THM:

$\sum_{m=0}^{\infty} A_m (x-c)^m$ POWER SERIES.

| $\lim_{m\to\infty} \left\| \frac{A_{m+1}}{A_m} \right\|$ | POWER SERIES CONVERGES | POWER SERIES DIVERGES | R |
|---|---|---|---|
| $L \neq 0$ | $\|x-c\| < \frac{1}{L}$ | $\|x-c\| > \frac{1}{L}$ | $\frac{1}{L}$ |
| $L = 0$ | EVERYWHERE | NOWHERE | $\infty$ |
| DIVERGES TO $+\infty$ | ONLY AT $x = c$ | FOR ALL $x \neq c$ | 0 |

EXAMPLE: $\sum_{m=0}^{\infty} 3^{-2m} \cdot m^2 (x-7)^m$

CENTER $c=7$ RADIUS OF CONVERGENCE?

$$\lim_{m\to\infty} \left| \frac{3^{-2m-2} \cdot (m+1)^2}{3^{-2m} \cdot m^2} \right| = \lim_{m\to\infty} 3^{-2} \left| \frac{(m+1)^2}{m^2} \right| = \frac{1}{9}$$

So $R=9$, P.S. CONVERGES FOR

$|x-7| < 9$, I.E. $-2 < x < 16$

EXAMPLE: $\sum_{n=0}^{\infty} \frac{(x-5)^n}{n!}$ (SPOILER $= e^{x-5}$)

CENTER $c=5$, RADIUS OF CONVERGENCE?

$$\lim_{n\to\infty} \left| \frac{n!}{n+1!} \right| = \lim_{n\to\infty} \frac{1}{n+1} = 0 \quad \text{SO} \quad R=\infty,$$

P.S. CONVERGES FOR ALL X.

EXAMPLE: $\sum_{n=0}^{\infty} 2^{n^2} x^n$

CENTER $x=0$, RADIUS OF CONVERGENCE?

$$\lim_{n\to\infty} \left| \frac{2^{(n+1)^2}}{2^{n^2}} \right| = \lim_{n\to\infty} 2^{(n+1)^2-n^2} = \lim_{n\to\infty} 2^{n^2+2n+1-n^2} =$$

$$\lim_{n\to\infty} 2^{2n+1} = \infty \quad \text{SO} \quad R=0 \quad \text{AND}$$

P.S. <u>ONLY</u> CONVERGES AT $x=0$.

EXAMPLE: $\sum_{m=0}^{\infty}(\pi_m+1)(x-c)^m$ $\pi_m = m$-th DIGIT OF $\pi$

THE LIMIT $\lim_{m\to\infty}\left|\frac{\pi_{m+1}+1}{\pi_m+1}\right|$ DOES NOT EXIST

(BECAUSE $\pi$ IS IRRATIONAL, ∴ DON'T WORRY ABOUT EXACTLY WHY)

BUT $\pi_m+1 \leq 10$ FOR ALL $m$ SO

$\sum_{m=0}^{\infty}(\pi_m+1)(x-c)^m$ CONVERGES ABSOLUTELY

BY COMPARISON WITH $\sum_{n=0}^{\infty} 10(x-c)^n$ FOR

$|x-c|<1$

AND $\pi_m+1 \geq 1$ FOR ALL $m$ SO

$\sum_{m=0}^{\infty}(\pi_m+1)(x-c)^m$ DIVERGES BY COMPARISON

WITH $\sum_{m=0}^{\infty}(x-c)^m$ FOR $|x-c|>1$

THM: LET $\sum_{m=0}^{\infty} A_m(x-c)^m$ BE A POWER SERIES. ONE OF THE FOLLOWING MUST HOLD, EITHER:

i) THRE IS $R>0$ S.T. $\sum_{m=0}^{\infty} A_m(x-c)^m$ CONVERGES FOR $|x-c|<R$ AND DIVERGES FOR $|x-c|>R$.

ii) $\sum_{m=0}^{\infty} A_m(x-c)^m$ CONVERGES FOR ALL $x$ $(R=\infty)$.

iii) $\sum_{m=0}^{\infty} A_m(x-c)^m$ CONVERGES FOR $x=c$ AND DIVERGES FOR ANY $x \neq c$ $(R=0)$.

EXAMPLE:

Q: WE KNOW THAT A POWER SERIES CENTERED AT $c=10$ CONVERGES AT $x=7$ BUT DIVERGES AT $x=15$. WHAT CAN WE CONCLUDE?

A: $R \geq |10-7| = 3$ AND $R \leq |10-15| = 5$

Q: WHAT IF WE ALSO KNEW THAT IT CONVERGES AT $x=5$?

$R \geq |10-5| = 5$ AND $R \leq |10-15| = 5$ SO $R=5$.