On Optimal Input Design for Model Predictive Control

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Abstract— **This paper considers a method for optimal input design in system identification for control. The approach addresses model predictive control (MPC). The objective of the framework is to provide the user with a model which guarantees that a specified control performance is achieved, with a given probability. We see that, even though the system is nonlinear, using linear theory in the input design can reduce the experimental effort. The method is illustrated in a minimum power input signal design in system identification of a water tank system.**

I. Introduction

Model predictive control (MPC) is a widely used model based control strategy in industry [1]. As its name entails, MPC predicts future states of the controlled process based on a model of the system. Given these predictions, MPC constructs the optimal control strategy which is then applied to the process. The performance of the controller is highly dependent on the quality of the model it is based on.

During the lifetime of the process, the accuracy of the model will degrade as the system is effected by wear and damages. Due to process–model mismatch, the control performance will decrease. If the level of performance sinks below a given threshold it may become necessary to re-estimate the model while the process is running. Thus, it is highly desirable to have an efficient and accurate method of identifying such models in an MPC context. Reducing the cost of the experiment is also often of importance and optimal input design has been shown to give significant reduction of the experimental effort [2].

In this contribution we build on the optimal input design for models used in MPC developed in [3] and [4]. We present a general method of performing optimal input design on a process controlled by MPC. The approach is illustrated on a water tank system. Given a model structure and a measurement of the control performance degradation, the method will provide the user with the optimal input signal to be used in the identification experiment.

In Section II we outline the basic ideas of application oriented experiment design presented in [5]. The specifics for the MPC case are outlined in Section III. Section III-C presents the experiment design procedure for MPC which is then illustrated by examples in Section V-D.

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II. Problem formulation

We will consider identification of models on the form

$$\mathscr{M}(\theta): \quad \begin{aligned} x_{t+1} &= F(\theta)x_t + G(\theta)u_t + v_t \\ y_t &= H(\theta)x_t + e_t \end{aligned} \tag{1}$$

where x_t is the state vector, $\{u_t\}_{t=1}^N$ is a known input sequence, v_t and e_t are zero-mean, Gaussian processes with covariance matrices R_v and R_e respectively and θ is an unknown parameter vector. We assume that there exists a parameter vector, say θ_o, such that the model (1) describes the true system, denoted $\mathscr{S}$.

The process is assumed to be controlled using MPC with the quadratic cost function

$$J(t) = \sum_{i=0}^{M} \left(\|\hat{y}_{t+i|t} - r_{t+i}\|_Q^2 + \|\Delta u_{t+i+1|t}\|_{R_1}^2 + \|u_{t+i+1|t}\|_{R_2}^2 \right), \tag{2}$$

where $\hat{y}_{t+i|t}$, $u_{t+i|t}$ and $\Delta u_{t+i|t}$ are i-step predictions of the output, input and input update of the system, respectively. The known reference trajectory is denoted r_t. The matrices Q, R_1 and R_2 are tunable weights. The norm $\|x\|_A$ is equal to $\sqrt{x^T A x}$. The cost function is minimized with respect to the input updates and the update Δu_{t+1} is applied to the process. The optimization is performed in each timestep in accordance with the receding horizon control philosophy.

A major advantage of MPC is the ability to handle signal and state constraints on the process in the controller. This, however, leads to that there is no explicit solution to the optimization problem in the controller [6]. We will see that this is a limiting factor in the experiment design and requires numerical calculations.

To find the output predictions that are used in the optimization, a model of the process is needed. The more accurate the model, the better the MPC performance. The degradation in performance due to using an inaccurate model will be formalized in the next section.

A. Application cost

The performance of a controller design based on a model of a process, will be directly related to the quality of the model. If θ_o were available for the design, the performance specifications would be met. However, for model estimates different from θ_o, the performance will degrade. The application cost relates model parameters to the performance degradation and is denoted $V_{app}(\theta)$.

We choose the cost function such that its minimal value is zero and occurs when the true parameter vector θ_o is used, i.e., $V_{app}(\theta_o) = 0$, $V'_{app}(\theta_o) = 0$ and $V''_{app}(\theta_o) \succeq 0$[1].

[1] $A \succeq B$ means that $A - B$ is a positive semi-definite matrix.

A maximal allowed performance degradation gives an upper limit

$$V_{app}(\theta) \leq \frac{1}{\gamma}, \tag{3}$$

for some real-valued positive constant γ. The parameters corresponding to acceptable performance degradation belong to the set

$$\Theta = \left\{ \theta \; \middle| V_{app}(\theta) \leq \frac{1}{\gamma} \right\}, \tag{4}$$

which we call the application set. This leads to the idea that the objective of system identification should be to deliver parameter estimates that belong to the application set.

We can make a convex approximation of Θ using a second order Taylor approximation. Hence, the inequality (3) can be approximated by

$$[\theta - \theta_o]^T V''_{app}(\theta_o)[\theta - \theta_o] \leq \frac{2}{\gamma}. \tag{5}$$

For sufficiently large γ, the set of acceptable parameter (4) can thus be approximated by the ellipsoidal set

$$\mathscr{E}_{app} = \left\{ \theta \;\middle|\; [\theta - \theta_o]^T V''_{app}(\theta_o)[\theta - \theta_o] \leq \frac{2}{\gamma} \right\}. \tag{6}$$

We will call this the application ellipsoid.

In [4] the scenario approach [7], [8], is presented as another possible approximation of the application set. The basis for that approach is to randomly select parameters that satisfy (3), called scenarios. If enough scenarios are used, the performance degradation can be guaranteed with high probability.

B. System identification

Let $\hat{\theta}$ be the estimated parameter vector of the model based on N input–output observations using the prediction error method (PEM). A well-known result from the theory of PEM for open-loop identification is the asymptotic (in sample size) Gaussian distribution

$$\sqrt{N}(\hat{\theta} - \theta_o) \sim \mathscr{N}(0, \mathbf{P}), \tag{7}$$

$$\mathbf{P}^{-1} = \mathrm{E}\left\{ \frac{d}{d\theta}\hat{y}(t,\theta) R_e^{-1} \frac{d}{d\theta}\hat{y}^H(t,\theta) \right\}, \tag{8}$$

of the estimates $\hat{\theta}$ [9]. The derivatives in the expression for $\mathbf{P}^{-1}$ should be evaluated at the true parameter values.

We can find confidence ellipsoids for the estimates given by

$$\hat{\theta}_N \in \mathscr{E}_{SI} = \left\{ \theta \;:\; [\theta - \theta_o]^T \mathbf{P}^{-1}[\theta - \theta_o] \leq \frac{\kappa}{N} \right\}, \text{ w. p. } \alpha. \tag{9}$$

The positive constant κ depends on the number of parameters to be estimated and the probability α. Its value can be obtained from the χ^2-distribution. This means that our estimated model will lie inside the system identification set, defined by (9), with probability α.

C. Input design

We want our estimated model parameters $\hat{\theta}$ to be acceptable with respect to control performance. Since the estimates are random variables, this is hard to guarantee. Therefore, we relax the condition and require only that the estimated parameters satisfy the control performance with some (high) probability. That is, we require the system identification set defined by (9) to be contained in the application set defined by (4), i.e.,

$$\mathscr{E}_{SI} \subseteq \Theta. \tag{10}$$

If we use the approximation (6), both sets are ellipsoids and the region constraint is equivalent to the linear matrix inequality (LMI)

$$\frac{N}{\kappa}\mathbf{P}^{-1} \succeq \frac{\gamma}{2}\, V''_{app}(\theta_o). \tag{11}$$

The LMI (11) together with (9) implies that our estimated parameters will lie in the application ellipsoid, i.e., $\hat{\theta}_N \in \mathscr{E}_{app}$, with at least probability α. This idea is presented in [5].

If we instead use the scenario approach, we replace (10) with the constraints

$$[\theta_k - \theta_o]^T \frac{N}{\kappa}\mathbf{P}^{-1}[\theta_k - \theta_o] \geq \gamma V_{app}(\theta_k),\; k = 1, \ldots, K, \tag{12}$$

where $\theta_k \in \Theta$ are samples taken from, say, a uniform distribution on Θ. For sufficiently large values of K this approximates the original constraint well. For more details on the scenario approach in general see [8] and for the use of scenario approach in identification see [4].

A natural objective of the input design is to minimize some experiment cost while guaranteeing that (10) holds. Experiment cost can, for instance, be experiment time, input power or input energy. The key is that the inverse covariance matrix $\mathbf{P}^{-1}$ can be expressed in the frequency domain as an affine function of the input spectrum. Hence, a linear parameterization, and any convex objective function of the input spectrum will lead to input design problems that are semi-definite programs. This has been extensively discussed in the literature, for a detailed discussion, see e.g., [10].

The application set gives the directions of high performance degradation with respect to model parameters. Thus, we can determine which linear combinations of elements of θ are important to estimate with high accuracy. The application set is linked to the identification through the input design. When the optimal input is applied to the system, the most sensitive parameter directions are excited while unimportant dynamics are not.

III. Identification for MPC

In this section we bring together the theory outlined in the previous section and present a scheme for optimal input design in an MPC context. There are two major challenges with the practical implementation of the method. The first is the fact that the optimal input design relies on knowledge of the true system parameters. These are, of course, not known at the time of input design. The two proposed ways

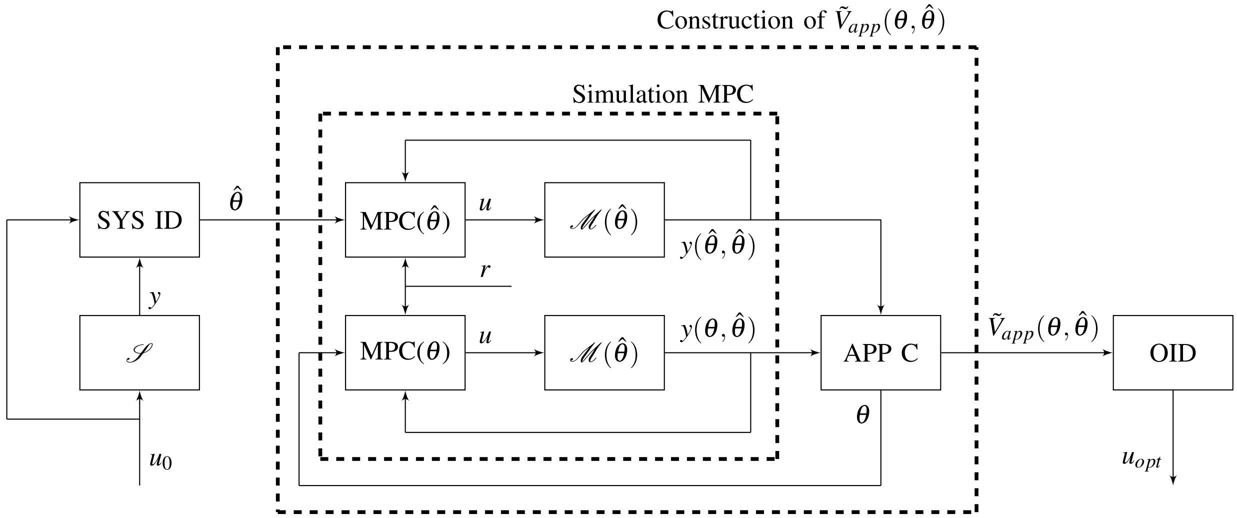

Fig. 1. The method of optimal input design for MPC. The true system ($\mathscr{S}$) is excited by a white noise sequence (u_0). Initial parameter estimates ($\hat{\theta}$) are obtained from system identification (SYS ID). The control performance and how it is effected by different values of the parameters (θ) is examined. This is done by simulating a model of the system ($\mathscr{M}(\hat{\theta})$) controlled with an MPC using $\hat{\theta}$ and another MPC using θ (MPC($\hat{\theta}$) and MPC(θ) respectively). Based on this, the approximate application cost ($\tilde{V}_{app}(\theta,\hat{\theta})$) is calculated (APP C). The application cost and initial estimate is then used in the optimal input design (OID) and the optimal input signal is obtained. The input is optimal if the system coincides with $\mathscr{M}(\hat{\theta})$ but might not be optimal for the system $\mathscr{S}$.

around this are to design inputs that are robust to parameter variations, e.g., [11], or to use an initial parameter estimate instead of the true parameters in the optimal input design. The latter approach will be considered here.

The second challenge relates to the use of time domain constraints in the MPC. There is, as of yet, no good way of including such constraints in the input design formulation that is considered here. The solution here is to include them in the calculation of the application cost but not to consider them in the identification part of the method. It may be possible to enforce some time domain constraints when the optimal signal is generated, e.g., [12].

A. Application cost

A reasonable application cost for the MPC case is the difference between the output of the process controlled by an MPC based on a model using $\theta \neq \theta_o$ and one based on θ_o, denoted $y_t(\theta)$ and $y_t(\theta_o)$, respectively. Therefore, we choose

$$V_{app}(\theta) = \frac{1}{N}\sum_{t=1}^{N} \|y_t(\theta_o) - y_t(\theta)\|, \tag{13}$$

which has the desired properties mentioned in Section II-A.

In an application, it is unlikely that one can evaluate (13) using outputs from the real process, as this would require controlling the process based on models with more or less arbitrary parameter values. Instead, we introduce an approximation of V_{app} where the true system is replaced with the linear model using estimated parameter values. This gives

$$\tilde{V}_{app}(\theta,\hat{\theta}) = \frac{1}{N}\sum_{t=1}^{N} \|y_t(\hat{\theta},\hat{\theta}) - y_t(\theta,\hat{\theta})\|, \tag{14}$$

where the first argument is the parameter used by the MPC and the second argument the parameter used in the linear model replacing the system, *cf. Simulation MPC* block in Figure 1.

The choice of acceptable performance degradation is highly application dependent. Here we consider the reference tracking capability of the MPC using a model with θ_o,

$$V(\theta_o) = \frac{1}{N}\sum_{t=1}^{N} \|y_t(\theta_o) - r_t\|^2, \tag{15}$$

and allow for a certain level of degradation, e.g., a 1 % degradation of the performance corresponds to

$$\gamma = \frac{100}{V(\theta_o)}. \tag{16}$$

B. Input design

The optimal input design should minimize the experimental cost while guaranteeing performance. Quantifying experiment cost is not obvious but some possibilities are

- input power, $\mathrm{var}(u)$,
- input energy, $N\,\mathrm{var}(u)$, or
- experiment length, N.

All measures have their own merits, however here we choose to focus on the first option. To formalize, we can write the full input design problem as

$$\min_{\phi_{u(\omega)}} \quad \mathrm{trace}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\phi_u(\omega)d\omega\right), \tag{17}$$

$$\text{s.t.} \quad \mathscr{E}_{SI} \subseteq \Theta, \tag{18}$$

$$\phi_u(\omega) \geq 0 \quad \forall\omega, \tag{19}$$

where $\phi_u(\omega)$ is the input spectrum. Depending on if we choose the ellipsoidal approximation of Θ or the scenario approach, the set constraint is replaced by (11) or (12), respectively. With linear parameterization of $\phi_u(\omega)$, this optimization can be written as a semi-definite program. For details on this, see e.g., [10].

C. Identification algorithm

We construct an optimal input design and identification method to estimate models to be used in MPC, where the true parameters in the expressions are replaced with estimates thereof. The proposed method is described by the following algorithm and further illustrated in Figure 1.

Algorithm

Step 0 Find an initial estimate of the model parameters using white noise as input in the system identification experiment.

Step 1 Find the application cost based on simulations of the model with the parameter estimates.

Step 2 Design the optimal input signal based on the application cost and parameter estimates.

Step 3 Find a new estimate of the model parameters using the optimal input signal in the system identification experiment.

Note: If a good initial guess of the parameters are available, e.g., through physical insight of the process, this guess can replace the initial estimation in Step 0.

The algorithm can be iterated so that the estimatew from Step 3 are used in Step 1 and 2 to calculate a new input design. As more and more data is used in the identification step and if there exists parameters θ_o such that $\mathscr{S} = \mathscr{M}(\theta_o)$, the estimates will converge to their true values. Therefore, one can expect the input design to converge to what would be obtained had θ_o been known. A discussion on this and a formal proof for the case with ARX systems are found in [13].

IV. WATER TANK PROCESS

We have implemented the method of system identification for MPC described in Section III on a version of the water tank process presented in [14]. It consists of four interconnected water tanks. The layout of the process is shown in Figure 2. The control objective is to regulate the water levels of the two lower tanks, according to a reference trajectory, using MPC. The process is nonlinear, however, a linearized and discretized model of the process is used in the MPC. We want to estimate the parameters of the linearized model.

A. Process description

The four water tanks are connected to two pumps that can deliver water into the tanks. Two valves are used to control the amount of water that is pumped into the upper and lower tanks respectively. The input signals are the voltages of the two pumps and the outputs are the water levels in the two lower tanks. There are a number of physical constraints on the process, such as the input voltages to the pumps and the water levels in the tanks.

We derive a nonlinear model of the process from Torricelli's principle,

$$\begin{aligned}
\frac{dx_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gx_1} + \frac{a_3}{A_1}\sqrt{2gx_3} + \frac{\gamma_1 k_1}{A_1}u_1,\\
\frac{dx_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gx_2} + \frac{a_4}{A_2}\sqrt{2gx_4} + \frac{\gamma_2 k_2}{A_2}u_2,\\
\frac{dx_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gx_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2,\\
\frac{dx_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gx_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1,
\end{aligned}$$

where x_i is the water level in centimeters of tank i and u_j is the voltage in volt of pump j. The parameters of the process and their nominal values can be found in Table I.

TABLE I

PHYSICAL PARAMETERS OF THE FOUR TANK PROCESS.

Parameter	Nominal	Description
a_i	{0.17 0.15 0.11 0.08} cm^2	area of outlet of tank i
A_i	15.5 cm^2	area of tank i
γ_j	0.625	parameter of valve j
k_j	4.14 cm^3/(sV)	parameter of pump j

B. Linear Model

We derive a linear and time discrete model of the process that will be used for predictions in the MPC. The nonlinear model is linearized around its equlibrium points, x^0 and u^0, giving the state space model

$$\frac{d\bar{x}}{dt} = \begin{bmatrix} \frac{-1}{\tau_1} & 0 & \frac{A_3}{A_1\tau_3} & 0 \\ 0 & \frac{-1}{\tau_2} & 0 & \frac{A_4}{A_2\tau_4} \\ 0 & 0 & \frac{-1}{\tau_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_3} \end{bmatrix} \bar{x}_t + \begin{bmatrix} \frac{\gamma_1}{A_1} & 0 \\ 0 & \frac{\gamma_2}{A_2} \\ 0 & \frac{(1-\gamma_2)}{A_3} \\ \frac{(1-\gamma_1)}{A_4} & 0 \end{bmatrix} \bar{u}_t, \tag{20}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \bar{x}_t + e_t, \tag{21}$$

where $\bar{x} = x - x^0$, $\bar{u} = u - u^0$ and $\tau_i = \frac{A_i}{a_i}\sqrt{\frac{2x_i^0}{g}}$. The measurement noise e_t is assumed to be zero-mean Gaussian with covariance matrix R_e. The equilibrium points of our process are $x^0 = [15\ 15\ 3\ 12]^T$ cm and $u^0 = [7.8\ 5.25]^T$ V.

The linear model is then discretized assuming zero-order hold sampling at a sampling rate of $T_s = 1$ Hz. The discretized model is used in the MPC.

The parameters to be estimated in the identification experiment are the physical parameters presented in Table I. The equilibrium points and gravity are considered known and hence the factor $\sqrt{2x_i^0/g}$ in τ_i, $i = 1, \ldots, 4$, is also known.

C. Control Strategy

The objective of the controller is to perform reference tracking of the water levels in the two lower tanks. The controller implemented is the MPC provided by the MPC Toolbox in Matlab. The MPC constructs an optimal control strategy by minimizing the cost function defined by (2), with the deviation variables used instead of $\hat{y}$, r, Δu and u, subject to the constraints of the process. These constraints are listed in Table II.

TABLE II

PHYSICAL CONSTRAINTS OF THE FOUR TANK PROCESS.

Parameter	Limit	Description
$x_{i,max}$	25 cm	maximum water level of tank i
$x_{i,min}$	0 cm	minimum water level of tank i
$u_{j,max}$	15 V	maximum voltage of pump j
$u_{j,min}$	0 V	minimum voltage of pump j

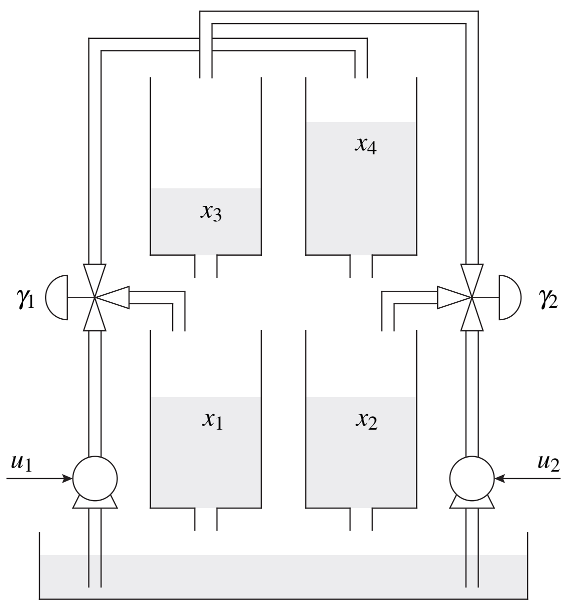

Fig. 2. The four tank MIMO process. Water is pumped from the reservoir into the four tanks. The voltages to the pumps are input signals and the levels in the two lower tanks, x_1 and x_2, are measured outputs. The setting of the two valves regulate how much water is pumped into the upper and lower tank respectively.

V. OPTIMAL INPUT DESIGN EXAMPLE

In this section we evaluate the proposed method on simulations of the water tank process presented in Section IV. The optimal input design is found and identified models are evaluated in control simulations.

A. Simulation Setup

The MPC prediction and control horizons are 10 time steps. There are no constraints on the input rate in the MPC control problem and the covariance matrix R_e is set to zero for the calculations of the scenarios in the input design. For all other settings we use the default values provided by the MPC Toolbox in Matlab (R2010a). The optimal input design problem is (17) using the scenario approach, i.e., constraint (12). The problem is solved with $R_e = 10^{-3} I_{\{2\times 2\}}$, $N = 400$ and κ from the $\chi^2(10)$-distribution with $\alpha = 0.95$. The number of scenarios used is 3,000.

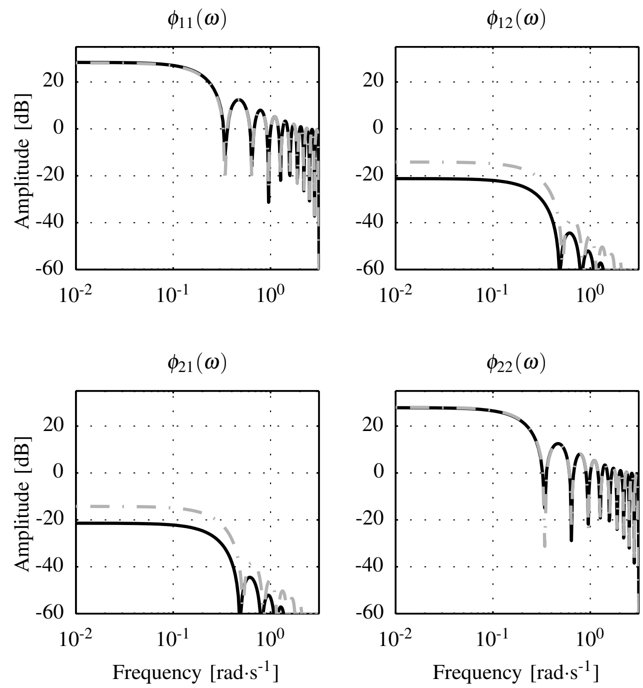

Fig. 3. The input spectra obtained using optimal input design based on θ_o (solid) and an initial estimate of the parameters (dashed). $\phi_{ij}(\omega)$ is the cross spectrum between u_i and u_j.

B. Input Design

We compare the optimal design obtained when using θ_o and one where the design is based on an initial estimate of the parameter values. These estimates were obtained using a zero mean white Gaussian excitation signal with variance 0.01. The optimal input spectra for 1 % performance degradation, i.e., γ given by (15) is shown in Figure 3. The optimization problem is implemented in CVX and solved using SDPT3 [15], [16].

We see that the optimal spectrum is clearly temporally colored but almost spatially white. The spectrum has high energy at low frequencies, indicating that the static gain of the system is important. This is expected since the application cost relates to reference tracking and therefore emphasizes the static gain. The design obtained from the initial estimate is very close to the optimal.

We also constructed the Hessian of the application cost, to see which directions of the model parameters give a high performance degradation. We can conclude that it is most important to estimate γ_1 with high accuracy. This seems reasonable since the pump corresponding to γ_1 is supplying water to tank 1 and tank 4, which have the largest outflows of the process. Thus, an error in γ_1 would highly effect the control performance. We can also see that it is important to estimate the parameters related to tanks 2 and 4 of the process, i.e., a_2, a_4 and γ_2.

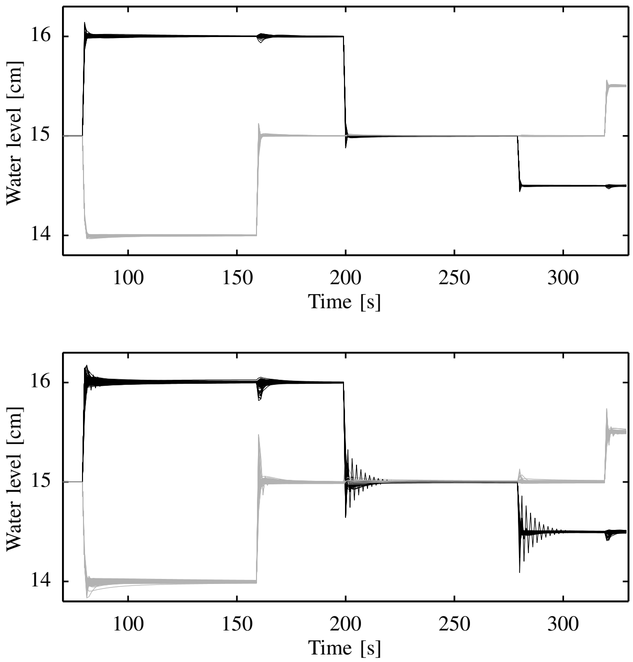

Fig. 4. The trajectories of the process controlled by MPC with models based on estimates from 100 identification experiments. In the upper plot the optimal input has been used in the identification whereas in the lower plot a white input has been used. For the upper plot 91 % of the trajectories satisfy the performance degradation requirements while only 15 % of the trajectories in the lower plot satisfy the requirements.

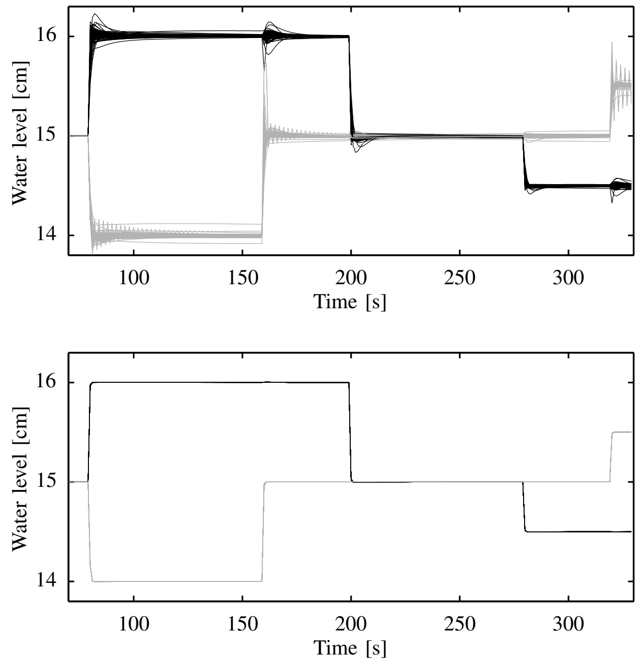

Fig. 5. The trajectories of the process controlled with MPC with models based on estimates from 100 different identification experiments. In the upper plot, only $N = 100$ samples are used to identify the system whereas in the lower plot $N = 10,000$ samples are used in the identification. The higher input power in the first case drives the system away from the linearization point, giving estimates with high variance. Both experiments have the same total input energy.

C. Control Performance Comparison

As a motivation to why we perform optimal input design, we will estimate the water tank process using an optimal input with minimum power and a white noise input with the same power. We then compare the performance of the MPC controllers based on these estimates. The simulation is performed with the setting specified in Section V-A and γ is defined by (16). The resulting output trajectories can be seen in Figure 4.

We can conclude that the optimal input signal outperforms white noise in terms of satisfying the specification on control performance. In total 91 % of the models estimated using the optimal input satisfy requirement (3) compared to only 15 % of the models estimated with white noise.

The reason that we do not reach the goal of 95 % acceptable models is, at least to some extent, explained by the fact that the identified system is nonlinear and no θ_o exists. If a linear system is estimated instead, using the same signal realizations, 94 % of the models are deemed acceptable, which is closer to the specifications.

D. Reducing Performance Degradation

We look at the effects of decreasing the upper limit on the performance degradation when a linear model is used for a nonlinear process. To decrease the application cost, we want estimates with lower variance. Had the process been linear, increasing input power, $\text{var}(u)$, or the experiment length, N, would both reduce the estimate variance. We can always trade power for experiment time or vice versa. However, when the process is nonlinear, increasing input power might drive the process too far from the linearization point for the model to be accurate. Therefore, one might have to increase experiment time to reduce the variance.

To investigate this for the four tank process, we conduct two experiments. In the first experiment we use $N = 100$ in the input design which gives a high input power solution. In the second experiment we use $N = 10,000$ which gives a low power solution. Note that both designs use the same input energy. We allow for 0.01 % performance degradation, i.e., $\gamma = 10,000/V(\theta_o)$. Figure 5 the resulting trajectories, when the estimated models from the two experiments are used in the MPC.

We see that increasing the power can degrade the quality of the estimates when the identified plant is nonlinear. If experiment time is allowed to increase, higher quality estimates are obtained. In total 85 % of the models from the low input power identification satisfy the requirements while none of the estimates from the high input power identification satisfy the requirements.

VI. Conclusion

We present a method for optimal input design for MPC. The identified model is guaranteed, with high probability, to give a prescribed control performance. The method thereby links the system identification and the intended use of the model.

The optimal input design method requires knowledge of the true parameter values. These are obviously not available. The proposed solution is to use an initial estimate instead. These can be obtained in an identification experiment or through knowledge of the system.

The proposed method requires an evaluation of the control performance degradation with respect to the model parameters. This evaluation may greatly effect the behavior of the system, thus preventing it from being performed on-line. We propose that it is instead based on simulations of a model of the system. We use an estimated linear model to approximate the process, even if the true system is nonlinear.

Our example shows that we have to be careful when trading power of the input signal with number of observations made in the identification experiment. This is because we use a linear model when designing the input but identify a nonlinear process. A high variance of the input signal may drive the process state far from its linearization point and thus the model will no longer be accurate.

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