# On Optimal Input Design in System Identification for Model Predictive Control

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We present a method of performing optimal input design on a process controlled by MPC. Given a model structure and a measure of control performance degradation, the method provides an optimal input signal to be used in the identification experiment.

## Optimal Input Design

**Model:**

$$\mathcal{M}(\theta): \quad \begin{aligned} x_{t+1} &= F(\theta)x_t + G(\theta)u_t + v_t, \\ y_t &= H(\theta)x_t + e_t, \\ v_t &\in \mathcal{N}(0, R_v),\ e_t \in \mathcal{N}(0, R_e) \end{aligned}$$

where $\theta \in \mathbb{R}^d$ is an unknown parameter vector. The true system is $\mathcal{S} = \mathcal{M}(\theta_0)$ for some $\theta_0 \in \mathbb{R}^d$.

**Application:** The relationship between $\theta$ and control performance degradation is given by $V_{app}(\theta)$. The allowed degradation is

$$V_{app}(\theta) \leq 1/(2\gamma),\ \gamma > 0,$$

yielding the approximative application set

$$\mathcal{E}_{app} = \{\theta \mid (\theta - \theta_0)^{\mathrm{T}} V''_{app}(\theta_0)(\theta - \theta_0) \leq 1/\gamma\}.$$

**Identification:** The estimates $\hat{\theta}_N$ are obtained using PEM based on $N$ observations. The confidence ellipsoid is

$$\hat{\theta}_N \in \mathcal{E}_{SI} = \{\theta \left| [\theta - \theta_0]^T \mathbf{P}^{-1} [\theta - \theta_0] \leq \chi^2_\alpha(d)/N\right\},\ \text{w. p. } \alpha,$$

where $\mathbf{P}$ is the Fisher information matrix.

**Optimization problem:**

$$\begin{aligned} \underset{\phi_{u(\omega)}}{\text{minimize}} \quad & \text{experiment cost} \\ \text{subject to} \quad & \mathcal{E}_{SI} \subseteq \mathcal{E}_{app} \\ & \phi_u(\omega) \geq 0 \quad \forall \omega \end{aligned}$$

where $\phi_u(\omega)$ is the input spectrum. An input is realized according to $\phi_{u(\omega)}$ and used in the identification experiment. The obtained $\hat{\theta}_N$ lie inside $\mathcal{E}_{app}$ with, at least, probability $\alpha$.

## Experimental Identification Algorithm

**Difficulties:**

- $\theta_0$ is unknown.
- $V_{app}(\theta)$ requires application based on models with more or less arbitrary $\theta$.

**Solutions:**

- Use estimates instead of $\theta_0$ in all the expressions.
- Evaluate $V_{app}(\theta)$ in simulation.

**Identification algorithm:**

**Step 0** Find an initial estimate of $\theta$.

**Step 1** Evaluate $V_{app}(\theta)$ based on simulations of the model with the parameter estimates.

**Step 2** Design the optimal input signal based on $\mathcal{E}_{app}$ and parameter estimates.

**Step 3** Find a new estimate of $\theta$ using the optimal input signal in the identification experiment.

## Control Design for Water Process

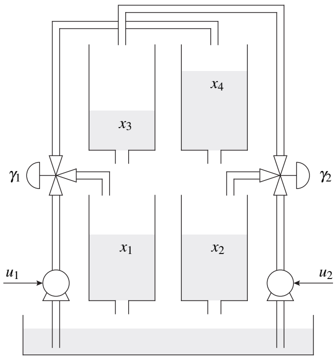


Figure 1: Water process. MPC controls the pump voltages, $u_1$ and $u_2$. Lower tank levels, $x_1$ and $x_2$, are outputs.

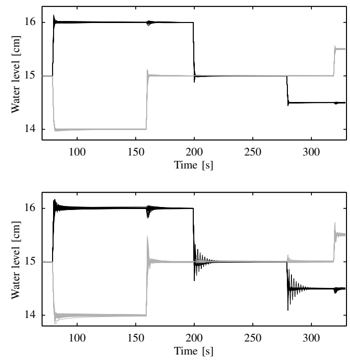


Figure 2: Trajectories. MPC based on models from 100 id. experiments. Optimal and white input are used in upper and lower plot, respectively.

**Application:** The control objective is reference tracking of the lower tank levels using MPC. Thus, we choose

$$V_{app}(\theta) = \frac{1}{N}\sum_{t=1}^{N} \|y_t(\theta_o) - y_t(\theta)\|.$$

**Experiment cost:** Input power, $\operatorname{trace}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\phi_u(\omega)d\omega\right)$.

**Result:** 91 % of the trajectories have acceptable degradation for optimal input while only 15 % have for white input.

## Conclusion

- Increased control performance.
- Linear framework applicable on nonlinear systems.

## References

[1] C. A. Larsson, M. Annergren, H. Hjalmarsson, “On Optimal Input Design in System Identification for Model Predictive Control”, 50th IEEE CDC, 2011, to appear.

[2] H. Hjalmarsson, “System Identification of Complex and Structured Systems”, European Journal of Control, 2009.