Fuel-efficient control of merging maneuvers for heavy-duty vehicle platooning

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Abstract— **The formation of groups of closely-spaced heavy-duty vehicles, known as platoons, reduces the overall aerodynamic drag and therefore leads to reduced fuel consumption and reduced greenhouse gas emissions. This paper focuses on the optimal control of merging maneuvers for the formation of a growing platoon. Hereto, the merging problem is formulated as a hybrid optimal control problem and an algorithm for the computation of optimal merging times and corresponding optimal vehicle trajectories is developed by exploiting an extension of Pontryagin's maximum principle. Moreover, a model predictive control approach on the basis of this algorithm is presented that makes the merging maneuvers robust to modelling uncertainties and external disturbances. The results are illustrated by evaluating a scenario involving three vehicles, which indicates fuel savings of up to 13% with respect to the vehicles driving alone.**

I. Introduction

The road freight transportation sector is facing large challenges due to increasing fuel prices and the need to reduce harmful greenhouse gas emissions. An approach to (partially) address these challenges is heavy-duty vehicle platooning, where groups (known as platoons) of vehicles with small inter-vehicular distances are formed to reduce the overall aerodynamic resistance. This cooperative approach towards freight transportation has been enabled by advances in wireless communication technology and it has been shown experimentally that the formation of platoons can yield a reduction in fuel consumption of up to $10\,\%$ (see [1] and [3]). Moreover, the operation of heavy-duty vehicles in a platoon has the potential for a better utilization of the existing road infrastructure due to the small inter-vehicular distances.

Most existing research on the topic of heavy-duty vehicle platooning has focused on topics related to the optimal control of vehicles in a platoon, which is motivated by the observation that automation is required to safely maintain the small inter-vehicular distances required to achieve a significant reduction in aerodynamic drag. Examples of such control strategies are given by [5], [8], [12], whereas other works focus on topics ranging from the influence of the inter-vehicular spacing policy [14] or road topography [15] to stability properties [9]. These works, however, have in common that they assume that the vehicles are already in a platoon. The coordination and formation of platoons has received considerable less attention in literature, with exceptions given by [6] and [16]. In [6], conditions are given under which it is beneficial for a heavy-duty vehicle to catch up with an existing platoon, whereas a large-scale optimization approach for the formation of platoons on a road network is presented in [16]. Thus, these works focus on deciding whether to form a platoon or not, but the actual formation of the platoons through the execution of merging maneuvers on the road is not considered.

The current paper addresses this topic by targeting the computation of fuel-optimal merging maneuvers, hereby assuming that the decision to form a platoon has already been made. In particular, a representative scenario is examined in which subsequent merging maneuvers are considered to form a growing platoon, but the results developed in this paper can be easily extended to cover more elaborate merging scenarios (e.g., by merging and formation of sub-platoons). The main contributions are as follows.

First, the merging problem is formally defined and cast as an optimal control problem aimed at the minimization of the total fuel consumption.

Second, an approach for the computation of the fuel-optimal trajectories is developed by exploiting theory on hybrid optimal control [13], [11]. Namely, as the dynamics of the platoon changes with the addition of vehicles, standard optimal control techniques can not be applied. Instead, the problem is decomposed into sections in which the composition of the platoon is unchanged, which corresponds to road sections without intersections from which vehicles can merge. For these sections, optimal velocity profiles are characterized by using the Pontryagin maximum principle, hereby assuming that the merging times are known. Next, the optimal merging times are characterized and an algorithm is presented to obtain the optmal merging times as well as the corresponding fuel-optimal trajectories for each vehicle. Heterogeneous platoons are allowed and the constraints imposed on vehicles in a platoon are explicitly addressed by considering the dynamics of the platoon as a whole.

Third, a model predictive control approach for control of merging maneuvers is developed based on the hybrid optimal control algorithm discussed above. By using this framework, the execution of the merging maneuvers becomes robust to modelling uncertainties as well as external disturbances. Moreover, it enables a decentralized implementation.

For a scenario involving three vehicles, it is shown that this approach leads to a reduction in fuel consumption of up to $13\,\%$ with respect to each vehicle driving alone. Moreover, it is noted that the techniques developed in this paper can also

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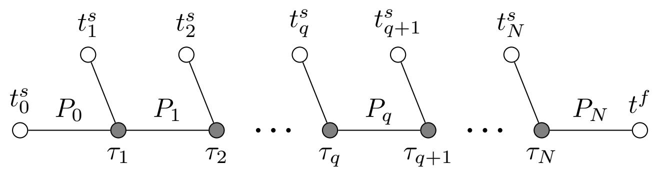

Fig. 1. Illustration of the road network for the merging problem for $N+1$ vehicles. The merging maneuvers are initiated from the open circles and starting times t_q^s, whereas the rightmost circle denotes the final destination with common final time t^f. Platoons P_q can be formed on overlapping parts of the vehicles' routes by merging with an existing platoon at the gray circles at merging times τ_q.

form the basis of an algorithm that decides on whether to form a platoon or not, as the fuel cost associated to merging maneuvers is explicitly computed.

The remainder of this paper is structured as follows. In Section II, the vehicle model is introduced and a detailed statement of the merging problem for platoon formation is given. A hybrid optimal control approach for the fuel-optimal execution of merging maneuvers is discussed in Section III, whereas Section IV presents an implementation of this optimization algorithm in the scope of model predictive control. Simulation results showing the feasibility and robustness of this approach are given in Section V. Finally, conclusions are stated in Section VI.

II. Vehicle model and problem statement

Consider $N+1$ heavy-duty vehicles modeled as

$$\begin{aligned} \dot{s}_i &= v_i, \\ m_i \dot{v}_i &= -c_{\mathrm{r}} g m_i - \tfrac{1}{2}\rho c_{\mathrm{d},i} A_i v_i^2 + u_i, \end{aligned} \tag{1}$$

with s_i and v_i representing the position and velocity of vehicle i, respectively, which are collected in the state $x_i = [\, s_i \;\, v_i \,]^{\mathrm{T}}$ for $i \in \{0, 1, \ldots, N\}$. The mass of vehicle i is given as m_i, whereas the first term on the right-hand side of the second equation represents the rolling resistance with rolling resistance coefficient c_{r} and g the gravitational acceleration. The second term models the air drag, which is dependent on the air density ρ, air drag coefficient $c_{\mathrm{d},i}$ and the frontal area A_i of vehicle i. The air drag coefficient is dependent on the interaction with other vehicles and decreases when vehicle i closely follows a preceding vehicle. This is represented as

$$c_{\mathrm{d},i} = \begin{cases} c_{\mathrm{d},i}^0, & \text{vehicle } i \text{ is alone or platoon leader,} \\ \eta_i c_{\mathrm{d},i}^0, & \text{vehicle } i \text{ is a platoon follower,} \end{cases} \tag{2}$$

where $0 < \eta_i < 1$ represents the air drag reduction due to platooning and $c_{\mathrm{d},i}^0$ the nominal drag coefficient. Finally, u_i represents the combined traction force due to the engine and the braking force.

In this paper, $N+1$ vehicles are considered that form a growing platoon through subsequent merging maneuvers, as schematically depicted in Figure 1. Each vehicle has a separate starting point $x_i^s = [\, s_i^s \;\, v_i^s \,]^{\mathrm{T}}$ with starting time t_i^s, but it is assumed that all vehicles have the same destination x^f (with final time t^f) and that all routes overlap. Specifically, the merging maneuvers have to be executed in such a way that the total fuel cost of all vehicles is minimized, as expressed in the cost function

$$J = \sum_{i=0}^{N} \int_{t_i^s}^{t^f} |u_i(t)|^2 \,\mathrm{d}t, \tag{3}$$

where it is recalled that the formation of platoons reduces the fuel consumption through a reduced aerodynamic drag, see (2). However, the formation of platoons imposes (when the length of the vehicles and their inter-vehicular distance is neglected) the following constraint:

$$x_i(t) = x_j(t), \;\; \forall\, i, j \in P_q, \; \forall\, t \in [\tau_q, \tau_{q+1}], \tag{4}$$

for all $q \in \{1, 2, \ldots, N\}$ and with $\tau_{N+1} = t^f$. In (4), $P_q = \{0, 1, \ldots, q\}$ is a set that collects the indices of the vehicles in the platoon after q vehicles have merged, and τ_q represent the merging times as indicated in Figure 1. These merging times correspond to the locations of the road intersections denoted as s_q^m. It is stressed that the minimization of the fuel cost (3) requires both the computation of the optimal merging times τ_q as well as the optimal vehicle trajectories $x_i(\cdot)$. A hybrid optimal control approach towards the computation of the optimal trajectories is given in the next section.

It is remarked that the formation of platoons might not be the most fuel-efficient approach towards traversing a number of vehicles to a common destination, as the cost of coordination required to form a platoon might be higher than the benefits offered by platooning. However, in this paper, it is assumed that the decision on forming a platoon has already been made. Nonetheless, by computing the optimal fuel cost for forming a platoon, the techniques developed in this paper might provide an approach towards making such a decision. Also, it is noted that, even though the specific case of Figure 1 is considered for ease of discussion, more complex merging schemes (e.g., through the formation and merging of sub-platoons) can be handled similarly.

III. Optimal control of merging maneuvers

The cost function (3) provides a characterization of the total fuel cost by summing the cost of each vehicle. However, the dynamics of a given vehicle changes after it has merged with the platoon due to (2), so that it is more convenient to express the cost function (3) equivalently as a sum over time intervals in which the dynamics (as well as the platooning constraints (4)) remain unchanged. This leads to

$$\begin{aligned} J = &\sum_{q=0}^{N} \sum_{i \in P_q} \int_{\tau_q}^{\tau_{q+1}} |u_i(t)|^2 \,\mathrm{d}t \\ &+ \sum_{q=0}^{N-1} \int_{t_{q+1}^s}^{\tau_{q+1}} |u_{q+1}(t)|^2 \,\mathrm{d}t, \end{aligned} \tag{5}$$

where $\tau_0 = t_0^s$ and $\tau_{N+1} = t^f$. Here, the terms on the first line represent the cost of driving the platoon P_q between intersection q and $q+1$ (with corresponding merging times τ_q and τ_{q+1}), whereas the second line gives the fuel cost of the individual vehicle approaching intersection $q+1$.

The form (5) of the cost function J will be used to obtain the fuel-optimal vehicle trajectories and corresponding merging times. In order to find this optimal solution, two subsequent steps are discussed. First, assuming that the merging times τ_q are given, the optimal vehicle and platoon trajectories are given. Second, the optimal merging times are determined and an algorithm for computing the optimal solution is discussed.

A. Optimization of trajectories

Under the assumption that the merging times τ_q are given, it is readily observed that the merging problem can be decomposed in subproblems of the optimal traversal of each edge (i.e., road segment) in Figure 1. Considering the traversal of the platoon $P_q = \{0, 1, \ldots, q\}$ between intersections q and $q+1$ leads to the cost function

$$\bar{J}_q = \sum_{i \in P_q} \int_{\tau_q}^{\tau_{q+1}} |u_i(t)|^2 \, \mathrm{d}t, \tag{6}$$

where it is recalled that the constraint (4) is required to hold. In order to ensure the satisfaction of this constraint, a platoon state $\bar{x}_q = [\, \bar{s}_q \;\, \bar{v}_q \,]^{\mathrm{T}}$ is defined such that all vehicles in the platoon P_q satisfy

$$\bar{x}_q = x_1 = x_2 = \ldots = x_q. \tag{7}$$

Similarly, after defining a platoon input $\bar{u}_q$ and total platoon mass $\bar{m}_q$ as

$$\bar{u}_q = \sum_{i \in P_q} u_i, \quad \bar{m}_q = \sum_{i \in P_q} m_i, \tag{8}$$

it follows from summing (1) for all $i \in P_q$, hereby using the constraint (4) as expressed through (7), that the dynamics of the platoon can be written as

$$\begin{aligned} \dot{\bar{s}}_q &= \bar{v}_q, \\ \bar{m}_q \dot{\bar{v}}_q &= -c_{\mathrm{r}} g \bar{m}_q - \tfrac{1}{2} \rho C_{\mathrm{d},q} \bar{v}_q^2 + \bar{u}_q, \end{aligned} \tag{9}$$

with $C_{\mathrm{d},q}$ a parameter characterizing the total air drag as

$$C_{\mathrm{d},q} = c_{\mathrm{d},0}^0 A_0 + \sum_{i=1}^{q} \eta_i c_{\mathrm{d},i}^0 A_i. \tag{10}$$

Moreover, by comparing the platoon dynamics (9) to the vehicle dynamics (1), it follows that the input $u_i(\cdot)$ of each vehicle in the platoon (i.e., satisfying (7)) can be obtained as $u_i = \alpha_{q,i} \bar{u}_q + \beta_{q,i} \bar{v}_q^2$, with

$$\alpha_{q,i} = \frac{m_i}{\bar{m}_q}, \quad \beta_{q,i} = \tfrac{1}{2} \rho \left(c_{\mathrm{d},i} A_i - \frac{m_i}{\bar{m}_q} C_{\mathrm{d},q} \right) \tag{11}$$

for all $i \in P_q$ and where $c_{\mathrm{d},i} = c_{\mathrm{d},i}^0$ for $i = 0$ and $c_{\mathrm{d},i} = \eta_i c_{\mathrm{d},i}^0$ otherwise. By exploiting this, the cost function (6) can be written as

$$\bar{J}_q = \sum_{i \in P_q} \int_{\tau_q}^{\tau_{q+1}} \left| \alpha_{q,i} \bar{u}_q(t) + \beta_{q,i} \bar{v}_q^2(t) \right|^2 \mathrm{d}t, \tag{12}$$

which is only dependent on the platoon state and input. More importantly, the introduction of the platoon state in (7) ensures the automatic satisfaction of the constraints (4), such that the optimal traversal of a road segment can be cast as the unconstrained optimal control problem

$$\begin{aligned} &\min\nolimits_{\bar{u}_q(\cdot)} \bar{J}_q \\ &\text{subject to:} \quad \begin{aligned} \dot{\bar{x}}_q &= \bar{f}_q(\bar{x}_q, \bar{u}_q), \\ \bar{x}_q(\tau_q) &= x_q^m, \; \bar{x}_q(\tau_{q+1}) = x_{q+1}^m, \end{aligned} \end{aligned} \tag{13}$$

where the vector field $\bar{f}_q$ is given by (9) as

$$\bar{f}_q(\bar{x}_q, \bar{u}_q) = \begin{bmatrix} \bar{v}_q \\ -c_{\mathrm{r}} g - \frac{1}{2\bar{m}_q} \rho C_{\mathrm{d},q} \bar{v}_q^2 + \frac{1}{\bar{m}_q} \bar{u}_q \end{bmatrix}. \tag{14}$$

In (13), $x_q^m = [\, s_q^m \;\, v_q^m \,]^{\mathrm{T}}$ defines the position of the intersection and the desired merging velocity.

The optimization problem (13) can be solved by exploiting the theory of optimal control (see, e.g., [7]) by introducing the Hamiltonian

$$\bar{H}_q(\bar{\lambda}_q, \bar{x}_q, \bar{u}_q) = \bar{\lambda}_q^{\mathrm{T}} \bar{f}_q(\bar{x}_q, \bar{u}_q) + l_q(\bar{x}_q, \bar{u}_q), \tag{15}$$

where $l_q(\bar{x}_q, \bar{u}_q) = \sum_{i \in P_q} |\alpha_{q,i} \bar{u}_q + \beta_{q,i} \bar{v}_q^2|^2$. By Pontryagin's maximum principle (see [10]), the optimal state trajectory $\bar{x}_q^*(\cdot)$ and optimal input function $\bar{u}_q^*(\cdot)$ satisfy

$$\dot{\bar{x}}_q^* = \frac{\partial \bar{H}_q}{\partial \bar{\lambda}_q} (\bar{\lambda}_q^*, \bar{x}_q^*, \bar{u}_q^*) = \bar{f}_q(\bar{x}_q^*, \bar{u}_q^*), \tag{16}$$

$$\dot{\bar{\lambda}}_q^* = -\frac{\partial \bar{H}_q}{\partial \bar{x}_q} (\bar{\lambda}_q^*, \bar{x}_q^*, \bar{u}_q^*), \tag{17}$$

for some function $\bar{\lambda}_q^*(\cdot)$ and satisfying the constraints $\bar{x}_q^*(\tau_q) = x_q^m$, $\bar{x}_q(\tau_{q+1}) = x_{q+1}^m$. The optimal input reads

$$\bar{u}_q^*(t) = \arg\min_u \bar{H}_q(\bar{\lambda}_q^*(t), \bar{x}_q^*(t), u) \tag{18}$$

for all $t \in [\tau_q, \tau_{q+1}]$ whereas the Hamiltonian is constant over the same time interval, i.e.,

$$\bar{H}_q(\bar{\lambda}_q^*(t), \bar{x}_q^*(t), \bar{u}_q^*(t)) = \text{const.} \tag{19}$$

The statement of the Pontryagin maximum principle allows for solving the optimal control problem (13) through the solution of the boundary value problem (16)–(18), where the latter can be obtained by the use of numerical methods such as the (multiple) shooting method, see, e.g., [2].

Even though the problem (13) is stated for a platoon, similar results can be obtained for vehicles that merge to the platoon. Namely, a vehicle $i = q+1$ that merges with the platoon P_q at τ_{q+1} can be regarded as a one-vehicle platoon. This leads to the optimal control problem

$$\begin{aligned} &\min\nolimits_{\tilde{u}_q(\cdot)} J_q \\ &\text{subject to:} \quad \begin{aligned} \dot{\tilde{x}}_q &= f_q(\tilde{x}_q, \tilde{u}_q), \\ \tilde{x}_q(t_{q+1}^s) &= x_{q+1}^s, \; \tilde{x}_q(\tau_{q+1}) = x_{q+1}^m, \end{aligned} \end{aligned} \tag{20}$$

with $\tilde{x}_q = x_{q+1}$ and $\tilde{u}_q = u_{q+1}$ and where the vector field f_q represents the dynamics (1) for vehicle $i = q+1$. The cost function J_q is in this case given as

$$J_q = \int_{t_{q+1}^s}^{\tau_{q+1}} |\tilde{u}_q(t)|^2 \, \mathrm{d}t. \tag{21}$$

As the optimization problem (20) can be regarded as a special case of (13), it is not discussed in more detail. The corresponding Hamiltonian and adjoint state will be denoted by H_q and $\tilde{\lambda}_q$, respectively.

B. Optimization of merging times

In the computation of the optimal vehicle and platoon trajectories in the previous section, it is assumed that the merging times τ_q are fixed and known. However, the merging times are in fact a variable that can be exploited in the optimization to further reduce fuel consumption. In order to do so, the theory of hybrid optimal control is used, which provides an extension of the Pontryagin maximum principle to systems that switch between modes in which the dynamics might be different. The platoon merging problem of Figure 1 falls in this class as the dynamics change after an additional vehicle is merged to the platoon.

Let q be the mode in which platoon P_q is preparing the merging with vehicle $q+1$. At the actual merging point at time τ_{q+1}, a larger platoon P_{q+1} is formed, representing a new mode $q+1$. Here, it is recalled that the optimal trajectory of the platoon P_q in mode q is given by the optimal control problem (13) with state $\bar{x}_q$ and Hamiltonian $\bar{H}_q$, whereas the trajectory of the vehicle is given through (20), with state $\tilde{x}_q$ and Hamiltonian H_q. Now, according to the hybrid maximum principle (see [13], [11]), the optimal solution satisfies the following continuity condition on the Hamiltonian at the switching instant τ_{q+1}:

$$\begin{aligned}\bar{H}_{q+1}&\big(\bar{\lambda}^*_{q+1}(\tau_{q+1}), \bar{x}^*_{q+1}(\tau_{q+1}), \bar{u}^*_{q+1}(\tau_{q+1})\big)\\ &= \bar{H}_q\big(\bar{\lambda}^*_q(\tau_{q+1}), \bar{x}^*_q(\tau_{q+1}), \bar{u}^*_q(\tau_{q+1})\big)\\ &\quad + H_q\big(\lambda^*_q(\tau_{q+1}), \tilde{x}^*_q(\tau_{q+1}), \tilde{u}^*_q(\tau_{q+1})\big). \qquad (22)\end{aligned}$$

C. Algorithm

The equality (22) provides a necessary condition for optimality of the switching instants and can be used as a basis for an algorithm to solve the optimal merging problem. The following algorithm is based on theoretical developments in [11], where convergence is proven.

1) Set $k=0$ and initialize the merging times as $\tau_q^{(k)}$, where $\tau_0^{(k)} = t_0^s$ and $\tau_{N+1}^{(k)} = t^f$.
2) Compute the optimal trajectories for the platoons by solving (13) for $q \in \{0, 1, \ldots, N\}$ and the optimal trajectories for the merging vehicles by solving (20) for $q \in \{0, 1, \ldots, N-1\}$, hereby using the merging times $\tau_q^{(k)}$.
3) For each merging maneuver, when the mode switches from q to $q+1$, compute an update for the merging time τ_{q+1} by solving
$$\begin{aligned}\tau_{q+1}^{(k+1)} &= \tau_{q+1}^{(k)}\\ &\quad - \varepsilon\big(\bar{H}_q(\tau_{q+1}^{(k)}) + H_q(\tau_{q+1}^{(k)}) - \bar{H}_{q+1}(\tau_{q+1}^{(k)})\big), \quad (23)\end{aligned}$$
for each $q \in \{0, \ldots, N\}$ and some step size $\epsilon > 0$. Here, the arguments in the Hamiltonians (as in (22)) should be replaced by the time instant at which the Hamiltonians should be evaluated.
4) Increment k to $k+1$ and repeat from 2) until convergence.

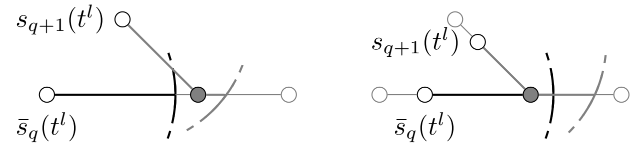

Fig. 2. The merging point at which platoon P_q (with position $\bar{s}_q$) and vehicle $q+1$ (with position s_{q+1}) is taken into account in the model predictive control problem at time t^l if the road intersection is in the prediction horizon (depicted by an arc) of both. Consequently, the merging point is not taken into account in the case presented in the left figure, but it is included in the problem formulation in the right figure.

IV. Model predictive control implementation

The hybrid optimal control approach of Section III allows for the computation of velocity trajectories for each vehicle in order to fuel-optimally form a platoon and traverse the desired road network. However, vehicles might not exactly track these optimal trajectories due to external disturbances (e.g., the influence of the road gradient and traffic) or due to modelling errors. The latter is particularly relevant as it is difficult in practice to estimate the vehicle mass and coefficients for road friction and aerodynamic drag.

In order to make the optimal control approach in Section III robust to such disturbances, a model predictive control framework is used. Model predictive control [4] relies on introducing a feedback mechanism by recalculating the optimal solution after each time step δt, herein considering a so-called prediction horizon with length h ($h \geq \delta t$).

The model predictive control algorithm can be informally stated as follows, where the perspective of vehicle i is taken.

1) Set $l=0$ and initialize $t^l = t_i^s$.
2) Compute a desired final state $x_i^{f,l}$ according to a desired average velocity v_{avg} to satisfy $s_i^{f,l} = s_i(t^l) + hv_{\text{avg}}$ and formulate the merging problem to include all intersections in this horizon. Use $x_j(\sigma)$ with $\sigma = \max\{t^l, t_j^s\}$, $j \neq i$, as initial conditions for the merging vehicles, i.e., taking their actual states if they are already on the road. Then, solve the resulting problem using the algorithm in Section III-C to obtain the optimal input $u_i^{*,l}$ on the interval $[t^l, t^l+h]$.
3) Implement the optimal input $u_i^{*,l}$ for the interval $[t^l, t^l+\delta t]$.
4) Set $t^{l+1} = t^l + \delta t$, increment l to $l+1$ and repeat from 2) until the final destination x^f is reached.

Apart from making the execution of the merging maneuvers robust to external disturbances and model uncertainties, the use of model predictive control also decentralizes the required computations over the vehicles. Here, it is noted that each vehicle $i \in \{0, 1, \ldots, N\}$ runs the algorithm discussed above. In this case, merging points are included in the optimization problem at time t^l if the road intersection is within the prediction horizon of both merging vehicles. This is schematically depicted in Figure 2. It is noted that a common final state needs to be agreed when both vehicles optimize their trajectories to form a platoon. This can be achieved by designating one of the vehicles as a platoon leader or through the use of consensus techniques.

TABLE I

Vehicle parameters for vehicles i according to the model (1) and their initial conditions $x_i^s = [s_i^s \;\; v_i^s]^{\mathrm{T}}$.

i	m_i [kg]	A_i [m^2]	$c^0_{\mathrm{d},i}$ [-]	t_i^s [s]	s_i^s [m]	v_i^s [m/s]
0	15000	10	0.5	0	-4500	25
1	14000	10	0.5	10	-4300	22
2	15000	11	0.5	63	-3000	21

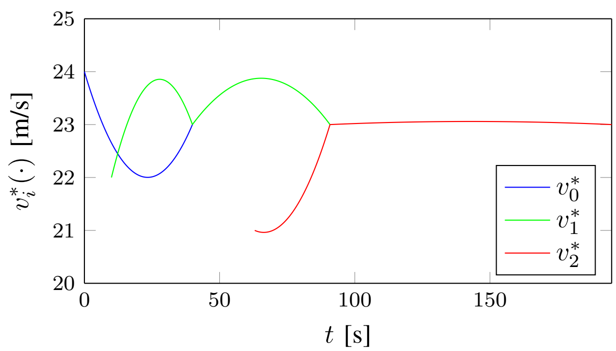

Fig. 3. Optimal velocity profiles $v_i^*(\cdot)$ for the merging problem in Figure 1 for $N = 2$ and the numerical values in Table I, as obtained by the algorithm in Section III-C.

V. Numerical Evaluation

The merging problem for three vehicles as in Figure 1 (with $N = 2$) is considered in this section, where the vehicle parameters and their initial conditions are given in Table I. Additional parameters are $c_{\mathrm{r}} = 0.01$, $g = 9.81\,\mathrm{m/s}^2$, and $\rho = 1.22\,\mathrm{kg/m}^3$. The coefficients η_i in (2) describing the reduction in aerodynamic drag due to platooning are chosen a $\eta_i = 0.5$ for all i. The desired merging points $x_q^m = [\, s_q^m \;\; v_q^m \,]^{\mathrm{T}}$ are at the road intersections s_q^m and have a desired merging velocity v_q^m given as

$$x_1^m = \begin{bmatrix} 3600 \\ 23 \end{bmatrix}, \quad x_2^m = \begin{bmatrix} 2400 \\ 23 \end{bmatrix}, \quad x_f = \begin{bmatrix} 0 \\ 23 \end{bmatrix} \tag{24}$$

where x^f is the desired final state at $t^f = 195\,\mathrm{s}$. If all vehicles optimize their velocity profiles independently (i.e., driving alone, without platooning), the optimal fuel cost is given as $J_{\text{ref}} = 4.87 \cdot 10^9\,(\mathrm{kg\,m})^2/\mathrm{s}^3$, which will be used as a reference throughout this section.

The application of the hybrid optimal control approach of Section III leads to the optimal merging times

$$\tau_1 = 40.0\,\mathrm{s}, \quad \tau_2 = 90.8\,\mathrm{s}, \tag{25}$$

and the optimal velocity profiles as depicted in Figure 3. It is clear from this figure that the vehicles indeed form a growing platoon satisfying the merging and final constraints (24). The corresponding optimal inputs are shown in Figure 4, which indicates the adaptation of the traction force to enable the formation of platoons. Moreover, in mode 2, where vehicles 0 and 1 form a platoon, it is clear that the required traction force for vehicle 1 is reduced due to a reduced aerodynamic drag. This is precisely the effect that is exploited in vehicle platooning and the root cause of the reduction in cost, which is computed as $J^*/J_{\text{ref}} = 0.866$ with J^* the optimal cost (5).

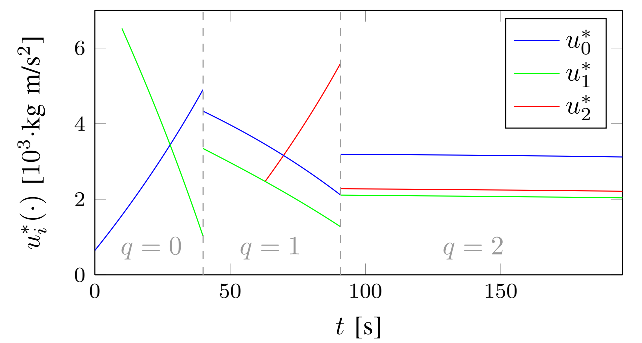

Fig. 4. Optimal input $u_i^*(\cdot)$ corresponding to the optimal velocity trajectories in Figure 3.

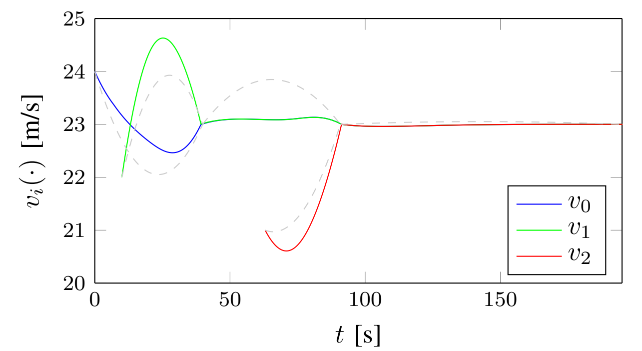

Fig. 5. Velocity trajectories for the model predictive control approach with optimization horizon $h = 35\,\mathrm{s}$. The same scenario as in Figure 3 is considered, where the trajectories of the latter are depicted in gray for comparison.

Thus, platooning amounts to a reduction in fuel consumption of roughly 13% in this scenario.

Next, the model predictive control approach of Section IV is applied to the same scenario, where an optimization horizon of $h = 35\,\mathrm{s}$ leads to the results in Figure 5. Due to the relatively short horizon length, the results differ from that in Figure 3, but the model predictive controller achieves the desired merging maneuvers and the merging times $\tau_1 = 39.3\,\mathrm{s}$ and $\tau_2 = 91.3\,\mathrm{s}$ are close to the optimal ones in (25). The corresponding cost J_{MPC} satisfies $J_{\text{MPC}}/J_{\text{ref}} = 0.893$ and is only slightly higher than the optimal cost J^*/J_{ref}. The dependence of the (relative) cost J_{MPC}/J^* on the horizon length h is investigated in Figure 6, which shows that the

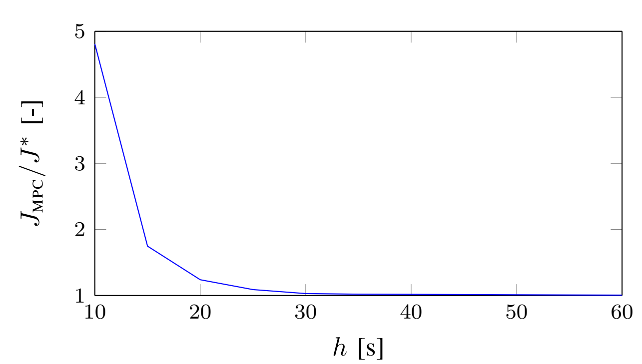

Fig. 6. Normalized fuel cost J_{MPC}/J for the model predictive control approach as a function of the horizon length h.

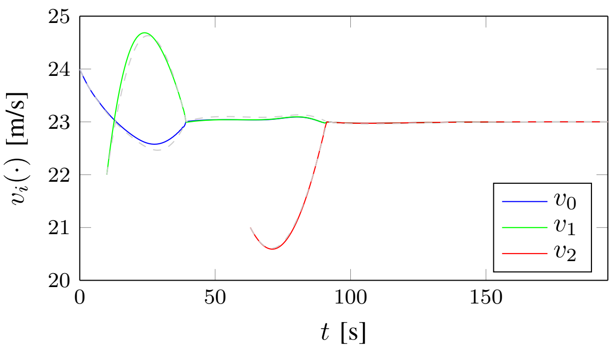

Fig. 7. Velocity trajectories for optimization horizon $h = 35\,\mathrm{s}$ and errors in the estimated vehicle masses. The grey dashed lines are the trajectories without uncertainties as in Figure 5.

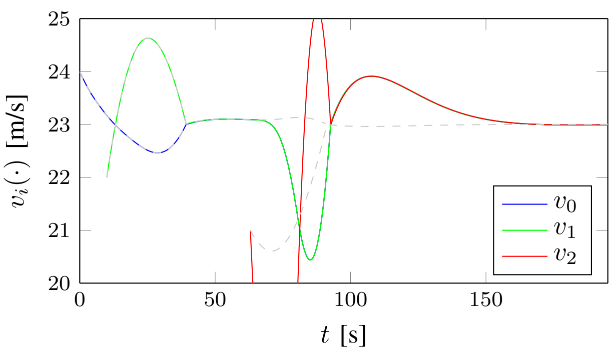

Fig. 8. Velocity trajectories for optimization horizon $h = 35\,\mathrm{s}$ and unknown disturbances acting in $t \in [63, 80]$ on the third vehicle. The grey dashed lines are the trajectories without disturbances.

optimal cost is approached for increasing horizon length. In fact, even relatively short horizon lengths (above 30 s) yield good results for this scenario, where it is recalled that the total length of the scenario is 195 s.

The influence of modelling uncertainties is assessed in Figure 7, which considers the same scenario as before. However, in Figure 7, the vehicle masses used in the optimization step in the model predictive control algorithm are taken as the estimated values $\hat{m}_0 = 12860\,\mathrm{kg}$, $\hat{m}_1 = 12000\,\mathrm{kg}$, and $\hat{m}_2 = 16000\,\mathrm{kg}$, whereas the masses in Table I are used in the simulation. The resulting trajectories deviate only marginally from the ones obtained with perfect model information, indicating the robustness of the model predictive control approach with respect to modelling uncertainties.

In order to analyze the effect of unmeasured external disturbances, a piecewise continuous disturbance

$$w_2(t) = \begin{cases} -1\,, & t \in [63, 80], \\ 0\,, & \text{otherwise}, \end{cases} \tag{26}$$

is added as a braking force on the vehicle with index 2. The results are given in Figure 8, in which it can be observed that the coordination between vehicles leads to a decreased velocity for the platoon of vehicles 0 and 1 in order to merge with vehicle 2. As a result, the merging time at the second intersection is changed slightly. Nonetheless, the merging maneuver is executed successfully, showing the robustness with respect to external disturbances.

VI. Conclusions

A hybrid optimal control approach was developed in this paper for the fuel-optimal control of merging maneuvers for the formation of heavy-duty vehicle platoons. This approach was extended towards a model predictive control formulation, ensuring that the merging maneuvers are robust with respect to disturbances.

The application of these techniques to a scenario involving three heavy-duty vehicles showed a reduction in fuel consumption of up to 13 %. Moreover, by explicitly computing the fuel cost of the formation of platoons, the results of this paper can also be used to evaluate decisions on whether to form a platoon or not. Ongoing work includes the experimental validation of the proposed approach.

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