a^b f(\tau)\,d\tau = F(x)\Big|_a^b$$

• THERE ARE MANY ANTIDERIVATIVES OF $f(x)$; ANY WILL WORK FOR THE PURPOSE OF INTEGRATING. THUS WE DEFINE THE INDEFINITE INTEGRAL OF $f(x)$

$$\int f(x)\,dx = F(x) + C$$

TO BE THE "GENERAL" ANTIDERIVATIVE OF $f(x)$; $F(x)$ IS A GIVEN ANTIDERIVATIVE AND C IS AN UNSPECIFIED CONSTANT.

EX: $\int \cos(x)\,dx = \sin(x) + C$

WE GET A TABLE OF ANTIDERIVATIVES BY "SWITCHING THE ORDER OF COLUMNS" IN A TABLE OF DERIVATIVES

$f(x)$	$\int f(x)\,dx$
1	$x + C$
$x^m,\ m \neq -1$	$\frac{1}{m+1}x^{m+1} + C$ EXERCISE: DERIVE FORMULA!
$\frac{1}{x}$	$\log\lvert x\rvert + C$ *
e^x	$e^x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x + C$
$\frac{1}{1+x^2}$	$\arctan x + C$

* ABSOLUTE VALUE IS CRUCIAL!

Ex: $\int_0^4 x^2 - 3x\,dx = \int_0^4 x^2\,dx - \int_0^4 3x\,dx$

$= \frac{x^3}{3}\Big|_0^4 - \frac{3x^2}{2}\Big|_0^4 = \frac{64}{3} - \frac{48}{2} = -\frac{8}{3}$

EX: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)\,dx = \left[\sin(x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)$

$= 1 - (-1) = 2$