Cooperative Optimal Coordination for Distributed Energy Resources

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***Abstract*— In this paper, we consider the optimal coordination problem for distributed energy resources (DERs) including distributed generators and energy storage devices. We propose an algorithm based on the push-sum and gradient method to optimally coordinate distributed generators and storage devices in a distributed manner. In the proposed algorithm, each DER only maintains a set of variables and updates them through information exchange with a few neighbors over a time-varying directed communication network. We show that the proposed distributed algorithm solves the optimal DER coordination problem if the time-varying directed communication network is uniformly jointly strongly connected, which is a mild condition on the connectivity of communication topologies. The proposed distributed algorithm is illustrated and validated by numerical simulations.**

I. Introduction

In the past decades, the power system has been undergoing a transition from a system with conventional generation power plants and inflexible loads to a system with a large numbers of distributed generators, energy storages, and flexible loads, often referred to as distributed energy resources (DERs) [1]. DERs are smaller, highly flexible, and can be aggregated to provide power necessary to meet regular demand. As the electricity grid continues to modernize, DER can help facilitate the transition to a smarter grid.

In order to achieve an effective deployment among DERs, one needs to properly design the coordination among them. One approach is through a completely centralized control strategy, where a single control center accesses the entire network's information and provides control signals to the entire system. This centralized control framework may not be effective for large-scale power networks due to performance limitations, such as a single point failure, high communication and computational burden, and limited flexibility.

Recently, an alternative distributed approach has been proposed to overcome these limitations. In particular, each DER makes a local decision based on the information received from a few neighboring DERs over the underlying communication network. Most existing distributed DER coordination studies focus on a single type of DERs. For distributed generation (DG) coordination, various distributed algorithms based on the consensus theory [3], [4] have been proposed, see, e.g., [2], [5]–[13]. On the other hand, cooperative management for a network of energy storages (ESs) has been considered [14], [15].

However, only few works consider the distributed coordination of both distributed generators and energy storages [16]–[18]. In [16], the authors proposed a distributed algorithm based on the consensus and innovation method to coordinate DGs and ESs over multiple time periods in a microgrid. However, the charging/discharging efficiencies are not modeled. As shown in [19] and other existing studies, the optimal charging/discharging operation and the corresponding benefits from a storage device could vary significantly with its efficiencies. Therefore, in [17], [18], we have developed distributed DER coordination strategies, where charging/discharging losses are modeled.

Note that one common assumption in [16]–[18] is that the communication network for information exchange among DERs is undirected and time invariant. However, in practice, the information exchange may be unidirectional due to nonuniform communication powers and the communication network topology may vary due to unexpected loss of communication links. Thus, in this paper, we consider DER coordination for the case where the communication network is directed and time-varying. To handle these challenges, we propose a distributed algorithm based on the push-sum and gradient method [20] and show that the proposed distributed algorithm solves the optimal DER coordination problem if the time-varying directed communication network is uniformly jointly strongly connected. Compared with existing studies for undirected fixed connected topologies [16]–[18], this requirement is much more general since the communication links can be unidirectional and the network can be disconnected at any time instant as long as the joint graph over a period of time is strongly connected.

The remainder of the paper is organized as follows. In Section II, we formulate the optimal DER coordination problem as a multi-step optimization problem, whose objective function and various constraints are introduced. Section III presents a centralized Lagrangian-based approach to solve the optimal DER coordination problem, summarizes our previously developed distributed algorithm for DER coordination, and motivates the study of this paper. In Section IV, a fully distributed DER coordination algorithm is developed. Section V presents case studies and simulation results. Con-

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cluding remarks are offered in Section VI.

II. Problem Formulation of DER Coordination

In this paper, we consider a distribution network including N distributed generators and M energy storage devices. Without loss of generality, we assume that the first N devices are distributed generators and the last M devices are energy storages. The optimal coordination problem can be formulated as a multi-step optimization problem consisting of an objective function and various constraints, which will be introduced in Section II-A and Section II-B, respectively. In Section II-C, we formally present the multi-step optimization problem formulation for optimal DER coordination.

A. Objective Function

The objective function is defined as the sum of generators' costs over a number of time periods

$$\sum_{t=1}^{T}\sum_{i=1}^{N} C_i(p_{i,t}), \tag{1}$$

where T is the number of time periods, $p_{i,t}$ is the power of DG i during period t, and $C_i(p_{i,t})$ is the cost function of DG i for period t and represented as a quadratic function of power output [21], given by

$$C_i(p_{i,t}) = a_i p_{i,t}^2 + b_i p_{i,t} + c_i, \tag{2}$$

with $a_i > 0$.

B. Constraints

In this section, we will present various constraints.

1) System constraint: Power system operation requires power balance between supply and demand, i.e., the power from the DGs and ESs together need to meet a given demand over a period of T. Such a requirement can be represented by

$$\sum_{i=1}^{N+M} p_{i,t} - D_t = 0, \quad \forall t \in \mathcal{T}, \tag{3}$$

where $p_{i,t}$ for $i = N+1, \ldots, N+M$ is the power of ES i, D_t is the given total demand of period t, and $\mathcal{T} = \{1, \ldots, T\}$.

2) Constraints for DG: For each DG $i \in \mathcal{N} := \{1, \ldots, N\}$, there are two constraints due to physical limits. The first one is the capacity limit on how much power DG i can generate at each time period, denoted by

$$p_i^{\min} \le p_{i,t} \le p_i^{\max}, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}, \tag{4}$$

where $p_i^{\min}, p_i^{\max}$ for $i \in \mathcal{N}$ are the lower and upper bound of the power limits of generator i, respectively.

The second one is ramping up/down constraints

$$\Delta \underline{p}_i \le p_{i,t} - p_{i,t-1} \le \Delta \overline{p}_i, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}, \tag{5}$$

where $\Delta \underline{p}_i, \Delta \overline{p}_i$ are the lower and upper bound of ramping rates of generator i, respectively.

3) Constraints for ES: For each ES $i \in \mathcal{M} := \{N+1, \ldots, N+M\}$, there are a few constraints due to physical limits. The first one is due to the storage capacity

$$p_i^{\min} \le p_{i,t} \le p_i^{\max} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{M}, \tag{6}$$

where $p_i^{\min}, p_i^{\max}$ for $i \in \mathcal{M}$ are the lower and upper bound of the power limits of ES i, respectively.

The second one expresses the rate of change of energy stored in ES due to the charging/discharging efficiencies as given below

$$p_{i,t}^{\text{batt}} = \begin{cases} \frac{p_{i,t}}{\eta_i^+}, & \text{if } p_{i,t} \ge 0 \\ p_{i,t}\eta_i^-, & \text{if } p_{i,t} < 0 \end{cases} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{M}, \tag{7}$$

where $p_{i,t}^{\text{batt}}$ is the rate of change of energy stored in ES i at the end of period t, which is positive when ES is discharged, and η_i^+, η_i^- are discharging and charging efficiency of storage device i, respectively.

The third one captures the dynamics of energy stored in ES i. The energy stored in ES i evolves according to the following dynamics

$$E_{i,t} = E_{i,t-1} - p_{i,t}^{\text{batt}} \Delta T \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{M}, \tag{8}$$

where $E_{i,t}$ is the energy stored in ES i at the end of time period t and ΔT is the size of time step.

The fourth constraint restricts the energy stored in ES i to be between its lower and upper bounds

$$0 \le E_{i,t} \le E_i^{\max} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{M}, \tag{9}$$

where $E_i^{\max}$ is the energy capacity of ES i.

The last constraint specifies the energy stored in ES i at the end of the scheduling period. It is set to be equal to the initial energy state as shown in (10)

$$E_{i,T} = E_{i,0} \quad \forall i \in \mathcal{M}, \tag{10}$$

but can be set to other feasible values.

C. Optimization Problem

With the objective function and various constraints, we are now ready to formally present the optimization problem formulation for DER coordination as the following multi-step optimization problem:

$$\textbf{P:} \min_{p_{i,t}, p_{i,t}^{\text{batt}}, E_{i,t}} \sum_{t=1}^{T}\sum_{i=1}^{N} C_i(p_{i,t}), \tag{11}$$

subject to (3)-(10). Note that the initial values $p_{i,0}$ for $i \in \mathcal{N}$ and $E_{i,0}$ for $i \in \mathcal{M}$ are parameters in the optimization problem and are given a prior.

Our goal is to design a distributed algorithm that drives the network of DERs to an optimal solution of (11) over time-varying directed communication topologies. However, the optimization problem is difficult to solve even in a centralized manner since the feasible set for the storage device $i \in \mathcal{M}$, which is defined as

$$\Omega_{p_i} := \{p_i \in \mathbb{R}^T | (6) - (10) \text{ are satisfied}\}$$

where

$$p_i = \left(p_{i,1}, p_{i,2}, \ldots, p_{i,T}\right)' . \tag{12}$$

is in general not convex due to non-convex constraint (7).

As shown in [18], when $\eta_i^+\eta_i^- < 1$ (which holds for all real world storage devices), we can convert the original problem to its convex equivalency by defining

$$p_{i,t} = p_{i,t}^+ - p_{i,t}^-, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{M} \tag{13}$$

where

$$0 \le p_{i,t}^+ \le p_i^{\max},\ 0 \le p_{i,t}^- \le -p_i^{\min},\ t \in \mathcal{T}, \forall i \in \mathcal{M} \tag{14}$$

and replace constraint (7) by

$$p_{i,t}^{\text{batt}} = \frac{1}{\eta_i^+} p_{i,t}^+ - \eta_i^- p_{i,t}^- . \tag{15}$$

Hence, the original non-convex problem in (11) is equivalent to

$$\mathbf{P}' : \min_{p_{i,t}, p_{i,t}^+, p_{i,t}^-, p_{i,t}^{\text{batt}}, E_{i,t}} \sum_{t=1}^{T} \sum_{i=1}^{N} C_i(p_{i,t}), \tag{16}$$

subject to (3)-(5), (8)-(10), (14), and (15).

III. Preliminary Results

A. Lagrangian-based Approach

In order to develop a distributed coordination algorithm, we dualize problem $\mathbf{P}'$ with respect to constraint (3), which couples the operation of all DERs. The other constraints are not relaxed because there is no coupling among devices.

Let $\tilde{\Omega}_{\mathcal{M},i}$ be the set of all $p_i^+, p_i^- \in \mathbb{R}^T$ for which (8)–(10), (14), and (15) are satisfied, where $i \in \mathcal{M}$,

$$p_i^+ = \left(p_{i,1}^+, p_{i,2}^+, \ldots, p_{i,T}^+\right)'$$

and

$$p_i^- = \left(p_{i,1}^-, p_{i,2}^-, \ldots, p_{i,T}^-\right)' .$$

We also denote $\Omega_{\mathcal{N},i}$ as the set of all $p_i \in \mathbb{R}^T$ for which (4) and (5) are satisfied, where $i \in \mathcal{N}$ and

$$p_i = \left(p_{i,1}, p_{i,2}, \ldots, p_{i,T}\right)' .$$

Note that both $\tilde{\Omega}_{\mathcal{M},i}$ and $\Omega_{\mathcal{N},i}$ are convex polytopes since all constraints are linear. This together with the fact that the objective function in (16) is convex with respect to the power of each DG and the power of each storage and that constraint (3) is affine, implies that if we dualize the problem in (16) with respect to constraint (3), there is zero duality gap. Moreover, the dual optimal set is nonempty [22]. We can thus solve the primal problem in (16) by considering its dual problem. With some algebra, the dual problem can be decomposed into into $N+M$ local optimization problems:

$$\max_{\lambda \ge 0} \sum_{i=1}^{N+M} \Phi_i(\lambda), \tag{17}$$

where $\lambda = \left(\lambda_1, \ldots, \lambda_T\right)'$ and λ_t, $t = 1, \ldots, T$ are Lagrange multipliers associated with power balance constraints (3),

$$\Phi_i(\lambda) = \min_{p_i \in \Omega_{\mathcal{N},i}} \sum_{t=1}^{T} C_i(p_{i,t}) - \lambda'(p_i - D^i),\ i \in \mathcal{N}, \tag{18}$$

$$\Phi_i(\lambda) = \min_{\{p_i^+, p_i^-\} \in \tilde{\Omega}_{\mathcal{M},i}} -\lambda'(p_i^+ - p_i^- - D^i),\ i \in \mathcal{M}, \tag{19}$$

and $D^i \in \mathbb{R}^T$ are virtual local demands at each agent for all the periods such that $\sum_{i=1}^{N+M} D^i = D = \left(D_1, \ldots, D_T\right)'$. Therefore, for any given λ, the minimizer p_i for $i \in \mathcal{N}$ in (18) and $\{p_i^+, p_i^-\}$ for $i \in \mathcal{M}$ in (19) can be obtained in a distributed manner by solving a local optimization problem.

B. Previous Results and Motivation

In [18], we solve these $N+M$ optimization problems locally via a distributed algorithm. In the proposed algorithm, each node runs a local optimization algorithm with an estimate of the optimal dual variable λ_i. These estimates are updated using the consensus and gradient strategy, where the consensus part ensures that all estimates (consensus variables λ_i) asymptotically approach the same value based on only local information exchange, and the gradient part guarantees that the power balance is satisfied.

Note that the proposed distributed algorithm is limited to the case where the communication topology among DERs is undirected and fixed. However, in practice, the information exchange may be unidirectional and the communication network topology may vary due to unexpected loss of communication links. Therefore, it is desirable to develop distributed algorithms for DER coordination over directed and time-varying communication networks. This motivates the study in this paper. In particular, in this paper, the communication topology for DERs is modeled as a time-varying directed graph $\mathcal{G}(k) = (\mathcal{V}, \mathcal{E}(k))$, where the first N agents correspond to distributed generators and the last M agents correspond to storage devices, and the edge set models communications among these DERs which may change over time due to unexpected loss of communication links.

IV. Main Results

In this section, we develop a distributed algorithm for optimal DER coordination over time-varying directed communication networks. In Section IV-A, we propose a distributed algorithm based on the push-sum and gradient method [20] for optimally coordinating DGs with energy storages. In Section IV-B, we show that the proposed distributed algorithm with appropriately chosen step-sizes is convergent if the time-varying directed communication network is uniformly jointly strongly connected.

A. Distributed Push-Sum and Gradient Based Algorithm

To handle the challenges of directed and time-varying communication among DERs, we propose a distributed algorithm based on the push-sum and gradient method [20] developed recently for distributed optimization over time-varying directed networks. The proposed algorithm is given in Algorithm 1 and contains two stages. One needs to execute the iterations in Stage I to get the optimal solution for distributed generators and then use the obtained optimal solution for DGs to run the iterations in Stage II to get the optimal solution for energy storage devices.

In particular, in Stage I of Algorithm 1, at time step k, each agent $i \in \mathcal{V}$ maintains T-dimensional variables $v_i(k)$, $w_i(k)$, $y_i(k)$, $\lambda_i(k)$, $p_i(k)$, where $p_i(k)$ and $\lambda_i(k)$ are estimates of the primal solution (optimal powers of DGs and ESs) and dual optimal solution (optimal incremental cost), respectively. For example, $\lambda_i = \left(\lambda_{i,1}, \dots, \lambda_{i,T}\right)'$, where each $\lambda_{i,t}$ for $t = 1, \dots, T$ is the estimate of the optimal incremental cost (marginal price) for period t. Note that variables $v_i(k)$, $w_i(k)$ and $y_i(k)$ are the auxiliary variables.

At each time step k, each agent $i \in \mathcal{V}$ updates its variables $w_i(k)$, $y_i(k)$ and $\lambda_i(k)$ according to (20).

$$w_i(k+1) = \sum_{j \in \mathcal{N}_i^{\text{in}}(k) \cup \{i\}} \frac{v_j(k)}{d_j(k)+1}, \tag{20a}$$

$$y_i(k+1) = \sum_{j \in \mathcal{N}_i^{\text{in}}(k) \cup \{i\}} \frac{y_j(k)}{d_j(k)+1}, \tag{20b}$$

$$\lambda_i(k+1) = \frac{w_i(k+1)}{y_i(k+1)}, \tag{20c}$$

where $\mathcal{N}_i^{\text{in}}(k) = \{j \in \mathcal{V} \mid (j,i) \in \mathcal{E}(k)\}$ is the in-neighbor set of agent i, i.e., the set of all agents that can transmit information to agent i directly at time instant k, and the division in (20c) operates entry-wise.

Once the estimate of the optimal dual variable $\lambda_i(k+1)$ is computed by an agent $i \in \mathcal{V}$. If the agent is associated with a distributed generator, i.e., $i \in \mathcal{N}$, then it updates the variables $p_i(k)$ by solving the following local optimization problem, which is the minimization problem in (18) with λ replaced by an estimate of the dual variable λ_i,

$$p_i(k+1) = \underset{p_i \in \Omega_{\mathcal{N},i}}{\arg\min} \sum_{t=1}^{T} C_i(p_{i,t}) - \lambda_i(k+1)' p_i. \tag{21}$$

If the agent is associated with an energy storage device, then it updates the variables $p_i(k)$ by solving the following local optimization problem, which is the minimization problem in (19) with λ replaced by an estimate of the dual variable λ_i,

$$\begin{aligned} &\{p_i^+(k+1), p_i^-(k+1)\} \\ &\quad = \underset{\{p_i^+, p_i^-\} \in \tilde{\Omega}_{\mathcal{M},i}}{\arg\min} \lambda_i(k+1)'\left(p_i^- - p_i^+\right), && \text{(22a)} \\ &p_i(k+1) = p_i^+(k+1) - p_i^-(k+1). && \text{(22b)} \end{aligned}$$

Once the estimate of the optimal power $p_i(k+1)$ is obtained by an agent $i \in \mathcal{V}$, it updates the variables $v_i(k)$ according to (23)

$$v_i(k+1) = w_i(k+1) - \alpha_{k+1}(p_i(k+1) - D^i). \tag{23}$$

The step-size α_{k+1} satisfies the following conditions:

$$\begin{aligned} &\sum_{k=1}^{\infty} \alpha_k = \infty, \quad \sum_{k=1}^{\infty} \alpha_k^2 < \infty, \\ &\alpha_k \le \alpha_s \text{ for all } k > s \ge 1. \end{aligned} \tag{24}$$

The typical choice for a sequence α_k satisfying (24) is $\alpha_k = \frac{a}{k+b}$, where $a > 0$ and $b \ge 0$.

In order to implement Algorithm 1, at time instant $k \in \mathbb{Z}_+$, where $\mathbb{Z}_+$ is the set nonnegative integers, each agent $i \in \mathcal{V}$ needs to know its out-degree $d_i(k)$ and sends the quantities $\frac{v_i(k)}{d_i(k)+1}$ and $\frac{y_i(k)}{d_i(k)+1}$ to all its out-neighbors $j \in \mathcal{N}_i^{\text{out}}(k)$ for the update. Based on the information received from in-neighbors, each agent makes a local update (decision). For example, in Stage I of Algorithm 1, based on the received information, each agent $i \in \mathcal{V}$ first runs the update (20) to obtain an estimate of dual variable $\lambda_i(k+1)$. Knowing this value, the estimates of optimal powers are obtained by solving $N+M$ local optimization problems, i.e., (21) for $i \in \mathcal{N}$, and (22) for $i \in \mathcal{M}$. Finally, each agent $i \in \mathcal{V}$ runs the update (23). The above procedure is repeated until the error is small enough, in the sense that $\|\lambda_i(k) - \lambda_i(k-1)\| < \epsilon_1$ and $\max_{i,j \in \mathcal{V}} \|\lambda_i(k) - \lambda_j(k)\| < \epsilon_2$, where ϵ_1 and ϵ_2 are small constants depending on the desired accuracy. In initialization, $v_i(0)$ is assigned with an arbitrary vector and $y_i(0) = \mathbf{1}$ for all $i \in \mathcal{V}$, where $\mathbf{1}$ is the column vectors with all entries being 1.

B. Convergence Result

In this section, we will show that Algorithm 1 with properly chosen step-sizes is capable to solve the optimal DER coordination problem over a time-varying directed communication network which satisfies the following assumption.

Assumption 1. *The time-varying directed communication network $\mathcal{G}(k)$ is uniformly jointly strongly connected, i.e., the jointly communication network $\mathcal{G}([k_0, k_0 + B))$[1] is strongly connected for any $k_0 \ge 0$ with some integer $B > 0$.*

Theorem 1. *Under Assumption 1, distributed Algorithm 1 with the step-size α_k satisfying conditions in (24) solves the optimization problem (16). In particular, Stage I yields $\lim_{k\to\infty} p_i(k) = p_i^*$ for all $i \in \mathcal{N}$ and Stage II yields $\lim_{m\to\infty} p_i(m) = p_i^*$ for all $i \in \mathcal{M}$ provided that $p_i^{sol} = p_i^*$ for all $i \in \mathcal{N}$, where p_i^* for all $i \in \mathcal{V}$ is the centralized optimal solution of the optimization problem (16).*

Proof. The proof is omitted due to the space limitation. □

Remark 1. *Theorem 1 shows that the proposed distributed Algorithm 1 solves the optimal DER coordination problem over a time-varying directed communication network which is uniformly jointly strongly connected. This is a mild condition on the connectivity of communication topologies, since the network can be disconnected at any time instant as long as the jointly graph over a period of time is strongly connected. Therefore, the requirement on network topologies is more general compared to the fixed undirected connected topologies considered in the existing literature for distributed DER coordination [16]–[18].*

V. Case Studies

In this section, various case studies are performed to illustrate and validate the proposed algorithm for optimal DER coordination. The IEEE 6-bus system used [18] is adopted here, where Buses 1–4 are connected with distributed generators and Buses 5 and 6 are connected to

[1]The joint graph of $\mathcal{G}(k)$ in the time interval $[k_1, k_2)$ with $k_1 < k_2 \le \infty$ is denoted as $\mathcal{G}([k_1, k_2)) = \cup_{k\in[k_1,k_2)}\mathcal{G}(k) = (\mathcal{V}, \cup_{k\in[k_1,k_2)}\mathcal{E}(k))$.

Algorithm 1 Distributed DER coordination algorithm over a time-varying directed communication network

Input: The time-varying graph $\mathcal{G}(k) = (\mathcal{V}, \mathcal{E}(k))$, the step-size α_k, an arbitrarily assigned $v_i(0) \in \mathbb{R}^T$, and $y_i(0) = \mathbf{1} \in \mathbb{R}^T$ for all $i \in \mathcal{V}$.
Output: The optimal generation p_i^* for $i \in \mathcal{V}$.
Stage I
repeat
  for $i = 1$ **to** $N + M$ **do**
    Run the update (20).
    if $i \in \mathcal{N}$ **then**
      Run the update (21).
    else
      Run the update (22).
    end if
    Run the update (23).
  end for
  Update k as $k := k + 1$.
until Error small enough
for $i = 1$ **to** N **do**
  $p_i^{\text{sol}} = p_i(k-1)$.
  Return p_i^{sol}.
end for
Stage II
repeat
  for $i = 1$ **to** $N + M$ **do**
    Run the update (20) with k replaced by m.
    if $i \in \mathcal{N}$ **then**
      Run the update $p_i(m+1) = p_i^{\text{sol}}$.
    else
      Run the update

$$\{p_i^+(m+1), p_i^-(m+1)\} = \underset{\{p_i^+, p_i^-\} \in \tilde{\Omega}_{\mathcal{M},i}}{\arg\min} \; \|p_i^+ - p_i^-\|^2 - \lambda_i(m+1)' \left(p_i^+ - p_i^-\right) \tag{25a}$$

$$p_i(m+1) = p_i^+(m+1) - p_i^-(m+1). \tag{25b}$$

    end if
    Run the update (23).
  end for
  Update m as $m := m + 1$.
until Error small enough
for $i = N + 1$ **to** $N + M$ **do**
  $p_i^{\text{sol}} = p_i(m-1)$.
  Return p_i^{sol}.
end for

energy storage devices. We also use the same parameters for DGs and ESs as those in [18]. This test system is used to study the performance of the proposed algorithm for both fixed directed communication networks and time-varying directed communication networks.

A. Fixed Directed Networks

We first demonstrate the performance of Algorithm 1 for the case where DERs exchange information over a fixed directed network, shown in Fig. 1. The demand to be supplied by these DERs is plotted in red in Fig. 2.

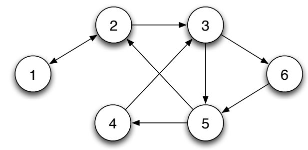


Fig. 1. Fixed directed communication network.

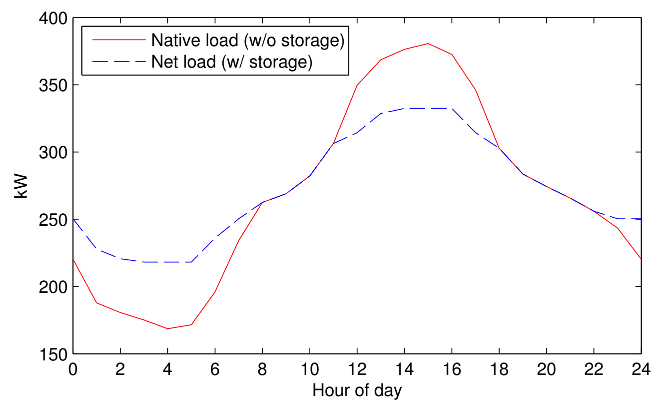


Fig. 2. Native load vs. Net load.

To coordinate four DGs with two storages over a 24-hour period, we apply the proposed Algorithm 1 with the step-size $\alpha_k = \frac{0.05}{k}$ and $\alpha_k = \frac{100}{k^{0.55}}$ for Stage I and Stage II, respectively. The obtained solution is the same as the centralized one. The blue curve in Fig. 2 is the resulting net load (load minus storage), which agrees with the result in [18]. Fig. 2 shows how two storage devices are coordinated to cut the peak and fill the valley. In particular, they are discharged during peak hours when the energy price is high and charged during off-peak hours when energy price is low.

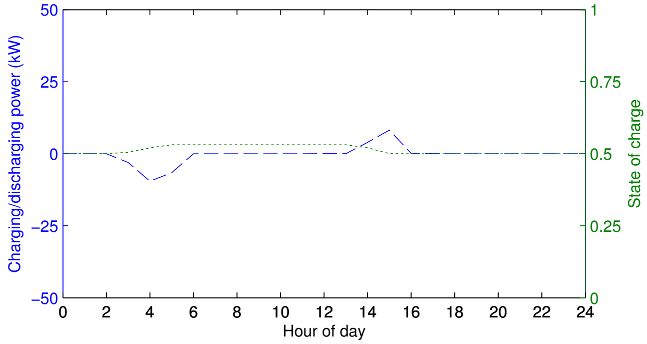
The power output and state of charge (SOC) for both storages are provided in Fig. 3, which is also in consistent with the result in [18].

B. Time-varying Directed Networks

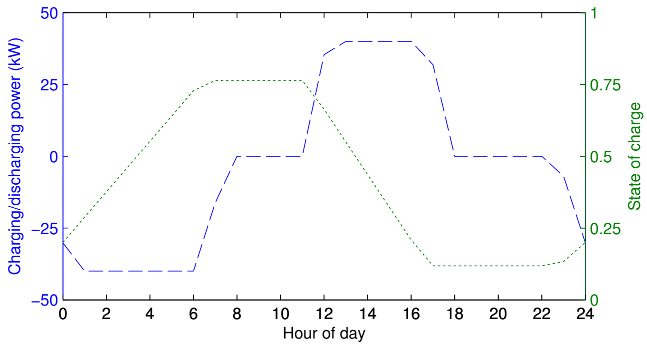
We next consider the case where DERs exchange information over a time-varying directed network $\mathcal{G}(k)$ switching among three fixed topologies $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$ shown in Fig. 4 at each time instant. In particular,

$$\mathcal{G}(k) = \begin{cases} \mathcal{G}_1, & \text{if } k \in [0,1) \cup \cdots \cup [3s, 3s+1) \cdots, \\ \mathcal{G}_2, & \text{if } k \in [1,2) \cup \cdots \cup [3s+1, 3s+2) \cdots, \\ \mathcal{G}_3, & \text{if } k \in [2,3) \cup \cdots \cup [3s+2, 3s+3) \cdots, \end{cases}$$

where $s \in \mathbb{Z}_+$. It is easy to check that each of the fixed topologies $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$ is not strongly connected. For example, there is no directed path from agent 2 to agent 1 in $\mathcal{G}_1$. However, the time-varying directed graph $\mathcal{G}(k)$ is uniformly jointly strongly connected since the joint graph $\mathcal{G}([k_0, k_0 + B))$ is strongly connected for any $k_0 \in \mathbb{Z}_+$ with $B = 3$. Thus, Assumption 1 is satisfied with $B = 3$.



(a) Battery 1



(b) Battery 2

Fig. 3. Charging (negative) and discharging (positive) power and state of charge.

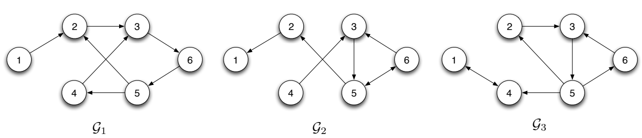


Fig. 4. Time-varying directed communication network.

According to Theorem 1, the proposed Algorithm 1 solves the optimal DER coordination problem.

By applying Algorithm 1 with the same step-sizes as those for the fixed communication network case, we find that the obtained solution agrees with the centralized one. The resulting net load (load minus storage) and the operation of storage devices are the same as those for the case of fixed directed networks. However, we have noticed that the convergence for this case is slower compared to the case of directed fixed networks.

VI. Conclusions

In this paper, we considered the optimal coordination problem of DERs, including distributed generators and energy storage devices. In the problem formulation, storage charging/discharging efficiencies were explicitly modeled. We proposed a distributed algorithm based on the push-sum and gradient method for optimal DER coordination. We showed that the proposed algorithm with appropriately chosen step-sizes solve the optimal DER coordination problem over time-varying directed communication networks that are uniformly jointly strongly connected. The performance of the proposed algorithm has been tested by various case studies. One future direction is to extend the proposed distributed algorithm to accommodate other communication effects, such as time delays and packet drops.

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