ROAD GRADE ESTIMATION RESULTS USING SENSOR AND DATA FUSION

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ABSTRACT

Advanced driver assistance systems for heavy duty vehicles, such as look-ahead cruise and gearshift controllers, rely on high quality map data. Current digital maps do not offer the required level of road grade information. This contribution presents an algorithm for on-board road grade estimation based on fusion of GPS and vehicle sensor data with measurements from previous runs over the same road segment. An incremental update scheme is utilized to ensure that data storage requirements are independent of the number of measurement runs. Results of the implemented system based on six traversals of a known road with three different vehicles are presented.

INTRODUCTION

Several algorithms in today's heavy duty vehicle (HDV) embedded systems are based on state information of the vehicle. Such state estimates are traditionally obtained from one or more sensors in the vehicle. In addition to the traditional applications new advanced driver assistance systems under development predict the future dynamics of the vehicle. These predictions combine the current state of the vehicle with, stored or sensed, information about the road ahead e.g., the topology, the curvature or the traffic situation.

Prediction of the behavior of the vehicle over significant distances requires high quality map data. Highway speed optimization for heavy duty vehicles often requires prediction of the vehicle dynamics for the next kilometre or more. The maps can either be bought or obtained by own measurements. In order to get adequate data quality from a single drive over the road it is necessary to use sophisticated measurement equipment. Combining sensor fusion, based on several ordinary sensors, and data fusion of multiple measurement runs obtained for the same road segment, it is possible to approach the required quality level. This paper outlines such a system suitable for HDVs which travel the same highways frequently and characterizes its performance. The system is based on standard mounted sensors and a GPS receiver.

RELATED WORK

How knowledge about the upcoming road topology can be used for optimizing the vehicle's speed profile with respect to fuel consumption has been presented in several recent contributions e.g., [1, 2, 3, 4].In this line of work knowledge about the road grade ahead of the truck is assumed to be available e.g., via a map. As previously mentioned, this contribution considers the task of developing a system for creating road grade maps for this type of application. The idea is to create a system that merges sensor data from several data sequences measured on a road segment into a map. An earlier version of the method described herein, without the enhancements derived from the latest experiments, can be found in [5]. That account contains more details about the models and Kalman filtering.

Similar ideas have been presented by Schroedel et. al. [6] and Brüntrup et. al. [7] where data mining is used to automatically create road network maps from a (large) collection of individual GPS traces. Neither of these sources however explicitly address road grade estimation or the possibility to use a vehicle model to improve estimation quality.

There are several different methods proposed in the literature for estimation of the road grade. One way is to use a sensor that is directly related to the grade. Such a solution is presented by Bae et al [8] where the grade is determined using a GPS receiver by calculating the ratio between the vehicle's vertical velocity and its horizontal velocity. A GPS receiver needs good satellite coverage to obtain decent estimates. This is however, a constraint that is difficult to sustain in areas with buildings, trees, tunnels or other large objects. There are also methods that recursively estimates the road grade without using direct sensor information. Lingman and Schmidtbauer [9] presents a method where the road grade is estimated using a Kalman filter-based on measured or estimated propulsion force, estimated retardation forces and measured velocity. Vahidi et. al. [10] presents a similar method where the grade is estimated using Recursive Least Squares-based on a simple motion model. An advantage with these methods is that no extra sensors are required. There are however certain occasions when these two methods fail, or have major difficulties, to deliver reliable estimates e.g., when the friction brakes are applied or when gearshifts are performed.

CONTRIBUTION

This paper presents a method to estimate the road grade-based on standard mounted HDV sensors and a GPS unit along with measured results from an implementation of the method. The performance of the system is investigated, and key considerations are highlighted with measured data. The behavior of the method when either GPS or vehicle model data are unavailable is presented. The combined road grade estimate from six measurements with three vehicles is compared to independent reference road data. A major contribution in the method is the spatial sampling of the sensor fusion estimate which through the estimate error covariance matrix enables data fusion of an arbitrary number of measurement series at difference time instants.

MODELS AND MEASUREMENTS

MODELS

In order to do model-based sensor fusion it is necessary to establish some basic relationships between the various signals that are available for measurement and the quantities to be estimated. The used model is divided into two parts. The first one describes the longitudinal movement of the vehicle. The second part describes the topology of the road by relating the altitude with the grade and the speed of the vehicle.

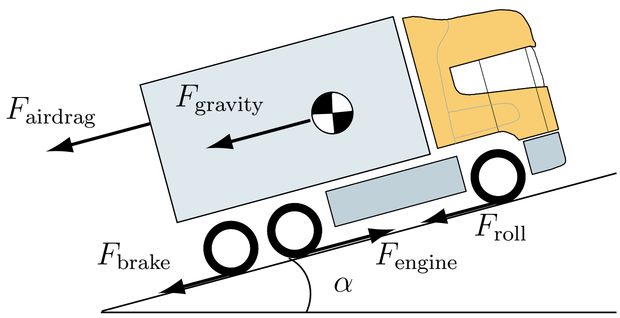

Figure 1: Longitudinal forces acting on the vehicle.

Information about the road grade can be obtained from the engine loading and velocity behavior of the vehicle. A principal sketch of the longitudinal forces acting on a HDV is shown in Figure 1. F_{engine} is the net pull force produced by the engine, F_{brake} is the applied brake force, F_{airdrag} is the air drag, F_{roll} is the rolling resistance and F_{gravity} is the gravity induced force. Using Newton's law of motion the force balance for the HDV in Figure 1 is given by

$$\dot{v} = \frac{1}{m_{\text{t}}} \left(F_{\text{engine}} - F_{\text{brake}} - F_{\text{airdrag}} - F_{\text{roll}} - F_{\text{gravity}} \right) \tag{1}$$

where m_{t} is the total accelerated mass, v is the velocity and α denotes the road grade. For more details on this model refer to [5, 11]

Two states are used to describe the topology of the road, the altitude z and the grade α. The dynamics for these two states are modeled as

$$\begin{aligned} \dot{z}(t) &= v(t) \sin \alpha(t) \\ \dot{\alpha}(t) &= 0 \end{aligned} . \tag{2}$$

Since the data fusion method used utilizes spatially sampled measurements the time domain relations (1) and (2) have to be expressed in the spatial domain instead. A combination of the spatial versions of (1)-(2) together with a first order Euler approximation yields a discrete spatially sampled model with sampling distance Δs as

$$\underbrace{\begin{bmatrix} v_k \\ z_k \\ \alpha_k \end{bmatrix}}_{x_k} = \underbrace{\begin{bmatrix} v_{k-1} + \Delta s \Delta v_k \\ z_{k-1} + \Delta s \sin \alpha_{k-1} \\ \alpha_{k-1} \end{bmatrix}}_{f(x_{k-1})} + \underbrace{\begin{bmatrix} w_k^v \\ w_k^h \\ w_k^\alpha \end{bmatrix}}_{w_k} \tag{3}$$

where process noise w_k has been added to capture the uncertainty in the model. Subscript k denotes the discrete sample number. Through modeling of the forces in Figure 1 the change in velocity Δv_k during the travel from the previous sample point is given by

$$\Delta v_k = c_1 \frac{M_{k-1}}{v_{k-1}} - c_2 v_{k-1} - c_3 \frac{1}{v_{k-1}} - c_4 \frac{\sin(\alpha_{k-1})}{v_{k-1}}$$

where $c_1, ..., c_4$ are vehicle parameters and M is the net driveline torque.

MEASUREMENTS

This section describes the measured quantities and their relation to the states in (3). Two sequences of data from measurements on the Swedish highway E4 are shown in Figure 2.

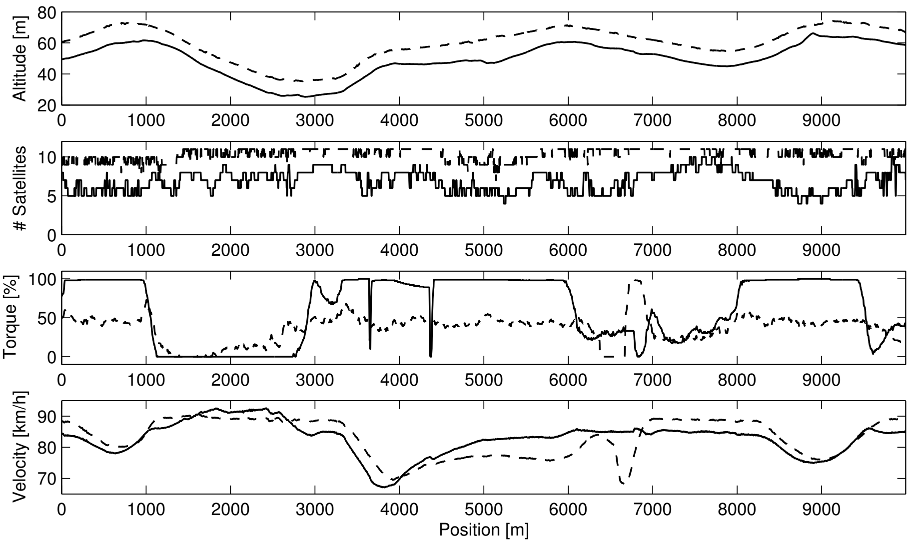

Figure 2: Source data from measurements 2 (solid) and 6 (dashed) on a segment of highway E4 south of Södertälje. From the top the plots show the GPS altitude signal, the number of active satellites, the engine torque and the measured mean front axle speed.

A GPS is used to record latitude, longitude, velocity, altitude and the number of active satellites. The latitude, longitude and velocity signals are used to resample the recorded measurements from the original time indexed to a distance indexed form. The altitude signal is used in the road grade estimation with the sensor model

$$z_k^{\mathrm{GPS}} = z_k + e_k^{\mathrm{zGPS}}. \tag{4}$$

Here e_k^{zGPS} is used to represent stochastic measurement noise.

From standard internal sensors in the vehicle the velocity, net engine torque, brake system usage, current gear and gear shifts are recorded. The vehicle velocity is denoted v^{veh} and is obtained from the wheel speed sensors. The filter measurement equation becomes

$$y_k = \begin{bmatrix} v_k^{\mathrm{veh}} \\ z_k^{\mathrm{GPS}} \end{bmatrix} = \underbrace{\begin{bmatrix} v_k \\ z_k \end{bmatrix}}_{h(x_k)} + \underbrace{\begin{bmatrix} e_k^{\mathrm{vveh}} \\ e_k^{\mathrm{zGPS}} \end{bmatrix}}_{e_k}. \tag{5}$$

ROAD GRADE ESTIMATION

Combining the previous sections the measurements and the state update are described by the state-space system

$$\begin{aligned} x_k &= f(x_{k-1}) + w_k \\ y_k &= h(x_k) + e_k \end{aligned} \tag{6}$$

cf., (3) and (5). It is assumed that the noise sources w_k and e_k can be represented by zero-mean white Gaussian noise processes. With this assumption, extended Kalman filtering [12] provides a method for estimation of the state vector x_k based on the measurements y_k. This method will be used to create an estimate of the road grade for each measurement run. The extended Kalman filter is defined by the system equations (6) together with the covariances for w_k and e_k. An overview of the data flow in the proposed method is given in Figure 3

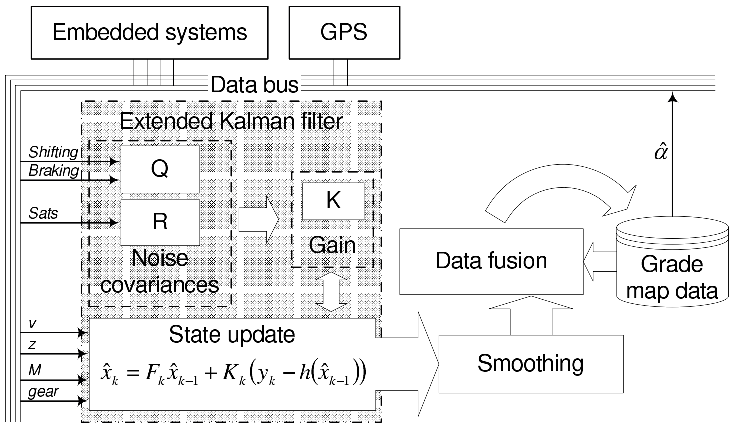

Figure 3: Overview of filter architecture for road grade estimation. Information from the vehicle's data bus is integrated using an extended Kalman filter, followed by a smoothing algorithm. The smoothed data are then merged with pre-existing data sequences. Information is delivered to the data bus from on-board embedded systems and a GPS receiver. The grade estimate is denoted $\hat{\alpha}$.

KALMAN FILTERING AND SMOOTHING

In extended Kalman filtering the non-linear system is linearized around the current trajectory. The standard recursions for Kalman filtering are then applied on the linearized system. These recursions are described by two update steps: a time update and a measurement update. In the first step, the time update, the state estimate $\hat{x}_{k-1}$ and the error covariance P_{k-1} are updated according to

$$\begin{aligned} F_k &:= \frac{\partial f}{\partial x}(\hat{x}_{k-1}) \\ \hat{x}_k &:= F_k \hat{x}_{k-1} \\ P_k &:= F_k P_{k-1} F_k^T + Q_k \end{aligned}$$

Event	Effect
Friction brake use	Driveline torque unknown
Auxiliary brake use	Uncertain driveline torque
Gear shifts	Unmodelled driveline dynamics
GPS signal masking	No or low quality GPS data

Table 1: Recorded events which are taken into account when determining the time varying process noise covariance (Q_k) and measurement noise covariance (R_k).

where Q_k is the covariance of the process noise w_k. The second step is a measurement update where the estimate is corrected based on the measurements according to

$$\begin{aligned} H_k &:= \frac{\partial h}{\partial x}(\hat{x}_k) \\ K_k &:= P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} \\ \hat{x}_k &:= \hat{x}_k + K_k(y_k - h(\hat{x}_k)) \\ P_k &:= P_k - K_k H_k P_k \end{aligned}$$

where R_k is the covariance of the measurement noise e_k.

By completing the measurement of a road segment before application of the sensor and data fusion, smoothing can be used to compensate for the filtering delay and include information from future sampling points in each estimate. In this work the Rauch-Tung-Striebel fixed point smoothing algorithm has been used. The algorithm is described in for example [13]. The smoothing is applied as a backwards recursion and the filtered estimates $\hat{x}_k$ and the estimated error covariance matrices P_k are used in the process. The smoothed state estimates $\hat{x}_k^{\mathrm{s}}$ and corresponding smoothed estimated error covariance matrices P_k^{s} are later used when several measurements are combined. The final state of the filter recursion is used as initialization for the smoothing backwards recursion

$$\begin{aligned} F_k^{\mathrm{s}} &:= P_k F_k^T P_{k+1}^{-1} \\ \hat{x}_k^{\mathrm{s}} &:= \hat{x}_k + F_k^{\mathrm{s}}(\hat{x}_{k+1}^{\mathrm{s}} - \hat{x}_{k+1}) \\ P_k^{\mathrm{s}} &:= P_k + F_k^{\mathrm{s}}(P_{k+1}^{\mathrm{s}} - P_{k+1}) F_k^{\mathrm{s}T}. \end{aligned}$$

COVARIANCE MATRICES

The implementation is now straightforward, with the exception of choosing noise covariances. To simply the design it is assumed that the covariance matrices are diagonal. The true system and measurement covariances may change while driving. An attempt to account for this is made by setting up rules for adjusting Q_k and R_k based on the events listed in Table . For detection of these events the recorded signals from the vehicle are used. The estimated time varying error covariance P_k^{s} will contain confidence information for the estimate at each sample point. This information is useful in the data fusion step.

Signal masking in the GPS-receiver, i.e. reduced number of available satellites due to obstacles in the vicinity has severe effects on the quality of the measurements of the GPS-receiver. Especially the precision of the altitude estimate is dependent on the number available satellites (and also their relative positions, although this effect is not considered here). Therefore the size of the variance of e^{zGPS} is varied based on the current number of visible satellites. When the satellite count drops below four, altitude determination is not possible with GPS, and a very high variance is set.

A similar reasoning can be applied for braking and shifting, with the addition that these affect the vehicle behavior and thus the process noise as well. It is difficult to estimate the brake force that acts on the vehicle. As a consequence, the model for the longitudinal dynamics becomes uncertain. A way to handle this is to increase the process noise w_k^{v}, whenever the brake system is engaged. Using wheel rotation to determine the vehicle speed becomes less reliable during braking, since the amount of slip changes and even lockups can occur. During gear shifts the produced torques in the driveline are difficult to model, large oscillations which are not included in the relatively simple driveline model may appear.

DATA FUSION

To optimally use all the road information in several measurement a large Kalman filter could be used. This approach would require all the data to be considered at the same time, but could be feasible in a one shot map creation scenario. In order to incrementally build a map however, it is desirable to have to store as little as possible from the source data for future iterations. In this work a weighted average is used to incrementally create a total estimate based on one estimate from previous measurements and one new measurement run. This limits the storage requirement to the current set of state estimates and their estimated error covariances. When a new measurement sequence becomes available it is used to find a new estimate of the grade profile, with an associated error covariance. The approach is inspired by the general fusion formula described in e.g. [14].

$$\begin{aligned} P_k^{\mathrm{f}} &:= ((P_k^1)^{-1} + (P_k^2)^{-1})^{-1} \\ \hat{x}_k^{\mathrm{f}} &:= P_k^{\mathrm{f}}((P_k^1)^{-1}\hat{x}_k^1 + (P_k^2)^{-1}\hat{x}_k^2). \end{aligned}$$

Here $\hat{x}_k^{\mathrm{f}}, P_k^{\mathrm{f}}$ denotes the resulting fused state and error covariances, and $\hat{x}_k^1, P_k^1$ and $\hat{x}_k^2, P_k^2$ denote the source quantities. Two of the states used in the road grade estimation, z_k and α_k, are directly related to the road. The third state, v_k, describes the measurement vehicle, and is not constant between measurements. Only the states describing the road are used in the data fusion process, giving $\hat{x}_k^{\mathrm{f}} = \begin{bmatrix}\hat{z}_k^{\mathrm{f}} & \hat{\alpha}_k^{\mathrm{f}}\end{bmatrix}^T$. If two overlapping data sequences are being merged $\hat{x}_k^1, P_k^1$ and $\hat{x}_k^2, P_k^2$ consist of the relevant elements of the smoothed estimates $\hat{x}_k^{\mathrm{s}}$ and P_k^{s}. If a new measurement is merged with an existing map (based on two or more previously merged overlapping data sequences) one of the source estimates is the smoothed new measurement data, and the other source estimate is the map.

The experiments show that using the error cross covariances in the off-diagonal elements of the error covariance matrices actually decrease the quality of the data fusion. This effect seems to come from the relatively large uncertainty in the absolute altitude estimate. As can be seen in Figure 2 (and deduced from GPS receiver specifications) the absolute error can be tens of meters even under good conditions. The relative altitude precision is generally much better. To prevent large absolute altitude uncertainties from interfering

Vehicle	Configuration	Weight [t]	Axles	Used for measurements
A	Tractor and semi-trailer	39	5	1,2,3
B	Tractor	13	2	4,5
C	Rigid truck	21	3	6

Table 2: Test vehicles used to collect experiment data for the proposed road grade estimation algorithm.

with the slope estimates currently only the error covariance matrix elements on the diagonal are used, effectively yielding a scalar fusion for each of the states. The estimated error covariance of a fused estimate will be lower than that of any of the source data sets. If the errors in different measurement runs are not independent as assumed, the fused P_k^{f} will be an underestimate. Segments of each source estimate with high error covariance estimates will have less weight in the calculation. As a result measurements obtained during periods of braking or loss of satellite coverage in one of the data sequences will not destroy higher quality information in the other. A problem to consider is that inherently difficult sections, such as downhill tunnel segments will still be very hard to estimate accurately, since any measurement will most likely contain low-quality estimates.

RESULTS

The measurements have been performed with three different Scania test vehicles driving on the same stretch of road. The three vehicles differ in their configuration and the experiments have been conducted on different days. Separate sets of parameters for the truck model in the EKF have been used for the different vehicles. Due to the differences between the vehicles types and varying weight, model errors are likely to vary somewhat between vehicles. The experimental data thus includes some of the uncertainties which would affect a real world system used in several vehicles with varying weight and environmental conditions. Key parameters for the test vehicles are given in Table 2. Most of the collected data have been logged from the on-board CAN bus. Since there is no standard mounted GPS in today's trucks an external VBOX GPS unit connected to a second CAN channel has been used. The measurements used in this paper represent 10km of the southbound E4 highway just south of Södertälje, Sweden.

ALGORITHM PERFORMANCE

Absolute vehicle position recorded from the GPS was used to syncronize the different measurements. Positioning errors and differing travel paths on the roadway limit the synchronization accuracy. A reference position from the first measurement has been used find a common starting point in all measurements. Apart from that fixed point only the distance traveled is used for positioning, which limits the useable segment length for study to 10-15 km. Longer segments will have drifted too much in the later part. All the figures in this paper are based on the same road segment and share a common distance scale for easy cross-referencing. An independent road grade profile measured using a specialized survey vehicle is used for validation of the results. This measurement lacks absolute coordinates in any dimension. The position along the road is found from minimizing the difference between the reference and estimated road profiles, and the relative altitude is obtained from numerical integration of the slopes. Figure 4 shows a close-up of the agreement between the reference road grade profile and the merged estimate based on all six measurement runs. There is a good overall agreement with a mean squared difference of 0.13% slope for this segment.

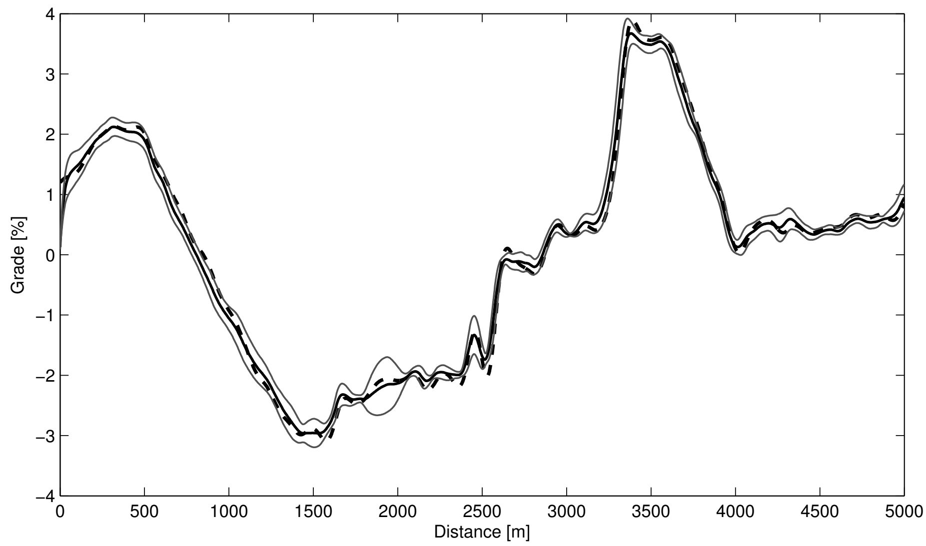

Figure 4: Comparison of final merged grade estimate (solid) with reference from specialized road grade measurement car (dashed). The thin lines flanking the road grade estimate illustrate the numerically calculated one standard deviation distance from the grade estimate, for each sample point based on the six measurements.

The test road contains one steep downhill slope section where ultimately vehicle A needs to apply a brake force to avoid over speeding. The steep portion of the slope lasts approximately from 1000 m to 2600 m. During the braking the torque in the vehicle model is described as unknown. Increased modeled process noise, Q_k, in the velocity state indicates less reliance on the accuracy of the vehicle model, and leads to a higher estimated slope error covariance. Further down the road around the 4000 m mark there is an uphill section where vehicle A needs to change gears. Vehicle C encounters some congestions and uses the brakes to slow down after about 6600 m. All these events and their influence on the estimated slope error covariance $P^{\mathrm{s}}_{k(3,3)}$ for measurements 3 and 6 are shown in Figure 5.

The descent described above ist the most challenging part of the studied road segment Figure 6 shows a close-up of all six smoothed grade estimates together with a simple mean and the final merged estimate from the proposed algorithm. Vehicle B and C are not heavy enough to need a brake force applied during the descent, and thus their estimates will have a higher weight in the merging. Particularly the second measurement yields a bad grade estimate in this section, and it can be seen that the effect from this on the merged result is less adverse than it is on the simple mean.

The GPS altitude information reduces the appearance of a grade estimate bias from mod-

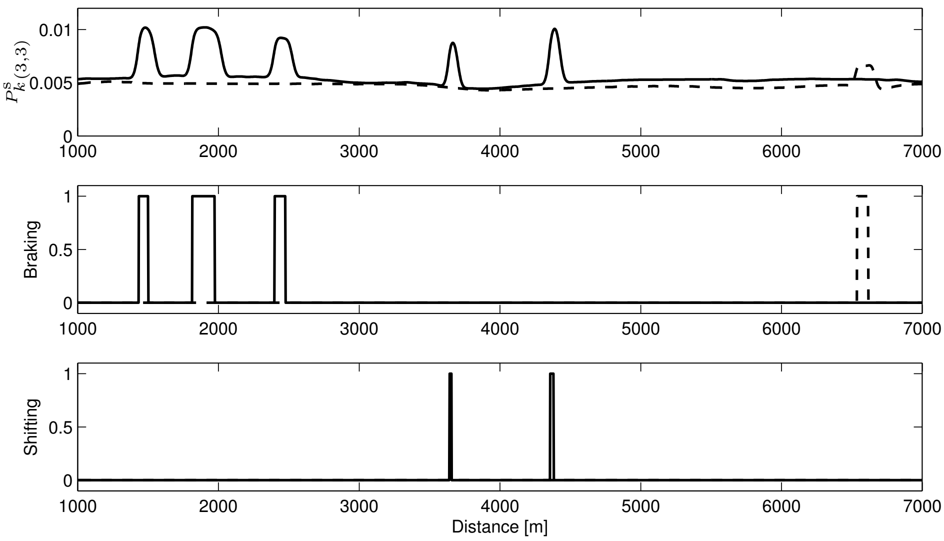

Figure 5: When braking or shifting gears the estimated slope covariance (top subfigure) will increase, which reduces the weight for the measurement in the merge step. The events used in the algorithm are indicated in the second (braking) and third (shifting) subfigures. The data shown are from measurements two (vehicle A, solid) and six (vehicle C, dashed).

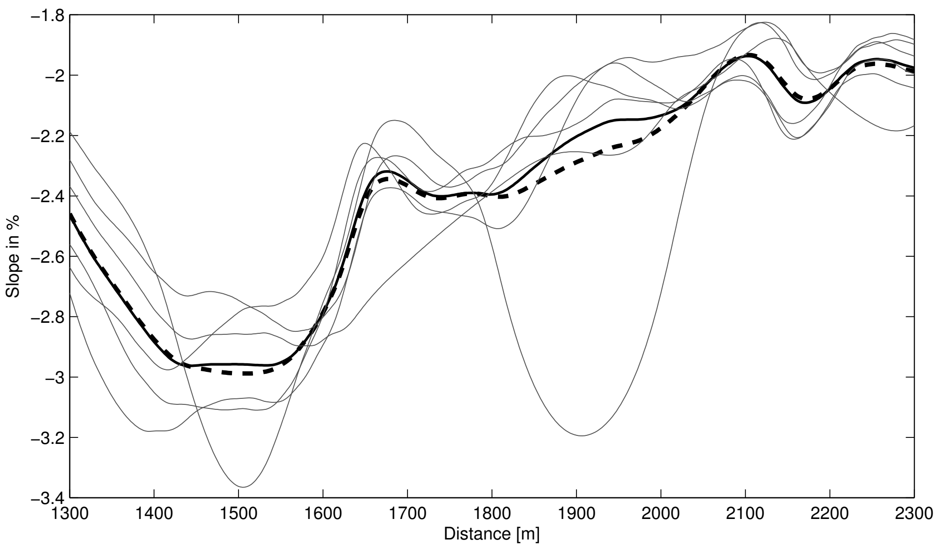

Figure 6: The final merged grade estimate (solid) is shown together with a simple mean of the individual smoothed estimates (dashed) for the difficult downhill section of the test road. When a vehicle brekes or shifts gears the estimated slope error covariance goes up, which reduces the weight in the merge. Out of the six smoothed grade estimates (thin, solid) the one based on measurement two is particularly at odds with the consensus, which is due to a GPS altitude signal disturbance during braking. During braking the GPS is more important than normal in the filter. Since the brake system was engaged, the disturbed estimate has a lower weight in the merge than in the simple mean, as can be seen from 1400 m to 1600 m and from 1800 m to 2000 m.

eling or model parameter errors. To investigate the influence of GPS altitude information on the grade estimate a version of the filter modified to use only the vehicle velocity as measured input has been used. A comparison of estimated road grade and altitude profiles obtained with the GPS enabled and disable is shown in Figure 7. The estimate from the filter without GPS information and the survey vehicle integrated reference altitude are initialized at the same altitude as the GPS based filter in order to make the comparison. The presented data are based on measurement two with the fully loaded tractor semi-trailer combination (vehicle A). This is the vehicle type best described by the vehicle model, the observed drift without GPS data with the other two vehicles was two to five times larger.

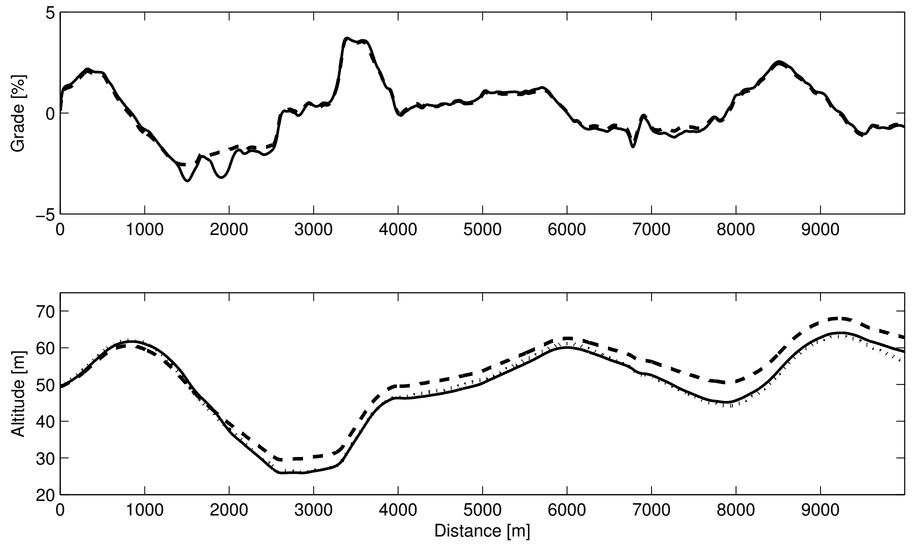

Figure 7: The first plot shows the road grade estimate for measurement two with (solid) and without (dashed) the GPS enabled. Without the GPS some model or parameter error on average causes a slight bias in the road grade. It can also be seen that a GPS signal disturbance while braking in the downhill section has a significant impact on the estimated grade. The second plot illustrates the grade estimate bias without GPS by showing the estimated altitude with (solid) and without (dashed) the GPS enabled together with the calculated altitude from the reference measurement (dotted).

CONCLUSIONS

The proposed method used on overlapping, vehicle sensor and GPS, data sequences can produce results which are very similar to a single pass reference measurement using a specialized measurement vehicle. The scheme to vary the process noise and measurement error covariance matrices depending on additional information from the measurement allows the filter to use the best information from multiple overlapping data sequences for estimating a particular road section. The performance of the algorithm depends on the accuracy of the altitude measurement of the GPS. An area of future work is to compare the performance when using a standard quality vehicle mounted GPS to the GPS unit used here. Further development of the methods used to select the covariances R_k and Q_k can hopefully increase the robustness of the method.

ACKNOWLEDGEMENTS

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