Delay-Sensitive Joint Optimal Control and Resource Management in Multiloop Networked Control Systems

Mohammad H. Mamduhi, Dipankar Maity, *Member, IEEE*, Sandra Hirche, *Fellow, IEEE*, John S. Baras, *Life Fellow, IEEE*, and Karl H. Johansson, *Fellow, IEEE*

***Abstract*—In the operation of networked control systems (NCSs), where multiple processes share a resource-limited and time-varying cost-sensitive network, communication delay is inevitable and primarily induced by, first, intermittent sensor sampling to restrict nonurgent transmissions, and second, resource management to avoid contentions, excessive traffic, and data loss. In a heterogeneous scenario, where control systems may tolerate only specific levels of sensor-to-controller latency, delay sensitivities need to be considered in the design of control and network policies to achieve the desired performance guarantees. We propose a cross-layer optimal co-design of control, sampling, and resource management policies for an NCS consisting of multiple stochastic linear time-invariant systems which close their sensor-to-controller links over a shared network. Aligned with advanced communication technology, we assume that the network offers a range of latency-varying transmission services for given prices. The performance of the local closed-loop systems is measured by a combination of linear-quadratic Gaussian cost and a suitable communication cost, and the overall objective is to minimize a defined *social cost* by all three policymakers. We derive optimal control, sampling, and resource allocation policies under different cross-layer awareness models, including constant and time-varying parameters, and show that higher awareness generally leads to performance enhancement at the expense of higher computational complexity. This trade-off is shown to be a key feature to select the proper interaction structure for the codesign.**

Manuscript received January 3, 2021; accepted March 7, 2021. Date of publication April 20, 2021; date of current version September 17, 2021. This work was supported in part by the Knut and Alice Wallenberg Foundation, in part by the Swedish Strategic Research Foundation, in part by the Swedish Research Council, and in part by the German Research Foundation within the Priority Program SPP 1914 – Cyber-Physical Networking. Recommended by Associate Editor W. X. Zheng. *(Corresponding author: Mohammad H. Mamduhi.)*

Mohammad H. Mamduhi and Karl H. Johansson are with the Division of Decision and Control Systems, KTH Royal Institute of Technology, 10044 Stockholm, Sweden (e-mail: mamduhi@kth.se; kallej@kth.se).

Dipankar Maity is with the Department of Electrical and Computer Engineering, University of North Carolina at Charlotte, Chapel Hill, NC 28223-0001 USA (e-mail: dmaity@uncc.edu).

Sandra Hirche is with the Chair of Information-Oriented Control, Technical University of Munich, 80333 Munich, Germany (e-mail: hirche@tum.de).

John S. Baras is with the Institute of Systems Research, The University of Maryland, College Park, MD USA (e-mail: baras@umd.edu).

Digital Object Identifier 10.1109/TCNS.2021.3074244

***Index Terms*—Cross-layer awareness model, joint optimal design, latency-varying transmission links, link capacity constraint, networked control systems.**

I. Motivation and Introduction

THE design and operation of networked control systems (NCSs), wherein multiple control loops exchange information between their sensors, controllers, and actuators via a common communication network, requires a major rethinking to respond to the growing requirements from current and future applications. The introduction of communication technologies that provide demand-driven serviceability with adjustable parameters and prices, together with novel approaches to virtually program network functions and adaptable network features, have created a significant potential to bring control and networking architectures to a whole new level [1] and [2]. This generally means moving from the traditional throughput-oriented and latency-minimizing data transmission with asymptotic-type performance guarantees, to smart data coordination schemes that consider real-time requirements and limitations of both the service providers and service recipients.

In the context of NCSs, this calls for novel sampling, control, and resource management architectures that incorporate the wide range of opportunities provided by the network infrastructure, such as adaptive service allocation, virtual programmability, adjustable channel reliability, and latency, to maximize quality-of-control (QoC), while minimizing the cost of network usage. Emerging NCS applications, such as networked cyber-physical systems (Net-CPS), autonomous driving, and Industry 4.0, often involve a large number of networked entities, each with time-varying requirements to fulfill specific tasks. The concept of “network” in such systems has gone beyond a simple shared communication channel to a general representation of evolving interlayer dependencies [3]. The state-of-the-art representation of networks creates a great potential to develop novel interactive approaches for real-time distributed sampling, networking, and control, such that the individual entities become aware of networking opportunities, and coupling constraints and incorporate them in decision-making, while the network is also aware of the demands and the task criticality of the entities and optimally allocate services and adjust the interdependencies.

A. Contributions

In this article, we propose jointly optimal communication and control policies for a general NCS model consisting of multiple delay-sensitive heterogeneous stochastic control systems closing their sensor-to-controller links via a shared communication network, under various interlayer awareness assumptions. Each subsystem is controlled by two local decision makers: a delay-sensitive controller that determines how fast state information should be sent to the plant controller, and a plant controller that maximizes control performance, measured by a linear-quadratic-Gaussian (LQG) cost. The network offers various transmission services, for fixed prices, through multiple capacity-limited channels each with a distinct and deterministic latency. Transmission requests from subsystems are arbitrated by a resource manager to avoid exceeding the link capacities. Resource arbitration is optimally performed such that the average sum of local LQG cost functions undergoes the minimum deviation compared to the resource-unlimited case, over a finite time horizon. We study scenarios each entailing a specific class of interlayer awareness (one-directional and bi-directional awareness of time-varying and constant parameters) among the three decision-makers, and derive the resulting jointly optimal policies. We show that performance of the joint design is associated with the level of delay-sensitivity tolerances and the awareness structure. In general, higher awareness results in lower local and social costs at the expense of higher computational complexity of the resulting optimization problem. We also observe that the extent of performance improvement is firmly tied to the particular awareness model. Our major contributions in this article are as follows.

1) Introducing a general model of NCS, including heterogeneous control loops and variety of network services, with evolving interactions between control and network layers leading to enhanced joint performance.
2) Investigating various awareness models for control and network layers and studying the interaction effects on the structure and performance of the optimal co-design.
3) Deriving jointly optimal policies from social optimization problems, including performance-complexity comparisons with respect to the awareness model.

B. Related Works

The problem of joint control and communication design in NCSs has been an active research topic for the last two decades in both control and communication communities [4], [5]. Two rather distinct perspectives in addressing it have evolved: from the communication perspective where maximizing quality-of-service (QoS) is the major objective, and requirements of control systems are often abstracted in the form of transmission rate, delay, and packet loss, with less attention given to the application dynamics and their real-time necessities [6], [7]. Numerous design methodologies are proposed, including protocols for QoS-enhacing medium access control (MAC) [8], [9]; resource allocation [10], [11]; scheduling and routing [12], [13]; and queuing management [14], [15]. On the other hand, from the control perspective, the aim is to maximize QoC, and the communication network is usually abstracted as maximum-rate and delay-negligible single-hop channels. Many design approaches for sampling, estimation, and control over shared networks are proposed to enhance QoC while reducing the rate of transmission, including event-triggered schemes [16]–[18], self-triggered schemes [19], [20], and adaptive/predictive data transmission and control models [21], [22]. For more sophisticated models of communication networks with data loss, delay, and resource constraints, attempts have been made mostly on codesign architectures that guarantee stability rather than optimality [23], [24]. Altogether, the efforts have often led to design frameworks that either consider no evolving cross-layer coupling or presume interactions in average form, with performance guarantees mostly valid in the asymptotic regime.

The design of cross-layer architectures for NCSs that consider active interactions between distributed components of control and communication layers to be aware of each other's conditions, capabilities, and requirements to achieve joint optimal quality-of-control-and-service, not only asymptotically but also over finite time horizons, is less studied in the literature. A major issue to address this is optimal timeliness, i.e., when is the best time to make a specific action such as sampling, transmission, or actuation. This problem is addressed in the control community mainly for data sampling over single-service communication support leading to optimal event-based technique to restrict unnecessary transmission [25], [26], and prioritized MAC protocols to distribute resources based on urgency [27], [28]. These approaches consider some measured or observed quantity of the control system, such as estimation error, as the triggering function. For multiple-loop nonscalar NCS, though, finding the optimal triggering law without major simplifications of the network layer is challenging. Moreover, resource allocation is often performed randomly or based on *a priori* given parameters but not based on dynamic awareness of interacting layers. In addition, the resulting performances of the proposed approaches are often addressed asymptotically over infinite horizon. To the best of our knowledge, a systematic approach that proposes a cross-layer optimal design of control, sampling, and resource management strategies to maximize QoC for multiloop NCSs with a shared network of various service opportunities is not presented in the literature.

C. Notations

We denote expectation, conditional expectation, transpose, floor, and trace operators by $\mathsf{E}[\cdot]$, $\mathsf{E}[\cdot|\cdot]$, $[\cdot]^\top$, $\lfloor\cdot\rfloor$, and $\mathsf{Tr}(\cdot)$, respectively. For $a \geq 0$, define the indicator $\mathbb{1}(a)=0$ if $a=0$, and $\mathbb{1}(a)=1$ if $a>0$. $X \sim \mathcal{N}(\mu, W)$ represents a multivariate Gaussian distributed random vector X with mean vector μ and positive definite covariance $W \succ 0$. The Q-weighted squared 2-norm of a column vector X is denoted by $\|X\|_Q^2 \triangleq X^\top Q X$. A time-varying column vector X_t^i includes an array of variables belonging to subsystem i at time t, while we define $X_{[t_1,t_2]}^i \triangleq \{X_{t_1}^i, X_{t_1+1}^i, \ldots, X_{t_2-1}^i, X_{t_2}^i\}$, and $X^i \triangleq \{X_0^i, X_1^i, \ldots\}$.

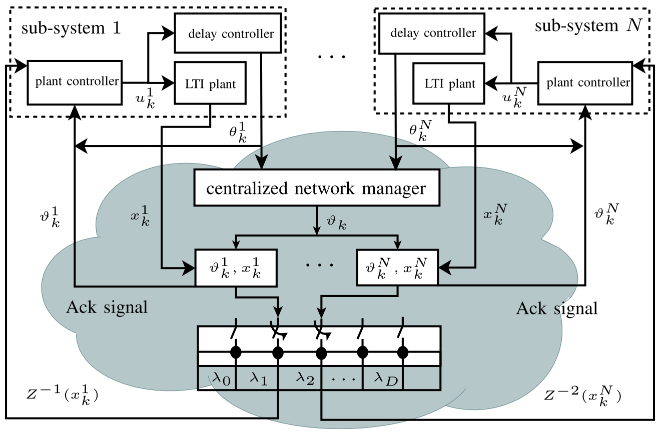

Fig. 1. Multiple LTI control loops exchange information with their respective controllers over a shared resource-limited communication network that can offer an array of latency-varying transmission services for different prices. (Z^{-d} is the delay operator).

II. NCS Model: Control and Communication Layers

We consider an NCS consisting of N synchronous stochastic linear time-invariant (LTI) controlled processes exchanging information over a common resource-limited communication network with resource management capabilities (see Fig. 1). Each process $i \in \mathrm{N} \triangleq \{1,\ldots,N\}$ comprises a physical plant $\mathcal{P}_i$, a delay-sensitivity controller $\mathcal{S}_i$, and a feedback control unit consisting of a state feedback controller $\mathcal{C}_i$ and an estimator $\mathcal{E}_i$. The dynamics of the plant $\mathcal{P}_i$, $i \in \mathrm{N}$, is described by the following stochastic difference equation:

$$x_{k+1}^i = A_i x_k^i + B_i u_k^i + w_k^i \tag{1}$$

where $x_k^i \in \mathbb{R}^{n^i}$ represents subsystem i's state vector at time-step $k \in \mathbb{N} \cup \{0\}$, $u_k^i \in \mathbb{R}^{m^i}$ denotes the corresponding control signal, $w_k^i \in \mathbb{R}^{n^i}$ the stochastic exogenous disturbance, and $A_i \in \mathbb{R}^{n^i \times n^i}$ and $B_i \in \mathbb{R}^{n^i \times m^i}$ describe the system and input matrices, respectively. To allow for heterogeneity, A_i and B_i matrices can be different across the NCS, i.e., $A_i \neq A_j$ and $B_i \neq B_j$, $i, j \in \mathrm{N}$. The disturbances are assumed to be random sequences with independent and identically distributed (i.i.d.) realizations $w_k^i \sim \mathcal{N}(0, \Sigma_w^i)$, $\forall k$ and $i \in \mathrm{N}$, and $\Sigma_w^i \succ 0$. The initial states x_0^i's are also presumed to be randomly selected from any arbitrary finite-moment distributions with variance $\Sigma_{x_0}^i$. For simplicity, we assume that the sensor measurements are perfectly noiseless copies of the state values.[1]

A. Communication System Model

To support the information exchange between each plant and its control unit, a resource-limited communication network provides cost-prone latency-varying transmission services. Precisely, the communication network consists of a set of multiple distinct one-hop transmission links, denoted by $\mathcal{L} \triangleq \{\ell_0, \ell_1, \ldots, \ell_D\}$, where ℓ_d represents the transmission link with deterministic service latency of d time-steps, and $|\mathcal{L}| = D+1$. Define the set $\mathcal{D} \triangleq \{0,1,\ldots,D\}$ and the vector $\Delta \triangleq [0,1,\ldots,D]^\top$. Hence, if x_k^i is sent to the controller $\mathcal{C}_i$ at time-step k through the transmission link ℓ_d with d-step delay, $d \in \mathcal{D}$, then x_k^i will be received at $\mathcal{C}_i$ at time-step $k+d$. A finite-valued service price $\lambda_d \in \mathbb{R}_{\geq 0}$ is assigned to each $\ell_d \in \mathcal{L}$ that is paid by the service recipient. Collectively, $\Lambda \triangleq [\lambda_0, \lambda_1, \ldots, \lambda_D]^\top$ denotes the link prices such that shorter delay induces higher price, i.e., $\lambda_0 > \lambda_1 > \cdots > \lambda_D \geq 0$.

Denote $c_d \in \mathbb{N}$ as the transport capacity of a certain link $\ell_d \in \mathcal{L}$, which entails the link ℓ_d has sufficient bandwidth resources to transport at most c_d number of data packets belonging to c_d number of distinct control systems. Being serviced with ℓ_d means that all those control systems will experience an equal delay of d time-steps. The data packet containing the state information of the ith control system includes a $\mathbb{R}^{n^i}$-valued vector of real numbers. The resource constraint on the number of data packets that can be serviced is stated as

$$c_d < N \quad \forall\, d \in \mathcal{D}. \tag{2}$$

Although not all subsystems can transmit through one certain link, we assume that the total capacity of all distinct transmission links is sufficient to service all subsystems, via multiple transmission links, at every time-step $k \in \{0,1,\ldots\}$, i.e.,

$$\sum_{d \in \mathcal{D}} c_d \geq N. \tag{3}$$

B. Distributed Policymakers and Decision Variables

We now introduce the three cross-layer policymakers and their corresponding decision outcomes for the underlying NCS, schematically depicted in Fig. 1, as follows.

1) Delay-Sensitivity: At the beginning of each sample cycle k, a local controller called "delay controller" decides on delay-sensitivity of its corresponding subsystem by selecting one of the transmission links $\ell_d \in \mathcal{L}$. We define the binary-valued vector $\theta_k^i \triangleq [\theta_k^i(0), \ldots, \theta_k^i(D)]^\top$ as the delay controller's decision variable of subsystem i at time-step k, where each element of θ_k^i is determined as follows:

$$\theta_k^i(d) = \begin{cases} 1, & \text{link } \ell_d \text{ selected to transmit } x_k^i \text{ at time } k \\ 0, & \text{link } \ell_d \text{ not selected} \end{cases}. \tag{4}$$

We assume that each local delay controller selects only one of the transmission links per time-step, therefore, we have

$$\sum_{d=0}^{D} \theta_k^i(d) = 1 \quad \forall\, k \in \{0,1,\ldots\}\; \forall\, i \in \mathrm{N}. \tag{5}$$

2) Control Input: The control unit of each local subsystem includes a feedback controller $\mathcal{C}_i$ and an estimator $\mathcal{E}_i$, which are assumed collocated. At every time k, the control command $u_k^i \in \mathbb{R}^{m^i}$ is the outcome of a causal and measurable law $\gamma_k^i(\cdot)$, given the available information at $\mathcal{C}_i$. In the absence of the state information x_k^i, the collocated estimator $\mathcal{E}_i$ may calculate the state estimate $\hat{x}_k^i$ if it is required for the computation of u_k^i.

3) Resource Allocation: Constraint (2) implies that if the number of requests to utilize a specific transmission link ℓ_d

[1] The results of this article extend, with lengthy but straightforward mathematical efforts, to noisy measurements if noise is an i.i.d. process.

exceeds the capacity c_d, not all requests can be accordingly serviced. Assume that a centralized network manager coordinates the resource allocation among subsystems. In case $\sum_{i=1}^{N} \theta_k^i(d) > c_d$ for a certain link ℓ_d, it decides which subsystems will be serviced via the link ℓ_d and which ones are reassigned to new transmission links. According to (3), no scheduled data packet is dropped due to capacity limitation, as there will be another transmission link with free capacity to be assigned. We define the binary-valued vector $\vartheta_k^i \triangleq [\vartheta_k^i(0), \ldots, \vartheta_k^i(D)]^\top$ as the decision outcome of the centralized resource allocation mechanism that determines implementable transmission links for subsystem i. The element $\vartheta_k^i(d) \in \{0,1\}$ is similarly defined as in (4), except that it is determined by the network manager after receiving the requests from all the subsystems. If at a time k, $\sum_{i=1}^{N} \theta_k^i(d) \leq c_d \forall d \in \mathcal{D}$, then $\vartheta_k^i = \theta_k^i \forall i \in \mathrm{N}$. Otherwise, if m requests are received for a certain link ℓ_d such that $m = \sum_{i=1}^{N} \theta_k^i(d) > c_d$, new transmission links will be assigned to $m - c_d$ of those requests. This means for every subsystem j of those c_d subsystems, $\vartheta_k^j = \theta_k^j$ holds. If a subsystem $\bar{j}$ belonging to the remaining set of $m - c_d$ subsystems had requested a certain link $\ell_{\bar{d}}$, but instead was serviced with a different link $\ell_{\tilde{d}}$, then $\vartheta_k^{\bar{j}}(\tilde{d}) \neq \theta_k^{\bar{j}}(\tilde{d})$ and $\vartheta_k^{\bar{j}}(\bar{d}) \neq \theta_k^{\bar{j}}(\bar{d})$, while for the rest of the links $\ell_d : \forall d \in \mathcal{D} \setminus \{\tilde{d}, \bar{d}\}$, we have $\vartheta_k^{\bar{j}}(d) = \theta_k^{\bar{j}}(d)$.

Since the ultimate link assignment is made by the network manager, state information received at the controller at time k, denoted by $\mathcal{Y}_k^i$, is determined by ϑ^i. Define $y_{k-d}^i(d) = x_{k-d}^i$ if $\vartheta_{k-d}^i(d) = 1$, and $y_{k-d}^i(d) = \emptyset$ if $\vartheta_{k-d}^i(d) = 0$, then

$$\mathcal{Y}_k^i = \{y_k^i(0), y_{k-1}^i(1), \ldots, y_{k-D}^i(D)\} \tag{6}$$

where, to avoid notational inconvenience, we define $\vartheta_{-1}^i(d) = \vartheta_{-2}^i(d) = \cdots = \vartheta_{-D}^i(d) = 0$ for all $d \in \mathcal{D}$.

Out-of-order delivery is a common phenomenon that may happen depending on the selected resource allocation policy. Assume state x_0^i is sent with delay 5 and x_1^i is sent with zero delay, then x_1^i will arrive before x_0^i. However, out-of-order arrival will be adequately handled while constructing the state estimate and computing the control input. If a stale state measurement arrives at the controller while a fresher one is available, or if both arrive simultaneously, the incorporation of the stale information will have no effect on improving the optimal control input, i.e., excluding the stale information in estimating the system state does not result in loss of optimality as the local control systems are fully observable. Note that the exclusion of stale measurements comes as the solution of the optimization problems in Section IV, and we do not need to assume dropout of stale information. Hence, without delving into the details, one can intuitively confirm that the optimal delay link profile should impose the least communication cost[2] for *outdated measurements*. This naturally emerges as the solution of the optimization problems described later.

[2]Due to the constraint (5) each subsystem is forced to pay a communication cost of at least λ_D per time-step.

III. Problem Formulation: Joint Optimization

In this section, we formulate a cross-layer joint optimization problem and discuss its structural characteristics with respect to to the policymakers. The three decision-makers are 1) local plant controllers that compute the control input u_k^i, $i \in \mathrm{N}$, at time-step k, 2) local delay controllers where the decision outcome θ_k^i determines the link $\ell_d \in \mathcal{L}$ through which x_k^i will be transmitted, and 3) resource manager to compute ϑ_k^i that determines whether θ_k^i can be accordingly serviced.

We assume that individual control systems have no knowledge of each other's parameters or decision variables. Let $\mathcal{I}_k^i$, $\bar{\mathcal{I}}_k^i$, and $\tilde{\mathcal{I}}_k$ denote the sets of accessible information for the plant controller, delay controller, and resource manager, respectively. (These sets are characterized in Section IV where the information structure at each policy maker is discussed.). Then, at every time k, the plant control, delay control, and resource allocation policies are measurable functions of the σ-algebras generated by their corresponding information sets, i.e., $u_k^i = \gamma_k^i(\mathcal{I}_k^i)$, $\theta_k^i = \xi_k^i(\bar{\mathcal{I}}_k^i)$, and $\vartheta_k = \pi_k(\tilde{\mathcal{I}}_k)$. Note that, γ^i and ξ^i represent local policies corresponding to a specific subsystem i, while π is computed centrally and includes the resource allocation profile for all $i \in \mathrm{N}$. The local objective function of each subsystem $i \in \mathrm{N}$, denoted by J^i, consists of its own LQG part plus the communication cost in average form over the finite horizon $[0, T]$, as follows:

$$J^i(u^i, \theta^i) = \mathsf{E}\left[\|x_T^i\|_{Q_2^i}^2 + \sum_{k=0}^{T-1} \|x_k^i\|_{Q_1^i}^2 + \|u_k^i\|_{R^i}^2 + \theta_k^{i\top}\Lambda\right] \tag{7}$$

where $Q_1^i \succeq 0$, $Q_2^i \succeq 0$, and $R^i \succ 0$ represent constant weight matrices for the state and control inputs, respectively.

The overall objective for the underlying NCS is to maximize the average performance of all subsystems under the resource constraint (2). This cannot simply be obtained by taking the average of the sum of the local cost functions (7) because the local decision variable θ_k^i might not be realized due to the resource limitations. More precisely, the time that a state information is received at a controller might not always be the time decided by its delay controller. In fact, the cost function (7) is achievable for a certain subsystem i only if $\vartheta_k^i = \theta_k^i \ \forall k \in [0, T]$. However, if the capacity of one or more transmission links are exceeded by the number of requests, the resource manager adjusts some of those requests, which eventually changes the realization of the control signal u_k^i and consequently the value of the local cost $J^i(u^i, \theta^i)$.

We formulate the system (commonly called social) cost J as the average difference between the sum of J^i's from the resource manager (given ϑ_k^i's) and local subsystems' (given θ_k^i's) perspectives, i.e., knowing $\vartheta_k = \pi_k(\tilde{\mathcal{I}}_k)$, we have

$$J = \frac{1}{N} \sum_{i=1}^{N} \mathsf{E}\left[J^i(u^i, \vartheta^i) - \min_{u^i, \theta^i} J^i(u^i, \theta^i)\right] \tag{8}$$

and J^i has been adjusted after resource allocation as

$$J^i(u^i, \vartheta^i) = \mathsf{E}\left[\|x_T^i\|_{Q_2^i}^2 + \sum_{k=0}^{T-1} \|x_k^i\|_{Q_1^i}^2 + \|u_k^i\|_{R^i}^2 + \vartheta_k^{i\top}\Lambda\right]. \tag{9}$$

Note that, $J^i(u^i,\theta^i)$ is computed locally independent of the decisions for subsystems $j \neq i$, while $J^i(u^i,\vartheta^i)$ is computed after central resource allocation is performed. The resources are allocated such that, with respect to the subsystems preferences, the closest possible services are provided and J is minimized.

In addition to the delay controllers that determine the real-time sensitivity of the control loops with respect to transmission latency, we introduce a latency-tolerance bound for each subsystem such that the allocated transmission links should remain within that given bound. To diversify this static sensitivity for each subsystem, we define α_i and β_i $(\in \mathcal{D})$ representing the maximum allowable delay tolerances. This specifies that a subsystem i can tolerate imposed deviations from the selected latency d only within the set $\{d-\alpha_i,\ldots,d,\ldots,d+\beta_i\}$.[3] In a real scenario, low latency-tolerance bounds would correspond to very precise machines with very fine sampling periods.

The ultimate goal is finding the optimal policies $\gamma_k^{i,*}(\mathcal{I}_k^i)$, $\xi_k^{i,*}(\bar{\mathcal{I}}_k^i)$, and $\pi_k^*(\tilde{\mathcal{I}}_k)$ that jointly minimize the social cost J

$$\min_{\gamma^i,\xi^i,\pi} J \tag{10a}$$

$$\text{s. t.} \quad u_k^i = \gamma_k^i(\mathcal{I}_k^i), \;\; \theta_k^i = \xi_k^i(\bar{\mathcal{I}}_k^i), \;\; \vartheta_k = \pi_k(\tilde{\mathcal{I}}_k) \tag{10b}$$

$$-\alpha_i \leq (\vartheta_k^i - \theta_k^i)^\top \Delta \leq \beta_i, \; i \in \mathrm{N} \tag{10c}$$

$$\sum_{j=1}^{N} \vartheta_k^j(d) \leq c_d, \; d \in \mathcal{D}, \; k \in [0, T-1]. \tag{10d}$$

Constraint (10c) specifies that if at time k, $\theta_k^i(d)=1$, then the network manager allocates an available resource only from the set of links $\{\ell_{\max\{0,d-\alpha_i\}},\ldots,\ell_{\min\{d+\beta_i,D\}}\}$ to subsystem i. The ultimate links from the allowable ones are selected by the resource manager such that the social cost J is minimized. Note that problem (10) might not have a feasible solution for all c_d. We derive a sufficient feasibility condition in form of a lower bound for the link capacities $c_d,\; d \in \mathcal{D}$, in Section IV.

Solving problem (10) is challenging due to the couplings between the decision variables. In fact, θ_k^i is the best choice, from the perspective of subsystem i, to make the balance between its LQG cost and communication price. However, delay controller decisions may go through changes because of resource limitations. Note that, the control input u_k^i is explicitly affected by θ_k^i in the absence of the resource limitations, but if $\vartheta_k^i \neq \theta_k^i$, then u_k^i will have a different realization. This means the realization of $u_k^{i,*}$ computed from problem (7) might be different from that being computed from problem (9) even if both are computed from the same control law. Moreover, any decision of ϑ_k^i is clearly θ_k-dependent. Further, θ_{k+1}^i might also be a function of $\vartheta_{[0,k]}^i$. Altogether, problem (10) is nontrivial due to interdependencies and cross-layer constraints, hence we need to identify relevant conditions under which it can be decomposed.

[3]To avoid notational inconvenience, the network manager only takes into account the feasible tolerances of this set that also belong to $\mathcal{D}$. Moreover, for a nontrivial set, we assume at least one nonzero α_i and β_j, $i,j \in \mathrm{N}$.

IV. Awareness Models and Optimal Codesign

Structural properties of the joint optimal policies are correlated with the cross-layer awareness model which characterizes the information sets $\mathcal{I}_k^i, \bar{\mathcal{I}}_k^i, \tilde{\mathcal{I}}_k$. We discuss directed awareness models for two different sets of information under which the couplings between u_k^i, θ_k^i, and ϑ_k^i are examined: "constant model parameters" and "dynamic variables." In the rest of the article, awareness of the constant model parameters for the network layer, if assumed, entails the knowledge of $\{A_i, B_i, Q_1^i, Q_2^i, R^i, \Sigma_w^i, \Sigma_{x_0}^i\} \forall i \in \mathrm{N}$. Note that, $\{\alpha_i,\beta_i\}$'s are known to the network layer. The local delay and plant controllers are also assumed to have the knowledge of their own model parameters $\{A_i, B_i, Q_1^i, Q_2^i, R^i, \Sigma_w^i, \Sigma_{x_0}^i, \alpha_i, \beta_i\}$ as well as the constant network parameters $\{\Lambda, \mathcal{L}\}$. In reality, information accessibility for each decision maker can be coordinated by a data center or through local servers.

To discuss awareness of dynamic variables, it is essential to have a clear picture of the order of generating variables in one sample cycle, e.g., $k \to k+1$. At the beginning of a sample time k, the system state x_k^i is updated according to the dynamics (1), and then the delay controller generates θ_k^i, based on the policy $\xi_k^i(\bar{\mathcal{I}}_k^i)$ to determine the transmission link through which x_k^i is to be communicated. System state x_k^i together with the service request θ_k^i is then forwarded to the network to be serviced. The resource manager receives this information from all subsystems and checks whether the number of requests for each link is exceeding its capacity. It then computes ϑ_k^i, according to the policy $\pi_k(\tilde{\mathcal{I}}_k)$, and x_k^i is transmitted through the link determined by ϑ_k^i. The control signal u_k^i is computed from the control law $\gamma_k^i(\mathcal{I}_k^i)$[4], x_{k+1}^i is afterward updated, and the pattern repeats over next samples.

At the controllers, the following awareness model of the dynamic variables is valid throughout the article. Knowledge of the model parameters of subsystem i is assumed for $\mathcal{C}_i$. Reminding (6), the information set $\mathcal{I}_k^i$ at time k is as

$$\mathcal{I}_k^i = \{\mathcal{Y}_0^i,\ldots,\mathcal{Y}_k^i,\theta_0^i,\ldots,\theta_k^i,\vartheta_0^i,\ldots,\vartheta_k^i,u_0^i,\ldots,u_{k-1}^i\}. \tag{11}$$

As in Fig. 2, the information set $\mathcal{I}_k^i$ in (11) specifies that the plant controllers are aware of the outcomes of the other two policies $\xi_{[0,k]}^i$ and $\pi_{[0,k]}^i$, from $t=0$ up to current time $t=k$. For that, we assume a dedicated low-bandwidth and error-free acknowledgment channel exists to inform the controllers at every time k about θ_k^i and ϑ_k^i (see Fig. 1). This practically can be done via broadcast or encoded acknowledgement signals.

To determine the awareness structure for the resource manager, we consider the following assumption.

Assumption 1: The resource allocation law π_k is rendered independent of the local plant control policies $\gamma_{[0,k-1]}^i$, $i \in \mathrm{N}$.

Assumption 1 declares a one-directional dependence between the plant control and resource allocation policies (see Fig. 2), i.e., γ_k^i's are explicit functions of ϑ_k^i, but π_k does not incorporate $u_{[0,k-1]}^i$'s, $i \in \mathrm{N}$, in determining ϑ_k^i. Although this results in the resource allocation being independent of local control laws, π_k

[4]In case the information set $\mathcal{I}_k^i$ is not updated, i.e., if no new state information belonging to subsystem i is scheduled to be delivered at time k, the control signal is updated based on a model-based estimation of x_k^i.

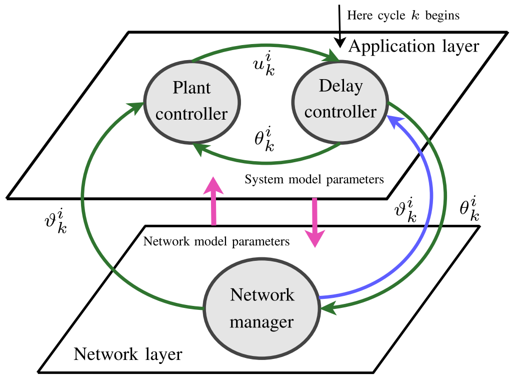

Fig. 2. Cross-layer interaction model: magenta arrows represent awareness of constant parameters. For network layer, awareness of system parameters, if assumed, includes $\{A_i, B_i, Q_1^i, Q_2^i, R^i, \Sigma_w^i, \Sigma_{x_0}^i, \alpha_i, \beta_i\}, \forall i \in \mathrm{N}$. For control loops, network parameters $\{\mathcal{L}, \Lambda\}$ are known. If ϑ_k^i is available for the delay controller (violet arrow), we call the delay-control policy *reactive*, otherwise, it is called *impassive*.

depends on $\theta^i_{[0,k]}$ which itself is effected by the control signals. In other words, the local delay controllers generate θ_k^i's such that an averaged equilibrium is achieved between maximizing the control performance and minimizing the communication cost. Since π_k is an explicit function of $\theta^i_{[0,k]}$'s, the effect of optimizing control performance is indirectly considered in resource allocation. Hence, the explicit dependence between the plant control and the resource manager policies that requires full knowledge of $u^i_{[0,k-1]}$'s, $i \in \mathrm{N}$ at the resource manager, is avoided. This assumption, nonetheless, leads to a considerable complexity reduction in computing the optimal policies π_k^* and $\gamma_k^{i,*}$ (Section IV-A).

Having Assumption 1, we introduce the dynamic variables included in the resource manager's information set $\tilde{\mathcal{I}}_k$, as

$$\tilde{\mathcal{I}}_k = \{\theta_0, \dots, \theta_k, \vartheta_0, \dots, \vartheta_{k-1}\}. \quad (12)$$

We also discuss the resource allocation with (Section IV-B) and without (Section IV-C) knowledge of the control systems model parameters. For the purpose of comparison, we discuss the scenario that the network manager does not take into account the local delay sensitivities in computing ϑ_k^i's, i.e., it allocates resources among subsystems knowing neither the constant $\{\alpha_i, \beta_i\}$'s nor $\theta^i_{[0,k]}$'s $\forall i \in \mathrm{N}$ (see Section IV-D). This is an important observation which shows how the local and social cost functions change with respect to the individual delay sensitivities.

For delay controllers, we introduce two design approaches, the so-called *impassive* and *reactive* delay control policies, each representing a distinct model of awareness of the dynamic variables (Fig. 2). We derive the resulting joint optimal delay control and resource allocation policies in Sections IV-B and IV-C. Before that, to determine the structure of the optimal plant control policy $\gamma_k^{i,*}$, $i \in \mathrm{N}$, we need to introduce the maximum amount of information that can be available at the ith delay controller at a time k.[5] The set $\bar{\mathcal{I}}_k^i$ contains, at most, information about the following dynamic variables:

$$\bar{\mathcal{I}}_k^i = \{\theta_0^i, \dots, \theta_{k-1}^i, \vartheta_0^i, \dots, \vartheta_{k-1}^i, u_0^i, \dots, u_{k-1}^i\}. \quad (13)$$

[5]Later we discuss that (13) corresponds to the reactive delay control approach and introduce the information set for the impassive approach.

A. Certainty Equivalence and Optimal Plant Controller

Having the sets $\mathcal{I}_k^i$, $\tilde{\mathcal{I}}_k$ and $\bar{\mathcal{I}}_k^i$ introduced in (11)–(13), and reminding Assumption 1, we state the following theorem.

Theorem 1: Given $\mathcal{I}_k^i$, $\tilde{\mathcal{I}}_k$, and $\bar{\mathcal{I}}_k^i$ in (11)–(13) and under Assumption 1, the optimal plant control law $\gamma_k^{i,*}$, $i \in \mathrm{N}$, with respect to (10) is of certainty equivalence form with the control inputs computed from the following linear state feedback law:

$$u_k^{i,*} = \gamma_k^{i,*}(\mathcal{I}_k^i) = -L_k^{i,*}\,\mathsf{E}[x_k^i|\mathcal{I}_k^i], \quad i \in \mathrm{N} \quad (14)$$

$$L_k^{i,*} = \left(R^i + B_i^\top P_{k+1}^i B_i\right)^{-1} B_i^\top P_{k+1}^i A_i \quad (15)$$

where $P_T^i = Q_2^i$ and P_k^i solves the below Riccati equation:

$$P_k^i = Q_1^i + A_i^\top\left[P_{k+1}^i - P_{k+1}^i B_i\left(R^i + B_i^\top P_{k+1}^i B_i\right)^{-1} B_i^\top P_{k+1}^i\right] A_i.$$

Proof: See Appendix A. ■

Remark 1: In the absence of constraint (2), the resource allocation becomes redundant as $\vartheta_k^i = \theta_k^i\ \forall i \in \mathrm{N}$, and $\forall k \in [0, T]$. Hence, from (32), we have $\min_{\gamma^i, \xi^i, \pi} J = 0$.

Corollary 1: Under the optimal certainty equivalence control law (14) and (15), the optimal cost-to-go $V_k^{i,*}$ equals

$$V_k^{i,*} = \|\,\mathsf{E}\left[x_k^i|\mathcal{I}_k^i\right]\|^2_{P_k^i} + \mathsf{E}\left[\|e_k^i\|^2_{P_k^i} + \sum_{t=k}^{T-1}\|e_t^i\|^2_{\tilde{P}_t^i}\middle|\mathcal{I}_k^i\right]$$

$$+ \sum_{t=k+1}^{T}\mathsf{Tr}(P_t^i \Sigma_w^i) \quad (16)$$

where $e_k^i \triangleq x_k^i - \mathsf{E}[x_k^i|\mathcal{I}_k^i]$, and $\tilde{P}_t^i = Q_1^i + A_i^\top P_{t+1}^i A_i - P_t^i$. Moreover, the estimator, at time-step k, is given as follows:

$$\mathsf{E}\left[x_k^i|\mathcal{I}_k^i\right] = \sum_{j=0}^{\min\{D,k+1\}} b_{j,k}^i\,\mathsf{E}\left[x_k^i|x_{k-j}^i, u_0^i, \dots, u_{k-1}^i\right] \quad (17)$$

where $b_{0,k}^i = \vartheta_k^i(0)$, and for all $j \in \mathcal{D}$ and $k \geq D$, we have

$$b_{j,k}^i = \prod_{d=0}^{j-1}\prod_{l=0}^{d}[1 - \vartheta_{k-d}^i(l)]\left[\sum_{d=0}^{j}\vartheta_{k-j}^i(d)\right] \quad (18)$$

and for, $k < D$, $b_{1,k}^i, \dots, b_{k,k}^i$'s are defined as in (18), $b_{k+1,k}^i = \prod_{d=0}^{k}\prod_{l=0}^{d}[1 - \vartheta_{k-d}^i(l)]$, and for notational convenience, we define $b_{k+2,k}^i = \dots = b_{D,k}^i = 0$, and $\mathsf{E}[x_k^i|x_{-1}^i, \mathcal{I}_k^i] \triangleq \mathsf{E}[x_k^i|\mathcal{I}_0^i]$.

Proof: The proof is similar to the proofs of Theorem 1 and Proposition 1 in [29] and hence omitted for brevity. ■

Remark 2: Theorem 1 shows that the optimal control law is certainty equivalence (14), yet $u_k^{i,*}$, i.e., the control law's realization, is computed based on $\mathsf{E}[x_k^i|\mathcal{I}_k^i]$ which is function on $\vartheta^i_{[k-D+1,k]}$; see (17). We discuss in the next section that, if the delay controller is impassive, $V_k^{i,*}$ is estimated according to $\theta^i_{[0,k-1]}$. Thus, if at a time $t \in [k-D, k-1]$, $\vartheta_t^i \neq \theta_t^i$, the delay controller computes $\mathsf{E}[V_k^{i,*}]$ as if θ_t^i is realized. Hence, $\mathsf{E}[V_k^{i,*}(\gamma^{i,*}, \xi^i)] \neq \mathsf{E}[V_k^{i,*}(\gamma^{i,*}, \pi)]$, despite similar $\gamma^{i,*}$ laws.

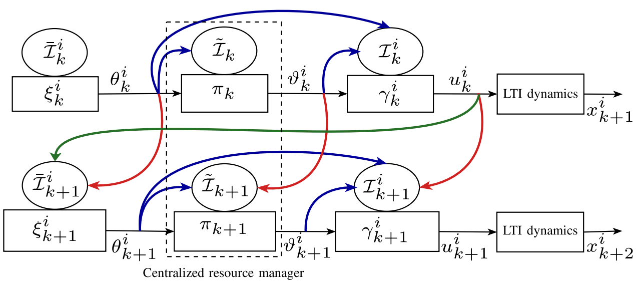

Fig. 3. Awareness model of the impassive delay control approach. Blue arrows represent policies' cross-awareness within one time-step. Red arrows show a policymaker's self-awareness. Green arrows depict state cross-awareness from one time-step to the next.

B. Optimal Delay Control and Resource Allocation Policies

We now derive optimal delay control and resource allocation policies $(\xi_k^{i,*}, \pi_k^*)$ under the following two awareness models of the *dynamic variables*. In this section, we assume the constant model parameters of all subsystems are accessible for the network manager. Resource allocation without knowledge of constant parameters is studied in Section IV-C.

1) Impassive Delay Control: We call the delay control policy an *impassive process* if the decision on θ_k^i's is made independent of $\vartheta_{[0,k-1]}^i$, i.e., the delay controller is passive with respect to the resource manager's decisions. Hence, it decides on θ_k^i's knowing nothing about possible reallocation by the resource manager. Therefore, the information set $\bar{\mathcal{I}}_k^i$ upon which $\theta_k^i = \xi_k^i(\bar{\mathcal{I}}_k^i)$ is computed impassively (see Fig. 3) becomes

$$\bar{\mathcal{I}}_k^i = \{\theta_0^i, \ldots, \theta_{k-1}^i, u_0^i, \ldots, u_{k-1}^i\}. \tag{19}$$

Note that, although $\vartheta_{[0,k-1]}^i$ is not incorporated in computing θ_k^i, the variable ϑ_k^i depends on $\{\theta_0, \ldots, \theta_k\}$. Moreover, the results of Theorem 1 hold for $\bar{\mathcal{I}}_k^i$ in (19), as we have $\bar{\mathcal{I}}_k^i \subseteq \mathcal{I}_k^i$.

Theorem 2: Consider problem (10) and let $\gamma^{i,*}, i \in \mathrm{N}$ follow the certainty equivalence law (14) and (15). Given $\bar{\mathcal{I}}_k^i$ and $\tilde{\mathcal{I}}_k$ in (19) and (12), the jointly optimal impassive delay control and resource allocation policies are offline solutions of the following constrained mixed-integer linear-programs (MILP)

$$\theta_{[0,T-1]}^{i,*} = \underset{\xi_{[0,T-1]}^i}{\operatorname{argmin}} \; J^i(\gamma^{i,*}, \xi_{[0,T-1]}^i(\bar{\mathcal{I}}_{[0,T-1]}^i)) =$$

$$\underset{\xi_{[0,T-1]}^i}{\operatorname{argmin}} \sum_{t=0}^{T-1} \left[\theta_t^{i^\top} \Lambda + \sum_{l=0}^{\tau_t^i} \sum_{j=l}^{\tau_t^i} \bar{b}_{j,t}^i \mathsf{Tr}(\tilde{P}_t^i A_i^{l-1^\mathsf{T}} \Sigma_w^i A_i^{l-1}) \right]$$

$$\text{s. t.} \quad \bar{b}_{j,t}^i = \prod_{d=0}^{j-1} \prod_{l=0}^{d} [1 - \theta_{t-d}^i(l)] [\sum_{d=0}^{j} \theta_{t-j}^i(d)],$$

$$\sum_{l=0}^{D} \theta_t^i(l) = 1, \; \sum_{j=0}^{\tau_t^i} \bar{b}_{j,t}^i = 1, \; \sum_{j=t+2}^{D} \bar{b}_{j,t}^i = 0 \tag{20}$$

and where $\tau_t^i \triangleq \min\{D, t+1\}$ and $\tilde{P}_t^i$ is defined in Corollary 1.

Proof: See the Appendix B. ■

Next, we propose a sufficient, but not necessary, capacity condition for $c_d, d \in \mathcal{D}$, ensuring that the reallocated resources remain within $\{\ell_{\max\{0,d-\alpha_i\}}, \ldots, \ell_d, \ldots, \ell_{\min\{d+\beta_i, D\}}\}$, and the MILP (21) as shown at the bottom of the next page is feasible. Selected c_d's should additionally satisfy constraints (2) and (3) to ensure that problem (10) is nontrivial, and to avoid packet drop out. We demonstrate in Section V that this condition is indeed conservative.

Corollary 2: The MILP problem (21) is feasible if (3) is satisfied and $\forall d \in \mathcal{D}$, the following sufficient condition holds:

$$c_d \geq \left\lfloor \frac{N}{1 + \frac{1}{N}[h_d(\alpha, \beta)]} \right\rfloor \tag{22}$$

with $h_d(\alpha, \beta) = \sum_{i \in N_1} \mathbb{1}(d\alpha_i) + \sum_{j \in N_2} \mathbb{1}((D-d)\beta_j) + \mathbb{1}(d) \sum_{l \in N_3} \mathbb{1}(d\alpha_l) + \mathbb{1}(D-d) \sum_{l \in N_3} \mathbb{1}((D-d)\beta_l)$, where we have $N_1 = \{i \in \mathrm{N} | \alpha_i \neq 0, \beta_i = 0\}$, $N_2 = \{j \in \mathrm{N} | \alpha_j = 0, \beta_j \neq 0\}$, and $N_3 = \{l \in \mathrm{N} | \alpha_l, \beta_l \neq 0\}$, with $|N_1| \cup |N_2| \cup |N_3| = N$.

2) Reactive Delay Control: We call the delay control policy *reactive* if the decisions on θ_k^i's are per-time made incorporating the knowledge of $\vartheta_{[0,k-1]}^i$. Thus, the information set $\bar{\mathcal{I}}_k^i$ upon which $\theta_k^i = \xi_k^i(\bar{\mathcal{I}}_k^i)$ is computed needs to contain $\vartheta_{[0,k-1]}^i$, hence $\bar{\mathcal{I}}_k^i$ coincides with (13).

Theorem 3: Consider the optimization problem (10). Let $\gamma^{i,*}, i \in \mathrm{N}$ follow the certainty equivalence law (14) and (15). Given the information sets $\bar{\mathcal{I}}_k^i$ and $\tilde{\mathcal{I}}_k$, respectively, in (13) and (12), the optimal reactive delay control law is computed online from the following constrained MILP:

$$\theta_{[k,T-1]}^{i,*} = \underset{\xi_{[k,T-1]}^i}{\operatorname{argmin}} \; J^i(\gamma^{i,*}, \xi_{[k,T-1]}^i(\bar{\mathcal{I}}_{[k,T-1]}^i)) =$$

$$\underset{\xi_{[k,T-1]}^i}{\operatorname{argmin}} \sum_{t=k}^{T-1} \left[\theta_t^{i^\top} \Lambda + \sum_{l=0}^{\tau_t^i} \sum_{j=l}^{\tau_t^i} \tilde{b}_{j,t}^i \mathsf{Tr}(\tilde{P}_t^i A_i^{l-1^\mathsf{T}} \Sigma_w^i A_i^{l-1}) \right]$$

$$\text{s. t.} \;\; \tilde{b}_{0,t}^i = \theta_t^i(0), \quad \tilde{b}_{j,t}^i \leq \sum_{l=0}^{j} \vartheta_{t-j}^i(l), \; j \in \{1, \ldots, \tau_t^i\}$$

$$\sum_{l=0}^{D} \theta_t^i(l) = 1, \; \sum_{j=0}^{\tau_t^i} \tilde{b}_{j,t}^i = 1, \; \sum_{j=t+2}^{D} \tilde{b}_{j,t}^i = 0, t \geq k \tag{23}$$

where τ_t^i and $\tilde{P}_t^i$ are similarly defined as in Theorem 2, and

$$\tilde{b}_{j,t}^i = \left[[1 - \theta_t^i(0)] \prod_{d=1}^{j-1} \prod_{l=0}^{d} [1 - \vartheta_{t-d}^i(l)] \right] \left[\sum_{d=0}^{j} \vartheta_{t-j}^i(d) \right]$$

with $\prod_{d=1}^{0} \prod_{l=0}^{d} [1 - \vartheta_{t-d}^i(l)] \triangleq 1$, for notation convenience.

Moreover, the optimal resource allocation law is computed online from the following constrained MILP:

$$\vartheta_{[k,T-1]}^* = \underset{\pi_{[k,T-1]}}{\operatorname{argmin}} \; \frac{1}{N} \sum_{i=1}^{N} \sum_{t=k}^{T-1} \Big[\vartheta_t^{i^\top} \Lambda$$

$$+ \sum_{l=0}^{\tau_t^i} \sum_{j=l}^{\tau_t^i} b_{j,t}^i \mathsf{Tr}(\tilde{P}_t^i A_i^{l-1^\mathsf{T}} \Sigma_w^i A_i^{l-1}) \Big]$$

$$\text{s. t.} \quad -\alpha_i \leq (\vartheta_t^i - \theta_t^{i,*})^\top \Delta \leq \beta_i, \; b_{j,t}^i \text{ as defined in (18)}$$
$$\sum_{i=1}^{N} \vartheta_t^i(d) \leq c_d, \; \forall d \in \mathcal{D}, \; t \in [k, T-1]. \tag{24}$$

Proof: Derivation of optimal policies in Theorem 3 follows similarly to that of Theorem 2 and hence omitted. The major differences are summarized in Remark 3. ■

Remark 3: In Theorem 3, the reactive delay controller is aware of $\vartheta_{[0,k-1]}^{i,*}$ and incorporates them in deciding $\theta_{[k,T-1]}^{i,*}$. Hence, unlike Theorem 2, here we solve a per-time-step MILP. Technically, the online nature of the MILP (23) is reflected in the time-varying $\tilde{b}_{j,t}^i$ that results in a time-varying $\theta_{[k,T-1]}^{i,*}$. Comparing it with $\bar{b}_{j,t}^i$ in Theorem 2, we see that for each time k, $\theta_{[k,T-1]}^{i,*}$ depends on $\vartheta_{[k-D,k-1]}^{i,*}$, while in Theorem 2, the same decision was dependent only on $\theta_{[k-D,k-1]}^{i,*}$. The MILP problem (24) also becomes online as it needs to satisfy the time-varying constraint $-\alpha_i \leq (\vartheta_t^i - \theta_t^{i,*})^\top \Delta \leq \beta_i$.

Remark 4: The optimal impassive delay control and resource allocation variables $(\theta_{[0,T-1]}^*, \vartheta_{[0,T-1]}^*)$ in Theorem 2 require offline MILPs (20) and (21) of complexity $\mathcal{O}(NdT)$, while the same variables of the reactive approach in Theorem 3 require online MILPs (23) and (24) of complexity $\mathcal{O}(NdT^2)$. This confirms that both approaches incur linear complexity growth with respect to the number of subsystems and the number of transmission links. However, complexity of the reactive approach grows quadratically with the time horizon T while the respective growth rate for the impassive approach is linear.

Remark 5: According to (17), the state estimation at the controller is performed using the freshest received state information, and hence, if an outdated state arrives while a fresher one is available, the former will not be used. In addition, both local and social objective functions (7) and (8) include communication costs. Therefore, to reduce the total cost, the delay controllers and the resource manager try to avoid transmission decisions that lead to out-of-order delivery of state information. This is reflected in the formulated MILPs in Theorems 2 and 3. This is, however, unavoidable due to constraint (5) that forces each subsystem to select one delay link $\ell_d \in \mathcal{L}$ while the maximum delay D is finite. Intuitively, many of transmissions with D-step delay would not have been executed if the subsystems had the option to remain open-loop and select *no transmission*. Hence, outdated information appearing at subsequent time-steps is discarded if a fresher data exists.

Corollary 3: Let the performance of the local policy codesign $(\gamma^{i,*}, \xi^{i,*}, \pi^*)$ for the impassive and reactive approaches be denoted, respectively, by $J_{\text{Im}}^{i,*}$ and $J_{\text{Re}}^{i,*}$, defined in (7), and also denote the social performance of the overall joint design (γ^*, ξ^*, π^*) by J_{Im}^* and J_{Re}^*, defined in (8). Let $\gamma^{i,*}$, $\xi^{i,*}$, and π^* of the impassive approach be computed as (14), (20), and (21), and of the reactive approach as (14), (23), and (24), respectively. Then, $J_{\text{Re}}^{i,*} \leq J_{\text{Im}}^{i,*}$ and $J_{\text{Re}}^* \leq J_{\text{Im}}^*$.

Proof: See Appendix C. ■

C. Optimal Resource Allocation Without Model Awareness

In an NCS, the individual entities may not be willing to share the specifications of their dynamical model or their objective functions with the communication service provider. Within our problem formulation, this essentially means that the network manager does not have the knowledge of constant parameters $\{A_i, B_i, Q_1^i, Q_2^i, R^i, \Sigma_w^i, \Sigma_{x_0}^i\}$, $i \in \mathrm{N}$. Technically, having no knowledge of the constant parameters (except α_i, β_i), the local cost functions J^i are not computable for the network manager, and hence the optimal resource allocation policy cannot be obtained from the problem (10a). More precisely, although the local policies $\gamma^{i,*}$'s and $\xi^{i,*}$'s can still be computed from (14), (20), and (23), for impassive and reactive approaches, respectively, π^* cannot be obtained from the either problems (21) and (24). Let the information set $\tilde{\mathcal{I}}_k$ the network manager be defined as in (12) but excluding the knowledge of the constant parameters of all subsystems except α_i, β_i's. Then, the best the network manager can perform is to allocate resources such that, given α_i, β_i's, the average deviation between the delay control and resource allocation decisions is minimized, which is the first term in the MILPs (21) and (24). Hence, the optimal resource allocation for the impassive approach will be obtained from

$$\vartheta_{[0,T-1]}^* = \underset{\pi_{[0,T-1]}}{\text{argmin}} \; \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T-1} \vartheta_t^{i\top} \Lambda$$
$$\text{s. t.} \quad -\alpha_i \leq (\vartheta_t^i - \theta_t^{i,*})^\top \Delta \leq \beta_i, \; i \in \mathrm{N}$$
$$\sum_{i=1}^{N} \vartheta_t^i(d) \leq c_d, \; \forall d \in \mathcal{D}, \; t \in [0, T-1] \tag{25}$$

$$\vartheta_{[0,T-1]}^* = \underset{\pi_{[0,T-1]}}{\text{argmin}} \; \frac{1}{N} \sum_{i=1}^{N} J^i(\gamma^{i,*}, \pi_{[0,T-1]}(\tilde{\mathcal{I}}_{[0,T-1]}))$$
$$= \underset{\pi_{[0,T-1]}}{\text{argmin}} \; \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T-1} \left[\vartheta_t^{i\top} \Lambda + \sum_{l=0}^{\tau_t^i} \sum_{j=l}^{\tau_t^i} b_{j,t}^i \mathsf{Tr}(\tilde{P}_t^i A_i^{l-1^\top} \Sigma_w^i A_i^{l-1}) \right]$$
$$\text{s. t.} \quad -\alpha_i \leq (\vartheta_t^i - \theta_t^{i,*})^\top \Delta \leq \beta_i, \; b_{j,t}^i \text{ as defined in (18),}$$
$$\sum_{i=1}^{N} \vartheta_t^i(d) \leq c_d, \; \forall d \in \mathcal{D}, \; t \in [0, T-1]. \tag{21}$$

and for the reactive approach, is obtained from

$$\vartheta^*_{[k,T-1]} = \underset{\pi_{[k,T-1]}}{\text{argmin}} \ \frac{1}{N}\sum_{i=1}^{N}\sum_{t=k}^{T-1}\vartheta_t^{i^\top}\Lambda$$

$$\text{s. t.} \quad -\alpha_i \le (\vartheta_t^i - \theta_t^{i,*})^\top\Delta \le \beta_i,\ i\in \mathrm{N}$$

$$\sum_{i=1}^{N}\vartheta_t^i(d) \le c_d,\ \forall d\in\mathcal{D},\ t\in[k,T-1] \qquad (26)$$

where $\theta_t^{i,*}$ in (25) is the solution of the impassive approach (20), while in (26) is solution of the reactive approach (23).

From (25) and (26), in the absence of the constant model parameters, the resource manager only optimizes the communication cost, and that the allocated resource to remain within the sensitivity constraint (10c). This results in a solution for ϑ that tends to select the transmission links that incur the least communication cost ignoring that such selections may severely affect the control cost. To counter that, in the reactive approach where the delay controller can adjust its link selection profile in response to the resource allocation policy, each system changes their $\theta_k^{i,*}$ drastically for the future time-steps to request for faster links aiming to reduce the control cost. Assume a system asked for a fast link, e.g., with delay zero, due to its task criticality; however, the network manager does not realize the urgency due to not being capable of estimating the control cost and allocates a higher latency transmission link (say $d=2$) which optimizes only the communication cost. The system will then be forced to select a low delay link again since its past request is not served accordingly. This approach thus leads to higher total cost of control and communication compared to the scenario that the resource manager knows the constant model parameters. Furthermore, when constant model parameters are assumed unknown, the reactive approach performs significantly better than its impassive counterpart since the systems will be generally unhappy of this agnostic resource allocation, and hence respond with a significantly different θ_k^* than the prescribed ϑ_k^* that leads to a very different ϑ_{k+1}^* than ϑ_k^*.

D. Delay-Insensitive Optimal Resource Allocation

For the purpose of benchmarking and comparing the two methods presented in the previous sections, we propose another ad hoc approach by extending the work of [29] to a multiagent scenario. More specifically, the approach presented in this section adopts a formulation that does not consider the delay sensitivity in the formulation, rather solely interested in the capacity constraint. This means that the resource manager ignores the knowledge of $\theta^i_{[0,k]}$ and $\{\alpha_i,\beta_i\}$'s, $i\in\mathrm{N}$, however, knows the constant model parameters of all subsystems. We define constant weights $w_i>0$ such that $\sum_{i=1}^{N} w_i=1$. The network manager then prioritizes each subsystem based on w_i and optimizes the MILP at every time-step k, i.e.,

Notice that since there is no coupling between ϑ_t and θ_t contrasting to the formulations in (24) and (26), $\vartheta^*_{[k,T-1]}$ can be found from $\vartheta^*_{[0,T-1]}$ without solving (27) as shown at the bottom of this page for all k. In fact, if $\vartheta^*_{[0,T-1]}$ is the solution of (27) for $k=0$, then the part $\vartheta^*_{[t,T-1]}$ of $\vartheta^*_{[0,T-1]}$ is the solution of (27) for any $k=t$. Furthermore, any feasible solution of (24) is a feasible solution for (27), and hence, often the delay-insensitive approach results in a lower social cost than the delay-sensitive MILP in (24). However, the lower social cost in this approach is obtained at the expense of higher deviations between the desired links and the allocated ones since no constraint of the form $-\alpha_i \le (\vartheta_t^i - \theta_t^{i,*})^\top\Delta \le \beta_i$ exists to restrict the deviation between ϑ_t^i and $\theta_t^{i,*}$. Hence, the social performance is expected to improve; however, certain individual subsystems suffer as their link allocation is far from the ones requested. This trade-off needs to be attended for the resource manager to be sufficiently responsive to timeliness sensitivity of local subsystems.

V. Simulation Results

We consider an NCS consisting of 10 homogeneous stable and 10 homogeneous unstable subsystems. The system and input matrices for the unstable and stable groups are $A^u=\begin{bmatrix}1.01 & 0.2\\ 0.2 & 1\end{bmatrix}$, $A^s=\begin{bmatrix}0.5 & 0.1\\ 0.6 & 0.8\end{bmatrix}$, and $B^u=B^s=\begin{bmatrix}0.1 & 0\\ 0 & 0.15\end{bmatrix}$, respectively. The disturbance is Gaussian distributed with mean and variance as $\mathcal{N}(0,1.5I_2)$. The LQG cost parameters for all subsystems are identically set as $Q_1^i=Q_2^i=R^i=I_2$, and $T=20$ is the total time horizon of the simulations.

The network supports the control loops via six transmission links with delays of $d\in[0,1,2,3,4,5]$ time-steps associated with the cost $\Lambda=[25,17,11,7,4,1]$. We assume $c_d=6$, $\forall d$, and $\alpha_i=\beta_i=3$, $\forall i\in\{1,\dots,20\}$. Note that $c_d=6$ satisfies the individual and total capacity constraints (2) and (3), however, does not meet the sufficient feasibility condition (22) for $d=\{0,5\}$[6] and yet is a valid choice for this simulation setup, which shows (22) is sufficient but not a necessary condition.

We illustrate the optimal delay control and link allocation for each subsystem using the discussed approaches: 1) with model awareness, 2) without model awareness, and 3) delay-insensitive approach, as presented in Sections IV-B, IV-C, and IV-D, respectively. For the first two approaches, we employ both *reactive* and *impassive* methods to perform optimal codesign and compare

[6]According to (22), $c_d\ge 10$ for $d=\{0,5\}$ and $c_d\ge 6$ for $d=\{1,2,3,4\}$.

$$\vartheta^*_{[k,T-1]} = \underset{\pi_{[k,T-1]}}{\text{argmin}} \sum_{i=1}^{N} w_i\,\mathsf{E}\Big[V_k^{i,*}(\gamma^{i,*},\pi^i) + \sum_{t=k}^{T-1}\vartheta_t^{i^\top}\Lambda\big|\tilde{\mathcal{I}}_k\Big] = \underset{\pi_{[k,T-1]}}{\text{argmin}} \sum_{i=1}^{N}\sum_{t=k}^{T-1} w_i\Big[\vartheta_t^{i^\top}\Lambda + \sum_{l=0}^{\tau_t^i}\sum_{j=l}^{\tau_t^i} b_{j,t}^i\,\mathsf{Tr}(\tilde{P}_t^i A_i^{l-1^\mathsf{T}}\Sigma_w^i A_i^{l-1})\Big]$$

$$\text{s. t.} \quad \sum_{i=1}^{N}\vartheta_t^i(d)\le c_d\ \forall d\in\mathcal{D},\ t\in[k,T-1]. \qquad (27)$$

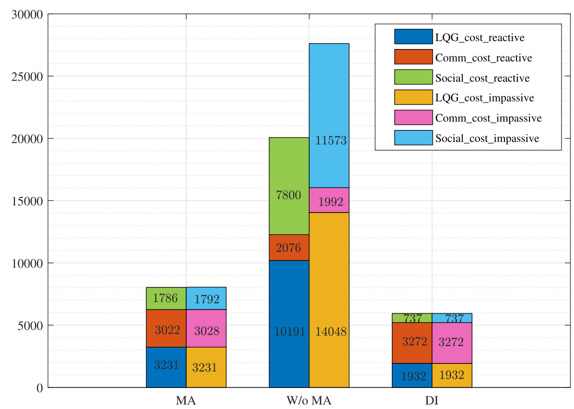

Fig. 4. Optimal LQG, communication, and social costs for different approaches. MA: with model awareness (Section IV-B), W/o MA: without model awareness (Section IV-C), DI: delay-insensitive (Section IV-D).

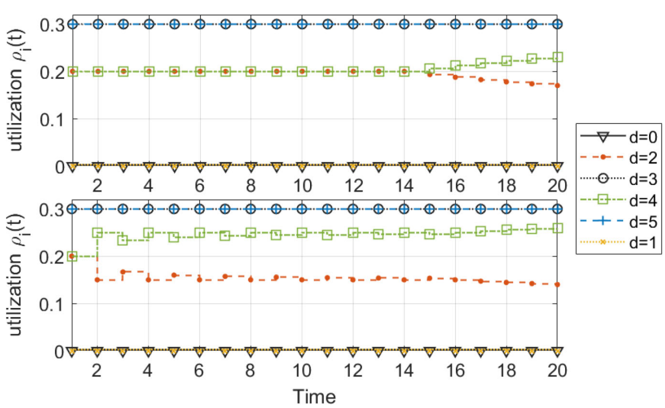

Fig. 5. Link utilization over time under capacity constraints without model awareness. Top: reactive method; bottom: impassive method.

their outcomes. As discussed in Corollary 3, we demonstrate that the reactive method performs no worse than the impassive method and may often perform significantly better, due to the dynamic coupling between θ and ϑ. Since such coupling does not exist in the delay-insensitive case, reactive and impassive methods yield identical results.

In Fig. 4, we illustrate the control, communication, and social costs for the abovementioned approaches, where the cost values are cumulative with respect to time, i.e., not time-averaged. We observe that the awareness of the constant model parameters leads to a significant performance improvement when compared with no model awareness scheme. However, as also discussed in Section IV-C, the superiority of the reactive approach over the impassive counterpart is far better for the case without model awareness. This can be observed in Fig. 4 for both local cost and the social cost values. In fact, one needs to contemplate whether to employ the reactive approach when the network is aware of the constant model parameters of the control systems, due to the insignificant overall performance augmentation achieved at the expense of the extra computational complexity imposed (see Remark 4).

Fig. 5 shows the transmission link utilization profile [defined in (28)] where we only provide the plot for the impassive and reactive scenarios when the network manager is not aware of the constant model parameters (Section IV-C).

$$\rho_i(t) = \frac{\#\text{ of utilization of link } i \text{ up to time } t}{N(t+1)}. \qquad (28)$$

According to (28), $\sum_{i=1}^{N} \rho_i(t) = 1$ at every time t, that is also reflected in Fig. 5. For the case without model awareness, the network manager only cares about the communication cost and hence the cheaper links are utilized, as can be seen in Fig. 5. Notice that link 3 is used more than link 4 due to the coupling constraints between θ_t and ϑ_t in (25) and (26). The subsystems which requested for the link ℓ_0 cannot be assigned to any link beyond ℓ_3 since $\beta_i = 3$. Thus, the majority of the requests for link ℓ_0 were assigned to ℓ_3 and the rest were assigned to ℓ_2 (ℓ_1 is more expensive). Similarly, the majority of the requests for ℓ_5 are assigned to ℓ_5 and the rest to ℓ_4, etc.

We also studied this problem for the case with model awareness, and we noticed that the difference in the link utilization is minor between the two impassive and reactive approaches (as also corroborated by the cost difference in Fig. 4). In fact, the link utilization, in this case, changes only after time $t = 15$. This observation brings out the question whether it makes sense to adopt the computationally expensive reactive approach over the simple impassive approach for this little improvement. Based on this observation, one may be tempted to adopt reactive approach in an intermittent fashion, i.e., instead of solving (24) for every k, do so at $k = t_1, t_2, \ldots, t_\ell$, where $0 < t_1 < \ldots < t_\ell < T$. An interesting yet challenging research question is how to determine $t_1, \ldots, t_\ell$. One may perhaps adopt an event-based strategy to solve for these quantities; we, however, leave this as a future research.

Next we study the average deviation between the requested θ^* and the allocated ϑ^*, computed by the following formula:

$$\Delta_i(t) = \frac{\sum_{i=1}^{N} \sum_{k=0}^{t} |(\vartheta_k^{i,*} - \theta_k^{i,*})^\top \Delta|}{N(t+1)}. \qquad (29)$$

We report the average deviation result for all three approaches in Fig. 6. The figure also shows that the average deviation is generally higher for the delay-insensitive approach compared to both delay-sensitive scenarios of reactive and impassive, confirming the explanations in Section IV-D.

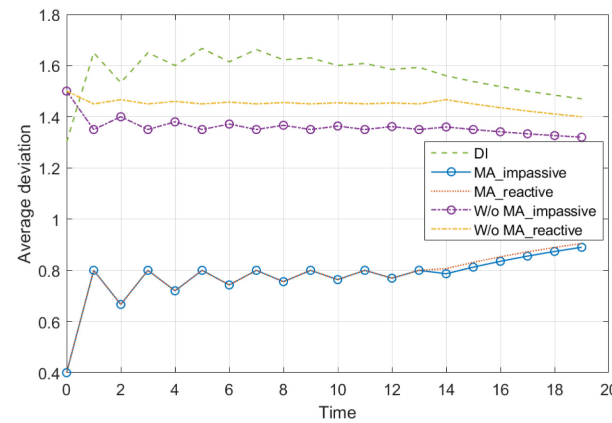

Fig. 6. Average deviation in the allocated links as computed by (29).

APPENDIX

A. Proof of Theorem 1

According to the information sets $\mathcal{I}_k^i$, $\tilde{\mathcal{I}}_k$ and $\bar{\mathcal{I}}_k^i$ in (11)–(13), and considering (7) and (8), we can restate (10) as

$$\min_{\gamma^i,\xi^i,\pi} J = \frac{1}{N}\sum_{i=1}^{N} \mathsf{E}\left[\min_{\gamma^i,\pi} J^i(u^i,\vartheta^i) - \right.$$
$$\left.\min_{\gamma^i,\xi^i} \mathsf{E}\left[\|x_T^i\|^2_{Q_2^i} + \sum_{k=0}^{T-1}\|x_k^i\|^2_{Q_1^i} + \|u_k^i\|^2_{R^i} + \theta_k^{i^\top}\Lambda\right]\right]. \quad (30)$$

where, for the first term of (30), we obtain the following due to the one-directional independence of ϑ_k^i from u_k^i

$$J^i(u^i,\vartheta^i) = \mathsf{E}\left[\mathsf{E}\left[\sum_{k=0}^{T-1}\vartheta_k^{i^\top}\Lambda\Big|\tilde{\mathcal{I}}_k\right]\right] +$$
$$\mathsf{E}\left[\mathsf{E}\left[\|x_T^i\|^2_{Q_2^i} + \sum_{k=0}^{T-1}\|x_k^i\|^2_{Q_1^i} + \|u_k^i\|^2_{R^i}\Big|\mathcal{I}_k^i,\tilde{\mathcal{I}}_k\right]\right].$$

We define $V_k^i = \|x_T^i\|^2_{Q_2^i} + \sum_{t=k}^{T-1}\|x_t^i\|^2_{Q_1^i} + \|u_t^i\|^2_{R^i}$. Since γ^i is a local policy and its decision outcome u^i is independent of all subsystems $j \neq i$, and moreover, π is independent of all γ_i's, the optimal cost-to-go can be expressed as

$$\min_{\substack{\gamma^i_{[k,T-1]}\\ \pi_{[k,T-1]}}} J^i(u^i,\vartheta^i) = \min_{\pi_{[k,T-1]}} \mathsf{E}\left[\min_{\gamma^i_{[k,T-1]}} \mathsf{E}\left[V_k^i\big|\mathcal{I}_k^i\right] + \right.$$
$$\left.\min_{\pi_{[k,T-1]}} \mathsf{E}\left[\sum_{t=k}^{T-1}\vartheta_t^{i^\top}\Lambda\big|\tilde{\mathcal{I}}_k\right]\Big|\tilde{\mathcal{I}}_k\right]. \quad (31)$$

For $J^i(u^i,\theta^i)$, we know $\bar{\mathcal{I}}_k^i \subseteq \mathcal{I}_k^i$ $\forall k$, from (11) and (13). Moreover, u_k^i and θ_k^i are measurable with respect to $\mathcal{I}_k^i$ and $\bar{\mathcal{I}}_k^i$, respectively. Therefore, employing the tower property,[7] and also using the law of total expectation,[8] we rewrite (7) as

$$J^i(u^i,\theta^i) =$$
$$\mathsf{E}\left[\mathsf{E}\left[\mathsf{E}\left[\|x_T^i\|^2_{Q_2^i} + \sum_{k=0}^{T-1}\|x_k^i\|^2_{Q_1^i} + \|u_k^i\|^2_{R^i} + \theta_k^{i^\top}\Lambda\Big|\mathcal{I}_k^i\right]\Big|\bar{\mathcal{I}}_k^i\right]\right].$$

Hence, introducing $C_k^i(u^i,\theta^i) = V_k^i + \sum_{t=k}^{T-1}\theta_t^{i^\top}\Lambda$, we obtain

$$\min_{\substack{\gamma^i_{[k,T-1]}\\ \xi^i_{[k,T-1]}}} J^i(u^i,\theta^i) = \mathsf{E}\left[\min_{\xi^i_{[k,T-1]}} \mathsf{E}\left[\min_{\gamma^i_{[k,T-1]}} \mathsf{E}\left[C_k^i(u^i,\theta^i)|\mathcal{I}_k^i\right]\Big|\bar{\mathcal{I}}_k^i\right]\right].$$

Finally, we can re-express (30) as

$$\min_{\gamma^i,\xi^i,\pi} J = \frac{1}{N}\sum_{i=1}^{N}\mathsf{E}\left\{\min_{\pi}\mathsf{E}\left[\min_{\gamma^i}\mathsf{E}\left[V_0^i\big|\mathcal{I}_0^i\right]\Big|\tilde{\mathcal{I}}_0\right]\right.$$
$$+\min_{\pi}\mathsf{E}\left[\sum_{k=0}^{T-1}\vartheta_k^{i^\top}\Lambda\Big|\tilde{\mathcal{I}}_0\right]$$
$$\left.-\min_{\xi^i}\mathsf{E}\left[\min_{\gamma^i}\mathsf{E}\left[V_0^i + \sum_{k=0}^{T-1}\theta_k^{i^\top}\Lambda\Big|\mathcal{I}_0^i\right]\Big|\bar{\mathcal{I}}_0^i\right]\right\}. \quad (32)$$

The sole γ^i-dependent term in the above expression is $\mathsf{E}[V_0^i|\mathcal{I}_0^i]$, and this term is minimized only by the control law γ^i. Therefore, for all $k \in [0, T-1]$, the following control law solves the inner optimization problem $\min_{\gamma^i}\mathsf{E}[V_0^i|\mathcal{I}_0^i]$

$$u^{i,*}_{[k,T-1]} = \gamma^{i,*}_{[k,T-1]}(\mathcal{I}_k^i) = \underset{\gamma^i_{[k,T-1]}}{\text{argmin}}\ \mathsf{E}\left[V_k^i|\mathcal{I}_k^i\right]$$
$$= \underset{\gamma^i_{[k,T-1]}}{\text{argmin}}\ \mathsf{E}\left[\|x_T^i\|^2_{Q_2^i} + \sum_{t=k}^{T-1}\|x_t^i\|^2_{Q_1^i} + \|u_t^i\|^2_{R^i}\big|\mathcal{I}_k^i\right]. \quad (33)$$

The last expression (33) is a standard LQG problem, and the optimal law $\gamma_k^{i,*}$ and gain $L_k^{i,*}$ in (14) and (15) are the solutions of (33). (Full derivation can be found in [29].)

B. Proof of Theorem 2

Having Assumption 1, and knowing $\bar{\mathcal{I}}_k^i \subseteq \mathcal{I}_k^i$, we begin from (32). Recall that $\vartheta^i_{[0,k-1]} \notin \bar{\mathcal{I}}_k^i$, hence, to decide θ_k^i, the delay controller presumes that the control signal is generated according to $\theta^i_{[0,k-1]}$ not $\vartheta^i_{[0,k-1]}$. We derived the optimal control policy in (32); therefore, the optimal impassive delay control policy $\xi_k^{i,*}(\bar{\mathcal{I}}_k^i)$ will be obtained by minimizing the local LQG cost function $J^i(u^{i,*},\theta^i)$, i.e., $\forall k \in [0, T-1]$

$$\theta^{i,*}_{[k,T-1]} = \underset{\xi^i_{[k,T-1]}}{\text{argmin}}\ \mathsf{E}\left[V_k^{i,*}(\gamma^{i,*},\xi^i) + \sum_{t=k}^{T-1}\theta_t^{i^\top}\Lambda|\bar{\mathcal{I}}_k^i\right]. \quad (34)$$

Recalling Remark 2, we compute $V_k^{i,*}(\gamma^{i,*},\xi^i)$ at the impassive delay controller side. From the estimator dynamics (17) and system dynamics (1), the estimation error e_k^i evolves as

$$e_k^i = \sum_{l=1}^{\tau_k^i}\sum_{j=l}^{\tau_k^i}\bar{b}^i_{j,k}A_i^{l-1}w^i_{k-l}$$

where $b^i_{j,k}$ in (17) is replaced by $\bar{b}^i_{j,k}$ because the delay controller has no knowledge about the variables $\{\vartheta_0^i,\ldots,\vartheta_{k-1}^i\}$. Since $\bar{\mathcal{I}}_k^i \subseteq \mathcal{I}_k^i$, it is, moreover, straightforward to compute $\mathsf{E}[\mathsf{E}[e_k^i e_k^{i^\top}|\mathcal{I}_k^i]|\bar{\mathcal{I}}_k^i] = \mathsf{E}[e_k^i e_k^{i^\top}|\bar{\mathcal{I}}_k^i]$, as follows:

$$\mathsf{E}[e_k^i e_k^{i^\top}|\bar{\mathcal{I}}_k^i] = \sum_{l=1}^{\tau_k^i}\sum_{j=l}^{\tau_k^i}\bar{b}^i_{j,k}\,\mathsf{E}\left[A_i^{l-1}w^i_{k-l}w^{i^\top}_{k-l}A_i^{l-1^\top}\right]$$
$$= \sum_{l=1}^{\tau_k^i}\sum_{j=l}^{\tau_k^i}\bar{b}^i_{j,k}A_i^{l-1}\Sigma^i_{k-l}A_i^{l-1^\top}$$

where $\Sigma^i_{k-l} = \Sigma^i_{x_0}$, $k<l$, and $\Sigma^i_{k-l} = \Sigma^i_w$, $k \geq l$. Having this and noting that $\bar{\mathcal{I}}_0^i = \{A_i, B_i, Q_1^i, Q_2^i, R^i, \Sigma_w^i, \Sigma_{x_0}^i\}$, we can

[7]For a random variable X defined on a probability space with sigma-algebra $\mathcal{F}$, if $\mathsf{E}[X] < \infty$, then for any two subsigma-algebras $\mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \mathcal{F}$, $\mathsf{E}[\mathsf{E}[X|\mathcal{F}_2]|\mathcal{F}_1] = \mathsf{E}[X|\mathcal{F}_1]$ *almost surely*.

[8]If the random variable X is $\mathcal{F}$-measurable, then $\mathsf{E}[\mathsf{E}[X|\mathcal{F}]] = \mathsf{E}[X]$.

rewrite $\mathsf{E}[V_0^{i,*}(\gamma^{i,*},\xi^i)|\bar{\mathcal{I}}_0^i]$ as follows:

$$\mathsf{E}\,[V_0^{i,*}(\gamma^{i,*},\xi^i)|\bar{\mathcal{I}}_0^i] = \|\mathsf{E}\,[x_0^i]\|^2_{P_0^i} + \sum_{t=1}^{T}\mathsf{Tr}(P_t^i\Sigma_w^i)$$
$$+\,\mathsf{Tr}\left(P_0^i\sum_{l=1}^{\tau_0^i}\sum_{j=l}^{\tau_0^i}\bar{b}^i_{j,0}A_i^{l-1^\top}\Sigma^i_{x_0}A_i^{l-1}\right)$$
$$+\sum_{t=0}^{T-1}\mathsf{Tr}\left(\tilde{P}_t^i\sum_{l=1}^{\tau_t^i}\sum_{j=l}^{\tau_t^i}\bar{b}^i_{j,t}A_i^{l-1^\top}\Sigma^i_{t-l}A_i^{l-1}\right). \tag{35}$$

As the only $\theta^i_{[0,T-1]}$-dependent term above is the last term, the problem (34) can be expressed, initiating from $k=0$, as

$$\theta^{i,*}_{[0,T-1]} = \underset{\xi^i_{[0,T-1]}}{\text{argmin}}\ \mathsf{E}\left[V_0^{i,*}(\gamma^{i,*},\xi^i) + \sum_{t=0}^{T-1}\theta_t^{i^\top}\Lambda|\bar{\mathcal{I}}_0^i\right] =$$
$$\underset{\xi^i_{[0,T-1]}}{\text{argmin}}\sum_{t=0}^{T-1}\left[\mathsf{Tr}(\tilde{P}_t^i\sum_{l=1}^{\tau_t^i}\sum_{j=l}^{\tau_t^i}\bar{b}^i_{j,t}A_i^{l-1^\top}\Sigma^i_{t-l}A_i^{l-1}) + \theta_t^{i^\top}\Lambda\right].$$

The constraints of problem (20) are all linear and θ_k^i is binary-valued, and hence the above problem is an MILP. Moreover, it is independent from both the noise realizations and $\vartheta_{[0,T-1]}$, and thus $\theta^*_{[0,T-1]}$ can be computed offline. The constraint $\sum_{l=0}^{D}\theta_t^i(l)=1$ ensures that only one delay link is selected per-time, while the last two constraints look after convenient indexes for $\bar{b}^i_{j,k}$ for $k\geq D$ and $k<D$ (see Corollary 1).

To find π^*, we use a similar procedure to that of computing $\xi^{i,*}$, except ϑ_k^i is now computed knowing the information $\{\theta^{i,*}_{[0,k]},\vartheta^{i,*}_{[0,k-1]}\}\ \forall i$. We compute $\mathsf{E}[V_0^{i,*}(\gamma^{i,*},\pi)|\tilde{\mathcal{I}}_0]$ that results in a similar expression as on the right side of the equality in (35) with the exception being $\bar{b}^i_{j,t}$ replaced by $b^i_{j,t}$. Hence, from (32), and the constrain (10c)–(10 d), we derive the optimal resource allocation offline from the following MILP:

$$\vartheta^*_{[k,T-1]} = \underset{\pi_{[k,T-1]}}{\text{argmin}}\ \frac{1}{N}\sum_{i=1}^{N}\mathsf{E}\left[V_k^{i,*}(\gamma^{i,*},\pi^i)+\sum_{t=k}^{T-1}\vartheta_t^{i^\top}\Lambda|\tilde{\mathcal{I}}_k\right]$$
$$= \underset{\pi_{[k,T-1]}}{\text{argmin}}\ \frac{1}{N}\sum_{i=1}^{N}\sum_{t=k}^{T-1}\left[\vartheta_t^{i^\top}\Lambda + \sum_{l=0}^{\tau_t^i}\sum_{j=l}^{\tau_t^i}b^i_{j,t}\mathsf{Tr}(\tilde{P}_t^iA_i^{l-1^\top}\Sigma_w^iA_i^{l-1})\right].$$

Since $\theta^{i,*}_{[0,T-1]}$ is computed offline from (20) independent of $\vartheta^i_{[0,T-1]}$, we can set $k=0$ above to complete the proof.

C. Proof of Corollary 3

The control policy $\gamma^{i,*}$ follows (14) for both impassive and reactive scenarios, so we only compare the optimal cost values of the joint policies $(\xi^{i,*},\pi^*)$ from Theorems 2 and 3. Define $(\bar{\theta}^{i,*},\bar{\vartheta}^{i,*})$ and $(\tilde{\theta}^{i,*},\tilde{\vartheta}^{i,*})$, respectively, as the joint optimal impassive and reactive delay control and resource allocation variables over time horizon $[0,T]$. First assume $\bar{\theta}^{i,*}=\tilde{\theta}^{i,*}$, then $\bar{b}^i_{j,t}=\tilde{b}^i_{j,t}\forall t$ must hold from (20) and (23), which leads to $\bar{\vartheta}^{i,*}=\tilde{\vartheta}^{i,*}$ from (21) and (24). As problems (20) and (23), and (21) and (24) coincide, we have $J^{i,*}_{\text{Re}}=J^{i,*}_{\text{Im}}$ and $J^*_{\text{Re}}=J^*_{\text{Im}}$.

Now assume $\bar{\theta}^{i,*}\neq\tilde{\theta}^{i,*}$. Since the information set $\bar{\mathcal{I}}^i_{[0,T-1]}$ in (19) for the impassive approach is a subset of its counterpart in (13) for the reactive approach, any optimal solution of problem (20) can also be obtained from problem (23) if it is optimal for the latter. Hence, if $\bar{\theta}^{i,*}\neq\tilde{\theta}^{i,*}$, then $\bar{\theta}^{i,*}$ is not the optimal solution of problem (23), which implies $J^{i,*}_{\text{Re}}(u^{i,*},\tilde{\theta}^{i,*})<J^{i,*}_{\text{Im}}(u^{i,*},\bar{\theta}^{i,*})$. Now let $\tilde{\vartheta}^{i,*}$ be the optimal solution of problem (24) such that $\tilde{\vartheta}^{i,*}\neq\bar{\vartheta}^{i,*}$ while $J^*_{\text{Re}}>J^*_{\text{Im}}$. Recall that $\bar{\vartheta}^{i,*}$ is the optimal resource allocation in response to $\bar{\theta}^{i,*}$ computed from (20), while we know if $\tilde{\vartheta}^{i,*}\neq\bar{\vartheta}^{i,*}$, then $\bar{\theta}^{i,*}\neq\tilde{\theta}^{i,*}$. Knowing this, together with $J^*_{\text{Re}}>J^*_{\text{Im}}$, implies that the joint policy $(\bar{\theta}^{i,*},\bar{\vartheta}^{i,*})$ outperforms $(\tilde{\theta}^{i,*},\tilde{\vartheta}^{i,*})$, which requires $J^{i,*}_{\text{Re}}(u^{i,*},\tilde{\theta}^{i,*})>J^{i,*}_{\text{Im}}(u^{i,*},\bar{\theta}^{i,*})$ to hold. This, however, contradicts the previous condition ensuring that if $\bar{\theta}^{i,*}\neq\tilde{\theta}^{i,*}$, then $J^{i,*}_{\text{Re}}(u^{i,*},\tilde{\theta}^{i,*})<J^{i,*}_{\text{Im}}(u^{i,*},\bar{\theta}^{i,*})$, and hence the condition $J^*_{\text{Re}}>J^*_{\text{Im}}$ cannot be realized if $\tilde{\vartheta}^{i,*}\neq\bar{\vartheta}^{i,*}$.

References

[1] E. Molina and E. Jacob, "Software-defined networking in cyber-physical systems: A survey," *Comput. Elect. Eng.*, vol. 66, pp. 407–419, 2018.

[2] B. Bordel, R. Alcarria, T. Robles, and D. Martín, "Cyber-physical systems: Extending pervasive sensing from control theory to the internet of things," *Pervasive Mobile Comput.*, vol. 40, pp. 156–184, 2017.

[3] J. S. Baras, "A fresh look at network science: Interdependent multigraphs models inspired from statistical physics," in *Proc. 6th Int. Symp. Commun., Control Signal Process.*, pp. 497–500, 2014.

[4] J. Baillieul and P. J. Antsaklis, "Control and communication challenges in networked real-time systems," *Proc. IEEE*, vol. 95, no. 1, pp. 9–28, Jan. 2007.

[5] S. Shakkottai, T. S. Rappaport, and P. C. Karlsson, "Cross-layer design for wireless networks," *IEEE Commun. Mag.*, vol. 41, no. 10, pp. 74–80, Oct. 2003.

[6] J. Bai, E. P. Eyisi, F. Qiu, Y. Xue, and X. D. Koutsoukos, "Optimal cross-layer design of sampling rate adaptation and network scheduling for wireless networked control systems," in *Proc. 3rd Int. Conf. Cyber-Phys. Syst.*, 2012, pp. 107–116.

[7] F. Xia, "QoS challenges and opportunities in wireless sensor/actuator networks," *Sensors*, vol. 8, no. 2, pp. 1099–1110, 2008.

[8] M. A. Yigitel, O. D. Incel, and C. Ersoy, "QoS-aware MAC protocols for wireless sensor networks: A survey," *Comput. Netw.*, vol. 55, no. 8, pp. 1982–2004, 2011.

[9] Y. Bi, L. X. Cai, X. S. Shen, and H. Zhao, "Medium access control for QoS provisioning in V2I communication networks," *Mobile Netw. Appl.*, vol. 18, no. 2, pp. 174–185, 2013.

[10] K. B. Letaief and Y. J. Zhang, "Dynamic multiuser resource allocation and adaptation for wireless systems," *IEEE Wireless Commun.*, vol. 13, no. 4, pp. 38–47, Aug. 2006.

[11] G. Song and Y. Li, "Utility-based resource allocation and scheduling in OFDM-based wireless broadband networks," *IEEE Commun. Mag.*, vol. 43, no. 12, pp. 127–134, Dec. 2005.

[12] Q. Zhang and Y. Zhang, "Cross-layer design for QoS support in multihop wireless networks," *IEEE Proc.*, vol. 96, no. 1, pp. 64–76, Jan. 2008.

[13] B. Li, Y. Ma, T. Westenbroek, C. Wu, H. Gonzalez, and C. Lu, "Wireless routing and control: A cyber-physical case study," in *Proc. 7th Int. Conf. Cyber-Phys. Syst.*, 2016, pp. 1–10.

[14] H. Susanto and B. G. Kim, "Congestion control with QoS and delays utility function," in *Proc. 22nd Int. Conf. Comput. Commun. Netw.*, 2013, pp. 1–5.

[15] H. Wang, C. Liao, and Z. Tian, "Providing quality of service over time delay networks by efficient queue management," in *Proc. IEEE 36th Conf. Local Comput. Netw.*, pp. 275–278, 2011.

[16] F. Forni, S. Galeani, D. Nesić, and L. Zaccarian, "Event-triggered transmission for linear control over communication channels," *Automatica*, vol. 50, no. 2, pp. 490–498, 2014.

[17] X. Wang and M. D. Lemmon, "Event-triggering in distributed networked control systems," *IEEE Trans. Autom. Control*, vol. 56, no. 3, pp. 586–601, Mar. 2011.

[18] D. Maity and J. S. Baras, "Optimal event-triggered control of nondeterministic linear systems," *IEEE Trans. Autom. Control*, vol. 65, no. 2, pp. 604–619, Feb. 2020.
[19] W. P. M. H. Heemels, K. H. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *Proc. 51st IEEE Conf. Decis. Control*, pp. 3270–3285, 2012.
[20] Y. Tang, H. Gao, and J. Kurths, "Robust H_∞ self-triggered control of networked systems under packet dropouts," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 3294–3305, Dec. 2016.
[21] K. Hashimoto, S. Adachi, and D. V. Dimarogonas, "Self-triggered model predictive control for nonlinear input-affine dynamical systems via adaptive control samples selection," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 177–189, Jan. 2017.
[22] S. Trimpe, "Predictive and self triggering for event-based state estimation," in *Proc. 55th IEEE Conf. Decis. Control*, pp. 3098–3105, 2016.
[23] W. P. M. H. Heemels, A. R. Teel, N. van de Wouw, and D. Nesić, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Autom. Control*, vol. 55, no. 8, pp. 1781–1796, Aug. 2010.
[24] K. Okano, M. Wakaiki, G. Yang, and J. Hespanha, "Stabilization of networked control systems under clock offsets and quantization," *IEEE Trans. Autom. Control*, vol. 63, no. 6, pp. 1708–1723, Jun. 2018.
[25] B. Demirel, V. Gupta, D. E. Quevedo, and M. Johansson, "Threshold optimization of event-triggered multi-loop control systems," in *Proc. 13th Int. Workshop Discrete Event Syst.*, pp. 203–210, 2016.
[26] D. Yue, E. Tian, and Q. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Trans. Autom. Control*, vol. 58, no. 2, pp. 475–481, Feb. 2013.
[27] B. W. Carabelli, R. Blind, F. Dürr, and K. Rothermel, "State-dependent priority scheduling for networked control systems," in *Proc. Amer. Control Conf.*, pp. 1003–1010, 2017.
[28] M. H. Mamduhi, A. Molin, D. Tolić, and S. Hirche, "Error-dependent data scheduling in resource-aware multi-loop networked control systems," *Automatica*, vol. 81, pp. 209–216, 2017.
[29] D. Maity, M. H. Mamduhi, S. Hirche, K. H. Johansson, and J. S. Baras, "Optimal LQG control under delay-dependent costly information," *IEEE Contr. Syst. Lett.*, vol. 3, no. 1, pp. 102–107, Jan. 2019.

Mohammad H. Mamduhi received the B.Sc. in mechanical engineering from the Sharif University of Technology, Tehran, Iran, in 2008, and M.Sc. degree in systems, control, and robotics, from KTH Royal Institute of Technology, Stockholm, Sweden, in 2010, and the Ph.D. degree in electrical and computer engineering from the Technical University of Munich, Germany, in 2017.

He is a Senior Scientist with the Automatic Control Laboratory, ETH Zürich, Switzerland. He was a Postdoctoral Researcher with the Division of Decision and Control Systems, KTH Royal Institute of Technology. His research interests include networked control systems, cyber-physical systems, and stochastic systems.

Dipankar Maity (Member, IEEE) received the B.E. degree in electronics and telecommunication engineering from Jadavpur University, India, in 2013, and the Ph.D. degree in electrical and computer engineering from the University of Maryland, College Park, MD, USA in 2018.

He is an Assistant Professor with the Department of Electrical and Computer Engineering, University of North Carolina at Charlotte, Chapel Hill, NC, USA. He was a Postdoctoral Fellow with the Georgia Institute of Technology, Atlanta, GA, USA. During his Ph.D., he was a Visiting Scholar at the Technical University of Munich (TUM) and at the KTH Royal Institute of Technology, Stockholm, Sweden. His research interests include temporal logic-based controller synthesis, control under communication constraints, intermittent-feedback control, stochastic games, and integration of these ideas in the context of cyber-physical systems.

Sandra Hirche (Fellow, IEEE) received the Diplom-Ingenieur degree from Technical University Berlin, Germany, in 2002, and the Doktor-Ingenieur degree from Technical University Munich (TUM), Germany, in 2005.

From 2005 to 2007, she was a JSPS Postdoctoral Scholar with the Fujita Laboratory, Tokyo Institute of Technology, Japan. She was an Associate Professor with TUM, in 2008. Since 2013, she has been TUM Liesel Beckmann Distinguished Professor and heads the Chair of Information-Oriented Control, Department of Electrical and Computer Engineering, TUM. She has authored/coauthored more than 200 papers in international journals, books, and refereed conferences. Her research interests include learning, cooperative, and networked control with applications in human–robot interaction, haptics, multirobot systems, and general robotics.

She has received multiple awards such as the Rohde & Schwarz PhD Award, the 2005 IFAC World Congress Best Poster Award, the Best Paper Award of the 2009 IEEE Worldhaptics, and the Outstanding Student Paper Award of the 2018 IEEE Conference on Decision and Control. In 2013, she was awarded with an ERC Starting Grant on the "Control based on Human Models" and in 2019 with an ERC Consolidator Grant on "Safe data-driven control for human-centric systems."

John S. Baras (Life Fellow, IEEE) received the Diploma in electrical and mechanical engineering from the National Technical University of Athens, Athens, Greece, in 1970, and the M.S. and Ph.D. degrees in applied mathematics from Harvard University, Cambridge, MA, USA, in 1971 and 1973, respectively.

Since 1973, he has been with the Department of Electrical and Computer Engineering, University of Maryland at College Park, MD, USA, where he is currently a Distinguished University Professor. He is also a Faculty Member of the Applied Mathematics, Statistics and Scientific Computation Program, and Affiliate Professor with the Fischell Department of Bioengineering, the Department of Mechanical Engineering, and the Department of Decision, Operations and Information Technologies, Robert H. Smith School of Business. Since 2013, he has been Guest Professor with the School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden. From 1985 to 1991, he was the Founding Director of the Institute for Systems Research (ISR). In 1990, he was appointed to the endowed Lockheed Martin Chair in Systems Engineering. Since 1992, he has been the Director of the Maryland Center for Hybrid Networks (HYNET), College Park, MD, USA, which he cofounded.

John S. Baras is a SIAM Fellow, AAAS Fellow, NAI Fellow, IFAC Fellow, AMS Fellow, AIAA Associate Fellow, Member of the National Academy of Inventors (NAI), and a Foreign Member of the Royal Swedish Academy of Engineering Sciences (IVA). He received the 1980 George Axelby Award from the IEEE Control Systems Society, the 2006 Leonard Abraham Prize from the IEEE Communications Society, the 2014 Tage Erlander Guest Professorship from the Swedish Research Council, a three-year (2014–2017) Senior Hans Fischer Fellowship from the Institute for Advanced Study of the Technical University of Munich, Germany, the 2017 Institute for Electrical and Electronics Engineers (IEEE) Simon Ramo Medal, the 2017 American Automatic Control Council (AACC) Richard E. Bellman Control Heritage Award, and the 2018 American Institute for Aeronautics and Astronautics (AIAA) Aerospace Communications Award. In 2016, he was inducted in the University of Maryland A. J. Clark School of Engineering Innovation Hall of Fame. In 2018, he was awarded a Doctorate Honoris Causa by the National Technical University of Athens, Greece.

Karl H. Johansson (Fellow, IEEE) received the M.Sc. degree in electrical engineering, and Ph.D. degree in automatic control from Lund University, Lund, Sweden, in 1992 and 1997.

He is Professor with the School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Sweden, and Director of Digital Futures, Sweden. He has held visiting positions at UC Berkeley, Caltech, NTU, HKUST Institute of Advanced Studies, and NTNU. His research interests include networked control systems and cyber-physical systems with applications in transportation, energy, and automation networks.

Dr. Johansson is a member of the Swedish Research Council's Scientific Council for Natural Sciences and Engineering Sciences. He has served on the IEEE Control Systems Society Board of Governors, the IFAC Executive Board, and is currently Vice-President of the European Control Association. He has received several best paper awards and other distinctions from IEEE, IFAC, and ACM. He has been awarded Distinguished Professor with the Swedish Research Council and Wallenberg Scholar with the Knut and Alice Wallenberg Foundation. He has received the Future Research Leader Award from the Swedish Foundation for Strategic Research and the triennial Young Author Prize from IFAC. He is Fellow of the Royal Swedish Academy of Engineering Sciences, and he is IEEE Control Systems Society Distinguished Lecturer.