COMPARISONS:

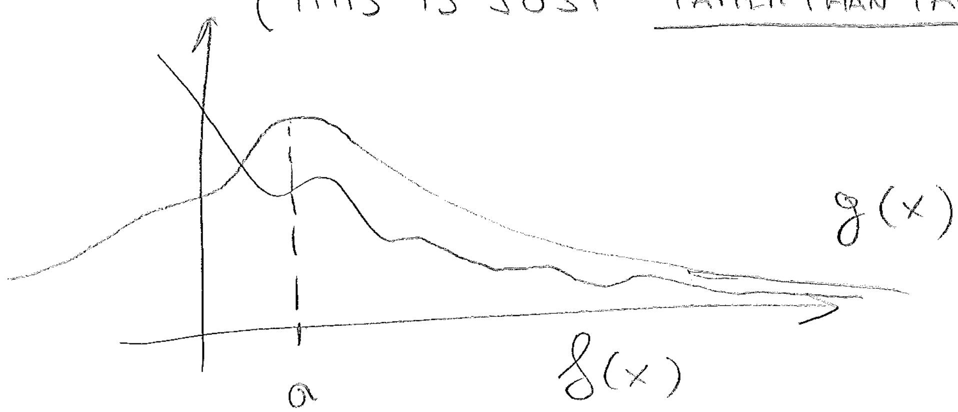
WE SAY THAT $\int_a^\infty f(x)$ CONVERGES (DIVERGES)

BY THE COMPARISON TEST WITH $g(x)$

IF WE HAVE A $g(x)$ S.T.

- $f(x)$ AND $g(x)$ HAVE THE SAME SIGN
- $|g(x)| \geq |f(x)|$ (RESP $|g(x)| \leq |f(x)|$)
- $\int_a^\infty g(x)$ CONVERGES (DIVERGES)

(THIS IS JUST "TAMER THAN TAME", "WILDER THAN WILD")



AREA UNDER $f(x)$ $\leq$ AREA UNDER $g(x)$

EXAMPLE:

$\int_1^\infty \frac{1}{x^2+x}\,dx$ CONVERGES BY COMPARISON

WITH $\int_1^\infty \frac{1}{x^2}$ AS $\frac{1}{x^2+x} < \frac{1}{x^2}$ FOR $x \geq 1$

SOMETIMES IT'S NOT SO EASY:

EXAMPLE: $\displaystyle\int_4^{\infty} \frac{2\sin x + 3}{x^2 - 3x}\, dx$

IDEA: IT SHOULD CONVERGE, AS THE DOMINANT TERM IS $\frac{1}{x^2}$

HOW TO DO IT: WE WANT TO COMARE IT WITH <u>A MULTIPLE</u> OF $\frac{1}{x^2}$. SO WE WANT $f(x) \leq \frac{C}{x^2}$

NUMERATOR: (WE NEED NUMER. $\leq$ SOMETHING)

$0 \leq 2\sin x + 3 \leq 2 + 3$

DENOMINATOR: (WE NEED DENOM. $\geq$ SOMETHING!)

HOW BIG IS $3x$ COMPARED TO x^2 FOR $x \geq 4$?

$\frac{3x}{x^2} = \frac{3}{x} \leq \frac{3}{4}$ SO $3x \leq \frac{3}{4}x^2$ AND

$x^2 - 3x \geq \frac{1}{4}x^2$

SO $$\frac{2\sin x + 3}{x^2 - 3x} \leq \frac{5}{\frac{x^2}{4}} = \frac{20}{x^2}!$$

AND IT CONVERGES BY COMPARISON.

STRONGER TOOL: WE ACTUALLY ONLY NEED THE COMPARISON TO HOLD FOR ALL <u>LARGE ENOUGH</u> VALUES OF X!

THM: (LIMIT COMPARISON)

SUPPOSE THAT $\lim_{x \to \infty} \frac{f(x)}{g(x)} = L \ (\neq \pm\infty)$

THEN:

- IF $\int_a^\infty g(x)\,dx$ CONVERGES, SO DOES $\int_a^\infty f(x)\,dx$.
- IF $L \neq 0$ MOREOVER IF $\int_a^\infty f(x)\,dx$ CONVER. SO DOES $\int_a^\infty g(x)\,dx$

EXAMPLE: $f(x) = \frac{5}{x^2 - 3x}$, $g(x) = \frac{1}{x^2}$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{5x^2}{x^2 - 3x} = 5 \quad \text{SO}$$

$\int_4^\infty \frac{5}{x^2 - 3x}\,dx$ CONV BY LIMIT COMP. WITH $\int_4^\infty \frac{1}{x^2}\,dx$.

BUT THIS LOOKS LIKE IT WOULDN'T WORK FOR $f(x) = \frac{2\sin x + 3}{x^2 - 3x}$

EX: $f(x) = \frac{2\sin(x) + 3}{x^2 - 3x}$, $g(x) = ?$

IDEA: $\frac{1}{x^2}$ IS KIND OF A "LARGE" BOUND, WE CAN TAKE SOMETHING SUCH AS $\frac{1}{x^{\frac{3}{2}}}$, AS $\int_4^\infty \frac{1}{x^{\frac{3}{2}}}\,dx$ STILL CONVERGES...

$$\underline{g(x) = \frac{1}{x^{\frac{3}{2}}}}$$

$$\lim_{x\to\infty} \frac{\frac{2\sin(x)+3}{x^2-3x}}{\frac{1}{x^{\frac{3}{2}}}} = \lim_{x\to\infty} \frac{x^{\frac{3}{2}}\underbrace{(2\sin(x)+3)}_{\leq 5}}{x^2 - 3x}$$

$$\leq \lim_{x\to\infty} \frac{5x^{\frac{3}{2}}}{x^2 - 3x} = 0$$

SO $\int_4^\infty \frac{2\sin(x)+3}{x^2-3x}\,dx$ CONVERGES BY LIMIT COMP. WITH $\frac{1}{x^{\frac{3}{2}}}$!