LQG Control and Scheduling Co-design for Wireless Sensor and Actuator Networks*

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***Abstract*—This paper studies a co-design problem of control, scheduling and routing over a multi-hop sensor and actuator network subject to energy-saving consideration. Sensors are observing multiple independent linear systems and transmit their data to actuators in which controllers are co-located. We formulate an optimization problem, minimizing a linear combination of the averaged linear quadratic Gaussian control performance and the averaged transmission energy consumption. Optimal solutions are derived and their performance is illustrated in a numerical example. Algorithms to reconfigure routing between sensors and actuators in case of link disconnection are also provided.**

***Index Terms*—LQG control, wireless sensor and actuator networks**

I. Introduction

The control of process plants with wireless sensors and actuators is of significant interest to process industries [1]. Process control over a wireless network offers advantages through enhanced and massive sensing, flexible deployment and operation, and more efficient maintenance. However, since wireless sensors have usually no inexhaustible or reliable energy sources, energy limitation of wireless sensors affect system performance. In this context, energy-aware protocols, real-time algorithms as well as empirical studies for optimizing the performance of wireless sensor networks have been discussed in [2]–[4]. Furthermore, there are a lot of theoretical results using event-triggering approach to reduce communication and energy usage for state estimation and control over wireless networks [5]–[8]. In [6], [7], event-triggered state estimation where communications are invoked based on estimation error covariance is proposed. LQG control based on covariance triggering is discussed in [8]. Under a related setup, [9], [10] consider sensor scheduling for remote state estimation. In [9], the authors study remote estimation of multiple linear systems where at most one sensor can communicate with the remote estimator at every time instance. In [10], a remote estimation problem over a multi-hop wireless network is discussed. However, despite the fact that a multi-hop wireless network architecture is accepted in some industrial standards such as wirelessHART [11], [12], it is still unclear how these event-triggering and sensor scheduling frameworks affect the performance of closed-loop systems. In this context, a co-design problem of control, scheduling, routing over multi-hop network are proposed [13], which minimizes L_2 gain of the error signal of the closed loop system with respect to a step reference.

*This work was supported in part by the VINNOVA PiiA project "Advancing System Integration in Process Industry," the Knut and Alice Wallenberg Foundation, the Swedish Strategic Research Foundation, and the Swedish Research Council.

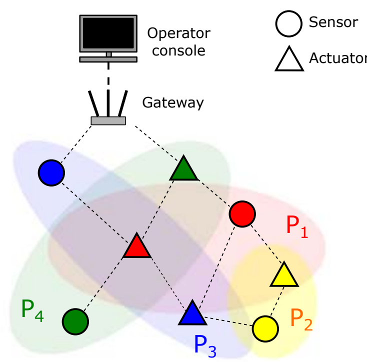

Fig. 1. Process control over wireless sensor and actuator network

In this paper, we study LQG control of multiple discrete-time linear systems with covariance-based triggering over a multi-hop sensor and actuator network (Figure 1). Here, sensors and actuators are distributed over a field and can communicate with their sensor and actuator neighbourhoods. We assume that the sensors and the actuators are smart enough to carry out regular estimation and control. This has been discussed as a future architecture for process automation [1]. In this system, each sensor communicates with the corresponding controller co-located with the actuator. In Figure 1, there are four plants (red, yellow, blue, and green) which are controlled by each local control loop consisting of a sensor and an actuator. To derive the LQG control gains and sensor schedules, we formulate an optimization problem which shows that the optimal solution is periodic. This implies that one can automatically determine a sampling time of the system, which otherwise is usually chosen by a heuristic [14]. We also offer algorithms implemented in the sensors and the actuators which can detect a network link disconnection and reroute its path when other paths are available.

The remainder of the paper is organized as follows. Sec-

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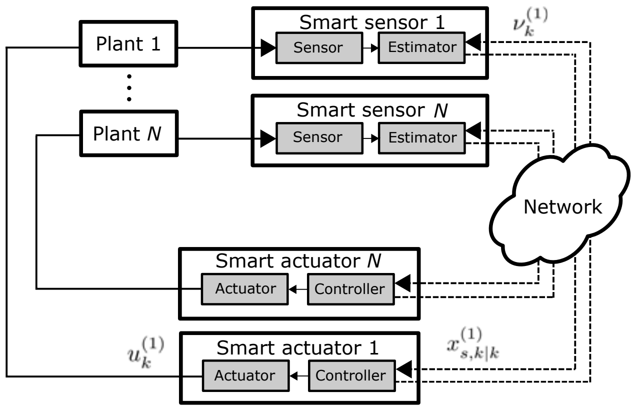

Fig. 2. System model

tion 2 describes the system including process and energy consumption models, and formulates the optimization problem. The optimal solution is discussed in Section 3. Algorithms for route reconfiguration are offered in Section 4. A numerical example is provided in Section 5. Section 6 presents the conclusion.

Notation: Throughout this paper, $\mathbb{N}$ and $\mathbb{R}$ are the sets of nonnegative integers and real numbers, respectively. The set of n by n positive definite matrices over the field $\mathbb{R}^{n\times n}$ is denoted as $\mathbb{S}^n_{++}$. For simplicity, we write $X > Y$ where $X, Y \in \mathbb{S}^n_{++}$, if $X - Y \in \mathbb{S}^n_{++}$.

II. Problem Formulation

A. System model

A diagram of the system model is shown in Figure 2. Consider N linear plants

$$x_{k+1}^{(i)} = A_i x_k^{(i)} + B_i u_k^{(i)} + w_k^{(i)}, \quad i \in \mathcal{N} \tag{1}$$

where $x_k^{(i)} \in \mathbb{R}^{n_i}$ is the state vector at time k, $u_k^{(i)} \in \mathbb{R}^{m_i}$ is the input, $w_k^{(i)} \in \mathbb{R}^{n_i}$ is zero-mean i.i.d. Gaussian noise with covariance W_i, and $\mathcal{N} = \{1, \dots, N\}$ is the plant index set, respectively. Each plant is monitored and controlled by a sensor-actuator pair $\mathcal{C}_i = \{s_i, a_i\}$. The sensors have measurements

$$y_k^{(i)} = C_i x_k^{(i)} + v_k^{(i)}, \quad i \in \mathcal{N} \tag{2}$$

where $y_k^{(i)} \in \mathbb{R}^{p_i}$ is the output, and $v_k^{(i)} \in \mathbb{R}^{p_i}$ is zero-mean i.i.d. Gaussian noise with covariance V_i, respectively. The pairs of N sensors and actuators are distributed over a field and connected through an underlying communication network denoted $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \bigcup_{i=1}^N \mathcal{C}_i$ is a sensor and actuator node set, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of communication links.

Define the information set available at sensor i at time k as

$$\mathcal{I}_{s,k}^{(i)} = \{y_0^{(i)}, \dots, y_k^{(i)}, u_0^{(i)}, \dots, u_{k-1}^{(i)}, \nu_0^{(i)}, \dots, \nu_k^{(i)}\}$$

where $\nu_k^{(i)} \in \{0, 1\}$ is decision variable such that $\nu_k^{(i)} = 1$ when the state estimate $\hat{x}_{s,k|k}^{(i)}$ is transmitted to actuator a_i. We assume that the transmission is carried out without failure until a link is disconnected. To detect the disconnection, decisions are made by each actuator and fed back to the corresponding sensor. Note that actuators are not required to transmit their decision at every time instance since the transmission is perfect and then the sensors can emulate the controllers.

The state estimate and the corresponding error covariance at sensor i are given by

$$\begin{aligned}
\hat{x}_{s,k|k-1}^{(i)} &\triangleq \mathbb{E}[x_k^{(i)} | \mathcal{I}_{s,k-1}^{(i)}], \quad \hat{x}_{s,k|k}^{(i)} \triangleq \mathbb{E}[x_k^{(i)} | \mathcal{I}_{s,k}^{(i)}] \\
P_{s,k|k-1}^{(i)} &\triangleq \mathbb{E}[(x_k^{(i)} - \hat{x}_{s,k|k-1}^{(i)})(x_k^{(i)} - \hat{x}_{s,k|k-1}^{(i)})^{\mathrm{T}} | \mathcal{I}_{s,k-1}^{(i)}] \\
P_{s,k|k}^{(i)} &\triangleq \mathbb{E}[(x_k^{(i)} - \hat{x}_{s,k|k}^{(i)})(x_k^{(i)} - \hat{x}_{s,k|k}^{(i)})^{\mathrm{T}} | \mathcal{I}_{s,k}^{(i)}].
\end{aligned}$$

In the same way, define the information set at actuator i at time k as

$$\begin{aligned}
\mathcal{I}_{a,k}^{(i)} = \{\nu_0^{(i)}, \dots, \nu_k^{(i)}, \nu_0^{(i)} \hat{x}_{s,0|0}^{(i)}, \dots, \nu_k^{(i)} \hat{x}_{s,k|k}^{(i)}, \\
u_0^{(i)}, \dots, u_{k-1}^{(i)}, \}
\end{aligned}$$

and the state estimate and the error covariance

$$\begin{aligned}
\hat{x}_{a,k|k-1}^{(i)} &\triangleq \mathbb{E}[x_k^{(i)} | \mathcal{I}_{a,k-1}^{(i)}], \quad \hat{x}_{a,k|k}^{(i)} \triangleq \mathbb{E}[x_k^{(i)} | \mathcal{I}_{a,k}^{(i)}] \\
P_{a,k|k-1}^{(i)} &\triangleq \mathbb{E}[(x_k^{(i)} - \hat{x}_{a,k|k-1}^{(i)})(x_k^{(i)} - \hat{x}_{a,k|k-1}^{(i)})^{\mathrm{T}} | \mathcal{I}_{a,k-1}^{(i)}] \\
P_{a,k|k}^{(i)} &\triangleq \mathbb{E}[(x_k^{(i)} - \hat{x}_{a,k|k}^{(i)})(x_k^{(i)} - \hat{x}_{a,k|k}^{(i)})^{\mathrm{T}} | \mathcal{I}_{a,k}^{(i)}].
\end{aligned}$$

B. Energy consumption

We introduce the energy consumption model used in [15], [16]. For data receiving and sending, a node consumes its energy

$$E_R = E_{\text{elec}} f, \quad E_S = E_{\text{elec}} f + E_{\text{amp}} d^2 f,$$

respectively, where f [bit] is an amount of data receiving or sending and d is a distance to a downstream node. Note that the energy consumption for sending depends on the link used. Denote $\theta_k^{(i)}((j,l)) : \mathcal{E} \to \{0, 1\}$ as the indicator function whether the data of sensor i is sent through link (j,l) at time k. If link (j,l) is used, then $\theta_k^{(i)}((j,l)) = 1$, otherwise 0. Then the energy consumption of node $j \in \mathcal{V}$ at time k is given by

$$\begin{aligned}
E_{j,k} = &\sum_{l:(l,j)\in\mathcal{E}} \Big[E_{\text{elec}} \sum_{i\in\mathcal{N}} c_i \theta_k^{(i)}((j,l)) \Big] \\
&+ \sum_{l:(j,l)\in\mathcal{E}} \Big[(E_{\text{elec}} + E_{\text{amp}} d_{jl}^2) \sum_{i\in\mathcal{N}} c_i \theta_k^{(i)}((j,l)) \Big]
\end{aligned} \tag{3}$$

where c_i [bit] is a constant amount of data transmitted from sensor i to actuator i. It is reasonable to assume that data flow is conserved such that for all $i \in \mathcal{N}$ and $k > 0$:

$$\sum_{l:(j,l)\in\mathcal{E}} \theta_k^{(i)}((j,l)) - \sum_{l:(l,j)\in\mathcal{E}} \theta_k^{(i)}((j,l)) = 0, \text{ if } j \neq s_i, a_i, \tag{4a}$$

$$\sum_{l:(j,l)\in\mathcal{E}} \theta_k^{(i)}((j,l)) - \sum_{l:(l,j)\in\mathcal{E}} \theta_k^{(i)}((j,l)) = \nu_k^{(l)}, \text{ if } j = s_i, \tag{4b}$$

$$\sum_{l:(j,l)\in\mathcal{E}} \theta_k^{(i)}((j,l)) - \sum_{l:(l,j)\in\mathcal{E}} \theta_k^{(i)}((j,l)) = -\nu_k^{(l)}, \text{ if } j = a_i, \tag{4c}$$

in order to guarantee that sensor data can reach the corresponding actuator.

C. Optimization problem

We formulate an optimization problem as LQG control with network node energy consumption to find the optimal feedback control, scheduling and routing. The problem is given by

$$\min_{\{\nu_k,u_k,\theta_k\}} \limsup_{T\to\infty} \frac{1}{T}\sum_{k=0}^{T-1}\Bigg[\sum_{i=1}^{N}(x_k^{(i)\mathrm{T}}Q_i x_k^{(i)} + u_k^{(i)\mathrm{T}}R_i u_k^{(i)}) + \sum_{j\in\mathcal{V}}\beta_j E_{j,k}\Bigg] \tag{5a}$$

$$\text{s.t. (4)}, \quad i\in\mathcal{N} \tag{5b}$$

with a weight factor $\beta_i > 0$, where $\nu_k = [\nu_k^{(1)},\dots,\nu_k^{(N)}]^{\mathrm{T}}$, $u_k = [u_k^{(1)\mathrm{T}},\dots,u_k^{(N)\mathrm{T}}]^{\mathrm{T}}$, and $\theta_k = [\theta_k^{(1)\mathrm{T}},\dots,\theta_k^{(N)\mathrm{T}}]^{\mathrm{T}}$ with $\theta_k^{(i)} = [\dots,\theta_k^{(i)}((j,l)),\dots]^{\mathrm{T}} \in \{0,1\}^{|\mathcal{E}|}$. Note that there is no controller which can access all the variables $\{\nu_k\}$, $\{u_k\}$, and $\{\theta_k\}$, but we will show in the next section that the optimal solution can be found by distributed optimization at each controller without loss of performance.

III. Optimal Controller and Scheduler

In this section, we discuss the optimality of problem (5). By equation (3), the last term of (5a) can be rewritten as

$$\begin{aligned}\sum_{j\in\mathcal{V}}\beta_j E_{j,k} &= \sum_{i\in\mathcal{N}}\Bigg[\sum_{(j,l)\in\mathcal{E}}(\beta_j E_{\text{elec}}+\beta_l E_{\text{elec}} + \beta_j E_{\text{amp}} d_{jl}^2)c_i\theta_k^{(i)}((j,l))\Bigg]\\ &\triangleq \sum_{i\in\mathcal{N}}\Bigg[\sum_{(j,l)\in\mathcal{E}}\alpha_{jl}c_i\theta_k^{(i)}((j,l))\Bigg] \triangleq \sum_{i\in\mathcal{N}} E_k^{(i)}\end{aligned} \tag{6}$$

where $E_k^{(i)}$ is a weighted total energy consumption for loop i transmission. Now, we have the following theorem.

Theorem 3.1: The optimal solution to problem (5) is obtained by solving the distributed optimization problem:

$$\min_{\{\nu_k^{(i)},u_k^{(i)},\theta_k^{(i)}\}} \limsup_{T\to\infty}\frac{1}{T}\sum_{k=0}^{T-1}\Big[x_k^{(i)\mathrm{T}}Q_i x_k^{(i)} + u_k^{(i)\mathrm{T}}R_i u_k^{(i)} + \nu_k^{(i)}\tilde{E}_i\Big] \tag{7}$$

where $\tilde{E}_i$ is the minimum-cost path for loop i when $\nu_k^{(i)} = 1$, i.e., $\tilde{E}_i$ is the optimal value of the problem:

$$\tilde{E}_i \triangleq \min_{\theta_k^{(i)}} \ \pi^{\mathrm{T}}\theta_k^{(i)} \quad \text{s.t. (4)} \tag{8}$$

where $\pi = [\dots,\pi_{jl},\dots]^{\mathrm{T}} \in \mathbb{R}^{|\mathcal{E}|}$ is given by $\pi_{jl} = \alpha_{jl}c_i$.

Proof: Using (6), the objective function (5a) is equivalent to the sum of the function

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{k=0}^{T-1}\Big[x_k^{(i)\mathrm{T}}Q_i x_k^{(i)} + u_k^{(i)\mathrm{T}}R_i u_k^{(i)} + E_k^{(i)}\Big]$$

up to $i = 1,\dots,N$. Since $E_k^{(i)}$ is only a function of $\theta_k^{(i)}$, and $x_k^{(i)}$ and $u_k^{(i)}$ are not affected by $\theta_k^{(i)}$, we can take any $\theta_k^{(i)}$ provided that (4) is satisfied. Thus, the optimal value of $E_k^{(i)}$ when $\nu_k^{(i)} = 1$ can be obtained by solving problem (8). ■

By Theorem 3.1, distributed optimization can achieve the optimality of problem (5).

Remark 3.2: Problem (8) is the shortest path problem which can be solved by polynomial-time algorithms [17]. The transmission paths are pre-calculated before starting the operation.

Remark 3.3: Problem (7) is a special case in [8] where the energy consumption is determined by problem (8) and where there is no packet drop.

To see the optimal solution of the distributed optimization problem (7), we state the following theorem.

Theorem 3.4: There exists a stationary solution to (7), and the solution $\{u_k^{(i)*}\}$ is given by

$$u_k^{(i)*} = -(B_i^{\mathrm{T}}S_iB_i + R_i)^{-1}B_i^{\mathrm{T}}S_iA_i\hat{x}_{a,k|k}^{(i)} \triangleq L_i^{(i)}\hat{x}_{a,k|k}^{(i)} \tag{9}$$

with

$$\hat{x}_{a,k|k}^{(i)} = \begin{cases} A_i\hat{x}_{a,k-1|k-1}^{(i)} + B_iu_{k-1}^{(i)}, & \text{if } \ \nu_k^{(i)} = 0\\ \hat{x}_{s,k|k}^{(i)} & \text{if } \ \nu_k^{(i)} = 1,\end{cases} \tag{10}$$

$$P_{a,k|k}^{(i)} = \begin{cases} A_iP_{a,k-1|k-1}^{(i)}A_i^{\mathrm{T}} + W_i, & \text{if } \ \nu_k^{(i)} = 0\\ \bar{P}_i, & \text{if } \ \nu_k^{(i)} = 1,\end{cases} \tag{11}$$

where $S_i \in \mathbb{S}_{++}^n$ is a solution of the Riccati equation

$$S_i = A_i^{\mathrm{T}}S_iA_i + Q_i - A_i^{\mathrm{T}}S_iB_i(B_i^{\mathrm{T}}S_iB_i+R_i)^{-1}B_i^{\mathrm{T}}S_iA_i$$

and $\bar{P}_i \in \mathbb{S}_{++}^n$ is a steady state of the error covariance at sensor i given by the standard Kalman filter. In addition, the stationary solution $\{\nu_k^{(i)*}\}$ is given by a threshold policy

$$\nu_k^{(i)*} = \begin{cases} 0, & \text{if } \ P_{a,k-1|k-1}^{(i)} < P_i^*\\ 1, & \text{otherwise},\end{cases} \tag{12}$$

where $P_i^* \in \mathbb{S}_{++}^n$ is the threshold matrix.

Proof: Follows from the proof of Theorem 3 in [8]. This is a special case when $\gamma_k = 1$ in [8]. ■

Remark 3.5: The controller (9) is a certainty equivalence controller and is optimal thanks to side-information available according to the control architecture in Figure 2 [18].

Remark 3.6: The schedule of $\nu_k^{(i)*}$ converges to the periodic solution when A_i is unstable. This follows from the fact that there exists $t_i \in \mathbb{N}$ such that $f_i^{t_i}(\bar{P}_i) = P_i^*$ where $f_i(X) \triangleq A_iXA_i^{\mathrm{T}} + W_i$ which is calculated numerically. See [7]. As in [7], [8], the optimal cost of problem (7) is given by

$$\mathrm{tr}\big(S_iQ_i\big) + \frac{1}{t_i+1}\Big[\mathrm{tr}\big((A_i^{\mathrm{T}}S_iA_i + W_i - S_i)\sum_{j=0}^{t_i}f^j(\bar{P}_i)\big) + \tilde{E}_i\Big].$$

IV. Link Disconnection and Route Reconfiguration

A controller recognizes a link disconnection when it fails to receive the new data from the sensor despite that $\nu_k^{(i)*} = 1$. In this case, the path is reconfigured by searching a new one. Let $\mathcal{P}_i = \{p_1^{(i)}, \ldots, p_j^{(i)}, \ldots, p_{M_i}^{(i)}\}$ be a set of possible paths form s_i to a_i where $p_j^{(i)} = ((s_i, \cdot), \ldots, (\cdot, a_i))$ is the jth minimum-cost path. Furthermore, let $\mathcal{M}_i = \{P_1^{(i)*}, \ldots, P_{M_i}^{(i)*}\}$ be the set of threshold matrices induced by each path. The sets $\mathcal{P}_i$ and $\mathcal{M}_i$ are assumed to be pre-set in s_i and a_i. Algorithms 1 and 2 provide the implementation of the proposed controller, scheduling, and routing reconfiguration. In the algorithms, if the controller detects a link disconnection, it changes its path to the second best one. If no other paths are available, the control loop goes to fail safe mode.

Algorithm 1 Iterative algorithm for smart sensor i

Calculate $\hat{x}_{s,k|k}^{(i)}$
if $\nu_k^{(i)} = 1$ **then**
  Send $\hat{x}_{s,k|k}^{(i)}$ along path $p_j^{(i)}$
end if
if New p_{j+l} received **then**
  Calculate $P_{a,k|k}^{(i)} = f^l(P_{a,k-1|k-1}^{(i)})$
  Set a new path $p_{j+l}^{(i)}$ and a threshold $P_{j+l}^{(i)*}$
else
  Calculate $P_{a,k|k}^{(i)}$ by (11)
end if
Calculate $\nu_{k+1}^{(i)}$ by (12)
$k \leftarrow k+1$

Algorithm 2 Iterative algorithm for smart actuator i

if $\nu_k^{(i)} = 1$ **and** $\hat{x}_{s,k|k}^{(i)}$ not received **then**
  Calculate $\hat{x}_{a,k|k}^{(i)} = A_i \hat{x}_{a,k-1|k-1}^{(i)} + B_i u_{k-1}^{(i)}$
  Calculate $P_{a,k|k}^{(i)} = f(P_{a,k-1|k-1}^{(i)})$
  if $j = M_i$ **then**
    Go to fail safe mode
  else
    Set a new path $p_{j+1}^{(i)}$ a threshold $P_{j+l}^{(i)*}$
    Send new path $p_{j+1}^{(i)}$ along a backward route of $p_{j+1}^{(i)}$
  end if
else
  Calculate $\hat{x}_{a,k|k}^{(i)}$ by (10)
  Calculate $P_{a,k|k}^{(i)}$ by (11)
end if
Calculate $u_k^{(i)}$ by (9)
Calculate $\nu_{k+1}^{(i)}$ by (12)
$k \leftarrow k+1$

V. Numerical Example

To illustrate our results, we consider a small network with $N = 3$ where sensors and actuators are distributed over a square field shown in Figure 3. The system parameters of the three plants are given by

$$A_1 = \begin{bmatrix} 1.3 & 0.5 \\ 0.2 & 0.9 \end{bmatrix}, \; A_2 = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.4 \end{bmatrix}, \; A_3 = \begin{bmatrix} 1.3 & 1.2 \\ 0 & 1 \end{bmatrix},$$

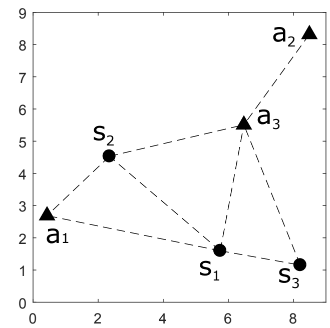

Fig. 3. Network with three sensor and actuator pairs over a field

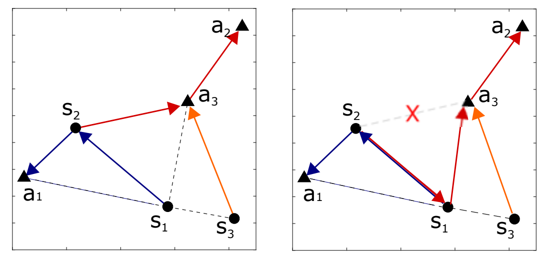

Fig. 4. The minimum-cost path for each loop before disconnection (left) and after disconnection (right)

$B_i = [1\ 2]^{\mathrm{T}}$, and $C_i = [1\ 1]$ for all $i = 1, 2, 3$. Furthermore, we have $W_i = 0.01I$, $V_i = 1$, $Q_i = I$, and $R_i = 1$ for all i. For communication parameters, we assume that $c_i = 4$ and $\beta_i = 5$ for $i = 1, 2, 3$, and $E_{\text{elec}} = E_{\text{amp}} = 1$. Under the given network, we can derive the minimum-cost path for each control loop as in Figure 4 (left). The optimal schedules are shown in Figure 5. We see that the solutions are periodic as stated in Remark 3.6. Sensor s_1 transmits its new estimate every eighth time instance, sensor s_2 every fourth time instance, and sensor s_3 every fifth time instance,

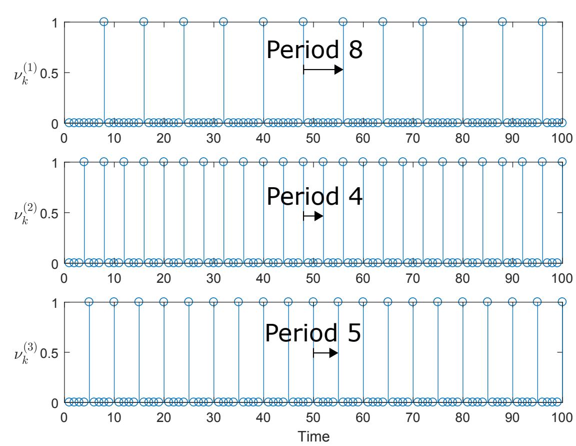

Fig. 5. Optimal schedules of three loops

	Proposed method	Every-time Communication
Averaged cost	$0.861 \cdot 10^4$	$3.022 \cdot 10^4$

TABLE I
COMPARISON OF AVERAGED COST BETWEEN THE PROPOSED METHOD AND THE EVERY-TIME COMMUNICATION CASE

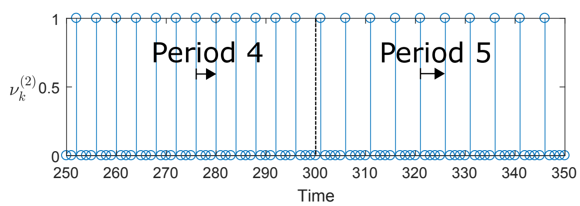

Fig. 6. Optimal schedule of loop 2 around rerouting at $k = 300$

respectively. The difference of the periods among the loops comes from the relation of the eigenvalues of A_i and the energy costs for transmission. The optimal averaged cost of the proposed method is shown in Table I compared with the case that when all the sensors communicate with the actuators at every time instance. We find that the proposed method obtains much lower cost than the every-time transmission case.

We also simulate the case that the link between s_2 and a_3 is disconnected at time $k = 300$ which leads to reroute the path between s_2 and a_2 to the second best path $s_2 \to s_1 \to a_3 \to a_1$ (Figure 4 (right)). The optimal schedule of loop 2 obtained by Algorithms 1 and 2 is indicated in Figure 6. Since it leads to more energy consumption, the period of loop 2 becomes five which is longer than the period before the disconnection. The averaged energy consumptions of sensors and actuators are shown in Figure 7. We found that the averaged energy consumption of s_1 increases after $k = 50$, since it is used as new intermediate node for loop 2.

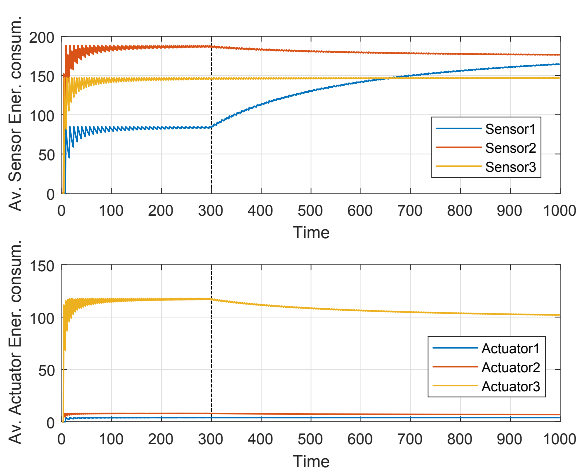

Fig. 7. Averaged energy consumption for sensor and actuator communication

VI. CONCLUSION

In this paper, we investigated the co-design framework of LQG control, sensor scheduling, and routing over a multi-hop sensor and actuator network by formulating the optimal problem which minimizes the infinite time averaged LQG control performance and energy consumption. We also proposed the algorithms for sensors and actuators to configure a new path when a link is disconnected. Possible future works will focus on the cases that communications have channel fading, delay and constraints regarding specific protocols such as wirelessHART.

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