Glocal Control for Network Systems via Hierarchical State-Space Expansion

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***Abstract*— In this paper, we propose a glocal (global/local) control method for large-scale network systems. The objective of the glocal control here is to suppress global propagation of a disturbance injected at a local subsystem based on the integration of different types of controllers, called global and local controllers. For the design of global and local controllers, we construct an aggregated model and a truncated model that can, respectively, capture global average behavior and local subsystem behavior of the network system of interest. Based on state-space expansion, called hierarchical state-space expansion, we show that a cascade interconnection of the aggregated model and the truncated model can be seen as a low-dimensional approximate model of the original network system, which has good compatibility with independent design of the global and local controllers. Furthermore, we show that appropriate integration of the global and local controllers can improve control performance with respect to global propagation of local disturbance. Finally, the effectiveness of the proposed method is shown through a numerical example of a benchmark model representing the bulk power system in the eastern half of Japan.**

I. INTRODUCTION

As technology advances, systems handled in control engineering have become larger and more complex. Typical examples include smart grids, where grid operators must maintain power balance among a number of generators and consumers while utilizing a large amount of renewable resources. As an approach to handle such large-scale complex network systems, distributed control [1], [2], [3] has been developed over the past few decades. Most distributed controller design methods need the entire system model. However, due to the complexity of large-scale systems, it is not reasonable to assume the availability of the exact model of the entire system. Instead, we, in practice, take an objective-based approach to system modeling and control, e.g., subsystem identification for local decentralized control and modeling of aggregate system behavior for global broadcast control. We integrate such different kinds of control to realize desired system behavior. Motivated by this practical controller design, a concept called *glocal (global/local) control* is introduced in [4]. However, it is still an open question to what system model is appropriate for systematic design of glocal control.

In [5], [6], the authors have proposed a retrofit controller design method for network systems. In this method, assuming that a preexisting network system is originally stable, we consider retrofitting a local decentralized controller that can improve control performance against disturbance that is directly injected only into a subsystem of the network system and we let such disturbance be called local disturbance. The key to designing such a retrofit controller is to use a state-space expansion technique, called *hierarchical state-space expansion*, that derives a higher-dimensional cascade realization of the original network system. The objective of the retrofit control is to design a local controller for the upstream part dynamics in the cascade realization that represents a truncated subsystem model, which we call a *local model*, while the downstream part dynamics representing the preexisting stable system is left untouched.

In this paper, as a generalization of the retrofit control, we propose a glocal control design method for large-scale network systems. The main idea for glocal control design is transforming the downstream part dynamics into a new coordinate system representing coherent and non-coherent dynamics. The coherent dynamics can be regarded as an aggregated model capturing global average behavior of the original system, called a *global model*. On the other hand, the non-coherent dynamics can be regarded as an error dynamics neglected though aggregation. In Section III, it is shown for a homogeneous network system that the non-coherent dynamics can be exactly eliminated, i.e., the global model is an exact low-dimensional model of the downstream part dynamics. This implies that the cascade interconnection of the local and global models, called a *glocal model*, corresponds to a low-dimensional realization of the original network system, which has good compatibility with glocal controller design.

We discuss similarities and differences between the proposed glocal control and *composite control*, which has been applied to many systems, e.g., power systems [8], [9], multi-robot networks [10], and transmission control protocols [11]. The composite control is based on the premise that the entire system dynamics is separated into slow and fast dynamics by using the singular perturbation technique [12]. Typically, the slow and fast dynamics are regarded as global and local models showing coherent and non-coherent behavior, respectively. However, it is not theoretically justified to derive such models by the time-scale separation. Indeed, the

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entire system possibly becomes unstable unless the time-scale separation is sufficient, due to unexpected interference between the controllers for fast and slow dynamics.

Finally, we refer to localized system level synthesis [13], [14], [15] as a different approach where the entire system model is not required for control of large-scale systems. In localized system level synthesis, optimal local controller synthesis is achieved for separable problems by making closed-loop responses to be finite impulse response and implementing the optimal controller with the desired system responses. While the applicability of that approach is limited to discrete-time systems instead of achieving optimality, our approach can be applied to continuous-time network systems.

This paper is organized as follows. In Section II, we formulate a glocal controller design problem. In Section III, we provide an illustrative example of the proposed glocal control and we develop a glocal control method for the homogeneous network systems. Numerical simulations for IEEJ (the Institute of Electrical Engineers of Japan) EAST 30-machine system [7], which is a benchmark model representing the bulk power system in the eastern half of Japan, are shown in Section IV and Section V draws conclusion.

A. Notation

We denote the N-dimensional all-ones vector by $\mathbf{1}_N$, the N-dimensional identity matrix by I_N, the image of a matrix A by $\mathrm{im}A$, the kernel by $\mathrm{ker}A$, the pseudo-inverse of a full-column rank matrix P by $P^{\dagger}$, the 2-induced norm of a matrix A by $\|A\|$, the Kronecker product by $\otimes$, the $\mathcal{L}_2$-norm of a square-integrable function $f(t)$ by $\|f\|_{\mathcal{L}_2}$ or $\|f(t)\|_{\mathcal{L}_2}$, the $\mathcal{H}_2$-norm and $\mathcal{H}_\infty$-norm of a transfer matrix $G(s)$ by $\|G(s)\|_{\mathcal{H}_2}$ and $\|G(s)\|_{\mathcal{H}_\infty}$, respectively, and the cardinality of a set $\mathcal{X}$ by $|\mathcal{X}|$. A map $\mathcal{F}(\cdot)$ is said to be a dynamical map if the triplet with $y = \mathcal{F}(u)$ solves a system of differential equations $\dot{x} = f(x,u), y = g(x,u)$ with some functions $f(\cdot)$ and $g(\cdot)$, and an initial value $x(0)$. A dynamical system is said to be stable if it is globally input-to-state stable in short.

II. PROBLEM FORMULATION

Consider an interconnected system each of whose subsystems is described by

$$\Sigma_i : \begin{cases} \dot{x}_i &= A_i x_i + L_i \sum_{j\in\mathcal{N}_i\cup\{i\}} \alpha_{i,j}\gamma_j + B_i u_i + R_i d_i \\ y_i &= C_i x_i \\ \gamma_i &= \mathit{\Gamma}_i x_i, \end{cases} \tag{1}$$

where x_i denotes the state, u_i denotes the external input signal, d_i denotes an unknown disturbance signal, y_i denotes the measurement output signal, γ_i denotes the interconnection output signal, $\alpha_{i,j}$ denotes a scalar weight coefficient, and $\mathcal{N}_i$ denotes the index set associated with the neighborhood of the ith subsystem. In the following, we suppose that the dimension of all subsystems is identical and it is denoted by n. The number of subsystems is denoted by N. Without loss of generality, we assume that $x_i(0) = 0$ for all $i \in \{1,\ldots,N\}$.

For this interconnected system, we suppose that just one local disturbance is injected to the system for simplicity though the following discussion can be easily extended to the case of multiple local disturbances. To this end, we give $d_1 = d_{\rm L},\ d_i = 0$ for all $i \in \{2,\ldots,N\}$. We assume that the local disturbance $d_{\rm L}$ is injected to the first subsystem. Our objective here is to suppress global propagation of the local disturbance in the framework of glocal control. More specifically, we suppose that a global input signal is injected to all subsystems in a broad cast manner and a local input signal is injected to the first subsystem. Such glocal inputs can be represented as

$$u_1 = u_{\rm G}/\sqrt{N} + u_{\rm L}, \quad u_i = u_{\rm G}/\sqrt{N}, \quad \forall i \in \{2,\ldots,N\}.$$

As measurement outputs to construct $u_{\rm G}$ and $u_{\rm L}$, we use the global and local outputs

$$y_{\rm G} = (y_1 + \cdots + y_N)/\sqrt{N}, \quad y_{\rm L} = y_1.$$

In this setting, the entire dynamics, having nN-dimension, is represented as

$$\Sigma : \begin{cases} \dot{x}_{\rm all} &= A_{\rm all}x_{\rm all} + B_{\rm G}u_{\rm G} + B_{\rm L}u_{\rm L} + R_{\rm L}d_{\rm L} \\ y_{\rm G} &= C_{\rm G}x_{\rm all} \\ y_{\rm L} &= C_{\rm L}x_{\rm all}, \end{cases} \tag{2}$$

where $x_{\rm all} := [x_1^{\mathsf T},\ldots,x_N^{\mathsf T}]^{\mathsf T}$, $A_{\rm all} = (A_{i,j})$ is given as

$$A_{i,j} = \begin{cases} A_i, & j = i \\ \alpha_{i,j}L_i\mathit{\Gamma}_j, & j \in \mathcal{N}_i \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

and the other matrices are given as

$$B_{\rm G} = \frac{1}{\sqrt{N}}\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix}, \quad B_{\rm L} = \begin{bmatrix} B_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad R_{\rm L} = \begin{bmatrix} R_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and

$$C_{\rm G} = \frac{1}{\sqrt{N}}\begin{bmatrix} C_1 & C_2 & \cdots & C_N \end{bmatrix}, \quad C_{\rm L} = \begin{bmatrix} C_1 & 0 & \cdots & 0 \end{bmatrix}.$$

We also assume that the entire system (2) is stabilizable and detectable.

In this paper, we do not assume that the complete system model of Σ is available for glocal controller design. Instead, we assume that a local subsystem model and an aggregated model are available for local and global controller design, respectively. This is formulated as follows. First, for the local controller design, the model of Σ_1 is assumed to be available, i.e., the system matrices A_1, B_1, C_1, L_1, $R_1, \mathit{\Gamma}_1$, and $\alpha_{1,j}$ for $j \in \mathcal{N}_1$ are assumed to be available. The model for local controller design, which we call here a local model, is described as

$$\mathbf{\Sigma}_{\rm L} : \begin{cases} \dot{\phi}_{\rm L} &= \boldsymbol{A}_{\rm L}\phi_{\rm L} + \boldsymbol{L}_{\rm L}\gamma_{\rm L} + \boldsymbol{B}_{\rm L}w_{\rm L} + \boldsymbol{R}_{\rm L}\eta_{\rm L} \\ z_{\rm L} &= \boldsymbol{C}_{\rm L}\phi_{\rm L}, \end{cases} \tag{4}$$

where $\phi_{\rm L}$ denotes the state of the local model, $\gamma_{\rm L}$ denotes the interconnection input signal, $w_{\rm L}$ and $z_{\rm L}$ denote the input and output signals for the attenuation of the disturbance $\eta_{\rm L}$. A simple choice of $\boldsymbol{\Sigma}_{\rm L}$, for example, can be given as

$$\boldsymbol{A}_{\rm L} = A_1,\ \boldsymbol{L}_{\rm L} = L_1,\ \boldsymbol{B}_{\rm L} = B_1,\ \boldsymbol{R}_{\rm L} = R_1,\ \boldsymbol{C}_{\rm L} = C_1. \tag{5}$$

However, there is a remaining degree of freedom to construct a more suitable model of $\boldsymbol{\Sigma}_{\rm L}$ based on the system matrices of Σ_1. We first ignore the interconnection input signal $\gamma_{\rm L}$, i.e., $\gamma_{\rm L} = 0$. Then we can design a local dynamical controller

$$K_{\rm L} : w_{\rm L} = \mathcal{K}_{\rm L}(z_{\rm L}; \boldsymbol{\Sigma}_{\rm L}) \tag{6}$$

such that the feedback system of (4) with $\gamma_{\rm L} = 0$ and (6) is stable and the control performance bound

$$\sup_{\|\eta_{\rm L}\|_{\mathcal{L}_2} \le 1} \|\phi_{\rm L}\|_{\mathcal{L}_2} \le \epsilon_{\rm L} \tag{7}$$

is satisfied for a given tolerance $\epsilon_{\rm L}$. In this formulation, we regard the local model $\boldsymbol{\Sigma}_{\rm L}$ as a design parameter to determine a specification of the local controller $K_{\rm L}$.

Provided that Σ_1 is isolated enough, i.e., the interconnection input signal $\sum_{j\in\mathcal{N}_1} \alpha_{1,j}\gamma_j$ in (1) is negligible, we can expect that the local control action $u_{\rm L} = \mathcal{K}_{\rm L}(y_{\rm L}; \boldsymbol{\Sigma}_{\rm L})$ with the choice of (5) works to stabilize the local subsystem Σ_1 of (2). However, in general cases, we do not have any guarantee not only for local subsystem control but also for entire system control. Moreover, the local control action may induce instability of the entire dynamics due to unexpected interference among subsystems.

The global input $u_{\rm G}$ is used to suppress interference propagation to other subsystems. For a model for global controller design, which we call a global model, we assume that an aggregated model of Σ described by

$$\boldsymbol{\Sigma}_{\rm G} : \left\{ \begin{array}{rcl} \dot{\phi}_{\rm G} &=& \boldsymbol{A}_{\rm G}\phi_{\rm G} + \boldsymbol{B}_{\rm G}w_{\rm G} + \boldsymbol{R}_{\rm G}\eta_{\rm G} \\ z_{\rm G} &=& \boldsymbol{C}_{\rm G}\phi_{\rm G}, \end{array}\right. \tag{8}$$

is available, where $\phi_{\rm G}$ denotes the state of the aggregated model, $w_{\rm G}$ and $z_{\rm G}$ denote the input and output signals for the attenuation of $\eta_{\rm G}$, which is regarded as a model of the interference signal. A simple model of $\boldsymbol{\Sigma}_{\rm G}$, for example, can be given as

$$\begin{array}{ll} \boldsymbol{A}_{\rm G} = \frac{1}{N}\sum_{i=1}^N \sum_{j=1}^N A_{i,j}, & \boldsymbol{B}_{\rm G} = \frac{1}{N}\sum_{i=1}^N B_i, \\ \boldsymbol{C}_{\rm G} = \sum_{i=1}^N C_i, & \boldsymbol{R}_{\rm G} = \frac{1}{N}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} A_{i,j}. \end{array} \tag{9}$$

There is a degree of freedom to determine a more suitable model of $\boldsymbol{\Sigma}_{\rm G}$. On the premise that an aggregated model is identified, we design a global dynamical controller

$$K_{\rm G} : w_{\rm G} = \mathcal{K}_{\rm G}(z_{\rm G}; \boldsymbol{\Sigma}_{\rm G}) \tag{10}$$

such that the feedback system of (8) and (10) is stable and the control performance bound of

$$\sup_{\|\eta_{\rm G}\|_{\mathcal{L}_2} \le 1} \|\phi_{\rm G}\|_{\mathcal{L}_2} \le \epsilon_{\rm G} \tag{11}$$

is satisfied for a given tolerance $\epsilon_{\rm G}$. Similarly to the local controller design, we regard the global model $\boldsymbol{\Sigma}_{\rm G}$ as a design parameter to determine a specification of the global controller $K_{\rm G}$.

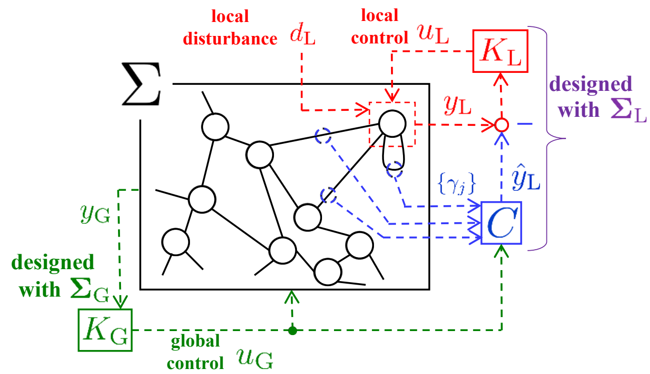

Fig. 1. Signal-flow diagram of the system.

Provided that the global model $\boldsymbol{\Sigma}_{\rm G}$ in (8) well approximates the average behavior of the original system Σ in (2), i.e., $\phi_{\rm G} \simeq \frac{1}{N}\sum_{i=1}^N x_i$, we can expect that the global control action $u_{\rm G} = \mathcal{K}_{\rm G}(y_{\rm G}; \boldsymbol{\Sigma}_{\rm G})$ works to suppress spatially global interference propagation. However, it is not clear how to give a reasonable global model $\boldsymbol{\Sigma}_{\rm G}$ for such interference propagation suppression. Moreover, the simultaneous use of global and local controllers may induce other unexpected interference even if each controller is well designed.

As an adopter between the global and local controllers, we introduce a dynamical compensator denoted as

$$C : \hat{y}_{\rm L} = \mathcal{C}\big(u_{\rm G}, \{\gamma_i\}_{i\in\mathcal{N}_1\cup\{1\}}\big), \tag{12}$$

which measures the input signal $u_{\rm G}$ from the global controller and the interconnection output signals $\{\gamma_i\}_{i\in\mathcal{N}_1\cup\{1\}}$ from the neighborhood subsystems to make a compensation signal $\hat{y}_{\rm L}$. This compensator is interconnected with the global and local controllers as

$$u_{\rm G} = \mathcal{K}_{\rm G}(y_{\rm G}; \boldsymbol{\Sigma}_{\rm G}), \quad \left\{ \begin{array}{rcl} \hat{y}_{\rm L} &=& \mathcal{C}\big(u_{\rm G}, \{\gamma_i\}_{i\in\mathcal{N}_1\cup\{1\}}\big) \\ u_{\rm L} &=& \mathcal{K}_{\rm L}(y_{\rm L} - \hat{y}_{\rm L}; \boldsymbol{\Sigma}_{\rm L}). \end{array}\right. \tag{13}$$

Note that the compensation signal $\hat{y}_{\rm L}$ is sent to the local controller.

To make discussion clearer, the assumptions for glocal controller design are summarized as follows.

Assumption 1: For the interconnected system Σ in (2), the following assumptions are made.

(i) For the design of a local controller $K_{\rm L}$ in (6) and the design of a compensator C in (12), a local model $\boldsymbol{\Sigma}_{\rm L}$ in (4) is only available.

(ii) For the design of a global controller $K_{\rm G}$ in (10), a global model $\boldsymbol{\Sigma}_{\rm G}$ in (8) is only available.

(iii) For the implementation of the global controller $K_{\rm G}$, the local controller $K_{\rm L}$, and the compensator C, the global output $y_{\rm G}$, the local output $y_{\rm L}$, and the interconnection outputs $\{\gamma_i\}_{i\in\mathcal{N}_1\cup\{1\}}$ are measurable, respectively.

Fig. 1 shows a signal-flow diagram of the glocal control system.

Then we address the following design problem of glocal control.

Problem 1: Consider the nN-dimensional interconnected system Σ in (2). Under Assumption 1, find a local controller $K_{\rm L}$ in (6) associated with an n-dimensional local model $\boldsymbol{\Sigma}_{\rm L}$ in (4), a global controller $K_{\rm G}$ in (10) associated with an n-dimensional global model $\boldsymbol{\Sigma}_{\rm G}$ in (8), and a compensator C in (12) such that the following requirements are satisfied.

– The resultant feedback system is stable under the interconnection of (13).
– If the control performance specifications in (7) and (11) are satisfied, then there exists a function $f:\mathbb{R}^2_+\to\mathbb{R}_+$ such that

$$\sup_{\|d_{\rm L}\|_{\mathcal{L}_2}\le 1}\|x_{\rm all}\|_{\mathcal{L}_2}\le f(\epsilon_{\rm L},\epsilon_{\rm G}) \tag{14}$$

where $f(\cdot,\epsilon_{\rm G})$ and $f(\epsilon_{\rm L},\cdot)$ are strictly monotone increasing for each fixed $\epsilon_{\rm G}$ and $\epsilon_{\rm L}$, respectively.

III. GLOCAL CONTROLLER DESIGN

A. Illustrative Example

To see an overview of our proposed glocal control design method, let us consider an example that models a simple homogeneous network system where $N=3$. In the example, we suppose that every subsystem is modeled as a scalar system with $A_i=-1, L_i=B_i=C_i=\Gamma_i=1$ for $i=1,2,3$, $R_1=2$, and $\alpha_{1,2}=\alpha_{1,3}=\alpha_{2,1}=\alpha_{3,1}=1, \alpha_{2,3}=\alpha_{3,2}=0, \alpha_{1,1}=-2, \alpha_{2,2}=\alpha_{3,3}=-1$ in (1). The system parameters of (2) are given by

$$\begin{aligned}
&A_{\rm all}=\begin{bmatrix}-3&1&1\\1&-2&0\\1&0&-2\end{bmatrix},\quad B_{\rm G}=\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix},\\
&B_{\rm L}=[1\ 0\ 0]^{\mathsf T},\quad R_{\rm L}=[2\ 0\ 0]^{\mathsf T},\\
&C_{\rm G}=\frac{1}{\sqrt{3}}[1\ 1\ 1],\quad C_{\rm L}=[1\ 0\ 0].
\end{aligned}$$

Suppose that the matrices $A_1, B_1, C_1, L_1, \Gamma_1, R_1$ and the coefficients $\alpha_{1,j}$ for $j=1,2,3$ are available for controllers design. We will, however, show that this homogeneous network system can be represented by a specific low-dimensional (two-dimensional) system having a cascade structure where the upstream part shows non-coherent (local) behavior of the first subsystem and the downstream part shows coherent (global) behavior.

First, as a local model, let us consider a one-dimensional system in the form of (4) with $\boldsymbol{L}_{\rm L}=0$, $\boldsymbol{B}_{\rm L}=1$, $\boldsymbol{R}_{\rm L}=2$, $w_{\rm L}=u_{\rm L}$, $\eta_{\rm L}=d_{\rm L}$, i.e.,

$$\dot{\phi}_{\rm L}=\boldsymbol{A}_{\rm L}\phi_{\rm L}+u_{\rm L}+2d_{\rm L}, \tag{15}$$

where $\phi_{\rm L}$ represents the local behavior directly affected by the inputs $u_{\rm L}$ and $d_{\rm L}$ and $\boldsymbol{A}_{\rm L}\in\mathbb{R}$ is a modeling parameter that we can choose. Since the state equation (15) is closed in the local system, $\phi_{\rm L}$ can be controlled and the disturbance $\eta_{\rm L}$ can be suppressed by the local control input $u_{\rm L}$. Next, to clarify the relationship between $\phi_{\rm L}$ and $x_{\rm all}$ and discover an effective design method for the global control input $u_{\rm G}$ in (2), we introduce the variable $\Phi_{\rm G}\triangleq x_{\rm all}-P\phi_{\rm L}$ where $P=[1\ 0\ 0]^{\mathsf T}$. By simple calculation, we see that $\Phi_{\rm G}$ obeys the dynamics

$$\begin{aligned}
\dot{\Phi}_{\rm G}&=A_{\rm all}\Phi_{\rm G}+(A_{\rm all}P-P\boldsymbol{A}_{\rm L})\phi_{\rm L}+B_{\rm G}u_{\rm G}\\
&=A_{\rm all}\Phi_{\rm G}+\begin{bmatrix}-3-\boldsymbol{A}_{\rm L}\\1\\1\end{bmatrix}\phi_{\rm L}+\frac{1}{\sqrt{3}}\mathbf{1}_3u_{\rm G}.
\end{aligned} \tag{16}$$

Here, we consider choosing the free parameter $\boldsymbol{A}_{\rm L}$ such that

$$\mathrm{im}(A_{\rm all}P-P\boldsymbol{A}_{\rm L})=\mathrm{im}(B_{\rm G}), \tag{17}$$

namely, $\boldsymbol{A}_{\rm L}=-4$ in this case. Then (16) can be rewritten as

$$\dot{\Phi}_{\rm G}=A_{\rm all}\Phi_{\rm G}+\mathbf{1}_3\left(\frac{1}{\sqrt{3}}u_{\rm G}+\phi_{\rm L}\right). \tag{18}$$

Note that this system is exactly reducible because its controllable subspace is only $\mathrm{im}(\mathbf{1}_3)$. Hence, by taking the subspace decomposition based on controllability with the coordinate transformation

$$\begin{bmatrix}\phi_{\rm G}\\\overline{\phi}_{\rm G}\end{bmatrix}\triangleq T\Phi_{\rm G},\quad T\triangleq\begin{bmatrix}\tilde{P}^{\dagger}\\\tilde{Q}^{\dagger}\end{bmatrix}$$

where $\tilde{P}=\mathbf{1}_3/\sqrt{3}, \tilde{P}^{\dagger}=\mathbf{1}_3^{\mathsf T}/\sqrt{3}$ and $\tilde{P}\tilde{P}^{\dagger}+\tilde{Q}\tilde{Q}^{\dagger}=I_3$, (18) can be transformed into the two independent systems,

$$\dot{\phi}_{\rm G}=-\phi_{\rm G}+u_{\rm G}+\phi_{\rm L}, \tag{19}$$

which represents average behavior of (18), and $\dot{\overline{\phi}}_{\rm G}=\tilde{Q}^{\dagger}A_{\rm all}\tilde{Q}\overline{\phi}_{\rm G}$. From the condition on initial values, $\Phi_{\rm G}(0)=0$ and hence $\overline{\phi}_{\rm G}(t)=0$ for all $t\ge 0$. Then $\Phi_{\rm G}=\tilde{P}\phi_{\rm G}$ holds. Finally, it follows that $x_{\rm all}=\tilde{P}\phi_{\rm G}+P\phi_{\rm L}$. This implies that we can describe the behavior of $x_{\rm all}$ by the cascade interconnection of the global model (19) and the local model (15). This stems from the particular choice of $\boldsymbol{A}_{\rm L}$ satisfying (17). Furthermore, it would be notable that in order to choose such an appropriate $\boldsymbol{A}_{\rm L}$ we need to know the first column of $A_{\rm all}$ only, namely, the interconnection from the first subsystem to the other subsystems, because the matrix P extracts only the corresponding column. In homogeneous network cases, if one knows the neighborhood of the first subsystem, the associated matrices from the first subsystem to other subsystems are available. Therefore, $\boldsymbol{A}_{\rm L}$ can be determined only with the global and local models.

Based on the local model (15) and the global model (19), $u_{\rm L}$ can be designed independently of $u_{\rm G}$ owing to the cascade structure, and moreover, we can design effective $u_{\rm G}$ only with the global model in which $\phi_{\rm L}$ is regarded as a disturbance input. A difficulty here is that, since $\phi_{\rm L}$ and $\phi_{\rm G}$ are virtual variables we cannot directly measure them as output signals for the controllers. We can actually overcome this difficulty by considering an implementable realization of the closed-loop system with the designed controllers and the detail is explained in the next subsection.

B. Homogeneous Network Systems

Let us consider homogeneous network systems. All subsystems are supposed to have the same dynamics except for local control inputs and the interconnection links among their neighborhoods. More specifically, we assume for (2) that the system matrices are given as

$$A_{\rm all} = I_N \otimes A_0 - \mathcal{L} \otimes (\alpha_0 L_0 \Gamma_0), \; B_{\rm G} = \mathbf{1}_N \otimes B_0/\sqrt{N},$$
$$B_{\rm L} = e_1 \otimes B_0, \quad C_{\rm G} = \mathbf{1}_N^{\sf T} \otimes C_0/\sqrt{N}, \quad C_{\rm L} = e_1 \otimes C_0$$

with matrices $A_0, B_0, C_0, L_0, \Gamma_0$ and a scalar α_0, where $e_1 \in \mathbb{R}^N$ is the canonical basis associated with the first coordination and $\mathcal{L}$ is assumed to be the graph Laplacian of the network among subsystems. Note that the pair $(A_{\rm all}, B_{\rm G})$ is uncontrollable, i.e., the corresponding system has an uncontrollable subspace. Moreover, we put an assumption:

Assumption 2: $\mathcal{N}_1 = \{2, \ldots, N\}$.

In other words, we assume that the first subsystem is directly connected to all the other subsystems.

For the homogeneous system (2), consider the following hierarchical state-space expansion [5], [6]:

$$\begin{cases} \dot{\phi}_{\rm L} = \hat{A}\phi_{\rm L} + B_0 u_{\rm L} + R_0 \eta_{\rm L} \\ \dot{\Phi}_{\rm G} = A_{\rm all}\Phi_{\rm G} + (A_{\rm all}P - P\hat{A})\phi_{\rm L} + B_{\rm G} u_{\rm G}, \end{cases} \tag{20}$$

with the zero initial condition $\phi_{\rm L}(0) = 0$ and $\Phi_{\rm G}(0) = 0$ where $P = e_1 \otimes I_n$ and $\hat{A} \in \mathbb{R}^{n\times n}$ can be any matrix. The following lemma holds.

Lemma 1: Consider the systems (2) and (20) with the zero initial conditions. For any $\hat{A} \in \mathbb{R}^{n\times n}$, if $\eta_{\rm L}(t) = d_{\rm L}(t)$ for all $t \geq 0$, then $x_{\rm all}(t) = \Phi_{\rm G}(t) + P\phi_{\rm L}(t)$ for all $t \geq 0$.

Proof: The proof is omitted due to the page limit. □

Letting the matrices in the local model (4) and the global model (8) for design of controllers

$$\begin{array}{llll} \boldsymbol{A}_{\rm L} = \hat{A}, & \boldsymbol{B}_{\rm L} = B_1, & \boldsymbol{R}_{\rm L} = R_1, & \boldsymbol{C}_{\rm L} = C_1, \\ \boldsymbol{A}_{\rm G} = A_0, & \boldsymbol{B}_{\rm G} = B_0, & \boldsymbol{R}_{\rm G} = B_0 X, & \boldsymbol{C}_{\rm G} = C_0, \end{array} \tag{21}$$

we have a *glocal (global/local)* model for (4) and (8)

$$\begin{cases} \dot{\phi}_{\rm L} = \boldsymbol{A}_{\rm L}\phi_{\rm L} + \boldsymbol{B}_{\rm L} u_{\rm L} + \boldsymbol{R}_{\rm L}\eta_{\rm L} \\ \dot{\phi}_{\rm G} = \boldsymbol{A}_{\rm G}\phi_{\rm G} + \boldsymbol{R}_{\rm G}\phi_{\rm L} + \boldsymbol{B}_{\rm G} u_{\rm G}. \end{cases} \tag{22}$$

The following theorem shows the equivalence between (20) and (22) with an appropriate choice of $\hat{A}$.

Theorem 1: Consider the systems (2) and (20) with the zero initial conditions. Let $\hat{A} = A_0 - (|\mathcal{N}_1| + 1)\alpha_0 L_0 \Gamma_0$, then $\Phi_{\rm G}(t) = \tilde{P}\phi_{\rm G}(t)$ for all $t \geq 0$ and hence $x_{\rm all}(t) = \tilde{P}\phi_{\rm G}(t) + P\phi_{\rm L}(t)$ for all $t \geq 0$ holds for any $u_{\rm G}, u_{\rm L}, d_{\rm L}$, and $\eta_{\rm L}$ when $d_{\rm L}(t) = \eta_{\rm L}(t)$, for all $t \geq 0$.

Proof: The proof is omitted due to the page limit. □

This theorem implies that the state of the original system can be exactly described by the states of the glocal model (22) for homogeneous network systems under Assumption 2 and the assumption on locality of disturbance. Consequently, $K_{\rm L}$ and $K_{\rm G}$ in (6) and (10) can be designed based on (22).

A problem here is that $u_{\rm G} = \mathcal{K}_{\rm G}(C_{\rm G}(\tilde{P}\phi_{\rm G} + P\phi_{\rm L}))$ and $u_{\rm L} = \mathcal{K}_{\rm L}(C_0\phi_{\rm L})$ cannot be generated with $\phi_{\rm L}$ and $\phi_{\rm G}$ because the states are virtual variables and the system is not directly implementable. We construct an implementable system as

$$\begin{cases} \dot{x}_{\rm all} = A_{\rm all}x_{\rm all} + B_{\rm G}\mathcal{K}_{\rm G}(y_{\rm G}) + B_{\rm L}\mathcal{K}_{\rm L}(y_{\rm L} - \hat{y}_{\rm L}) + R_{\rm L}d_{\rm L} \\ \dot{\hat{x}} \;= A_0\hat{x} + (P^\dagger A_{\rm all} - \hat{A}P^\dagger)x_{\rm all} + B_0 u_{\rm G}, \end{cases}$$

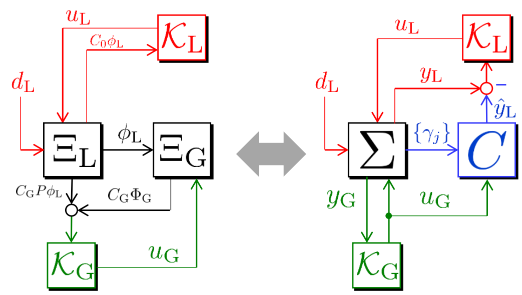

Fig. 2. Signal-flow diagram of the equivalent closed-loop systems where $\Xi_{\rm L}$ and $\Xi_{\rm G}$ represent the dynamics of $\phi_{\rm L}$ and $\Phi_{\rm G}$, respectively.

by applying the coordinate transformation $x_{\rm all} = \tilde{P}\phi_{\rm G} + P\phi_{\rm L}$, $\hat{x} = P^\dagger\tilde{P}\phi_{\rm G}$. From the relationship $(P^\dagger A_{\rm all} - \hat{A}P^\dagger)x_{\rm all} = L_0\sum_{j=1}^N \alpha_0\gamma_j$, we establish a compensator C in (12) to generate $\hat{x}$ as

$$\mathcal{C} : \begin{cases} \dot{\hat{x}} &= A_0\hat{x} + L_0\sum_{j=1}^N \alpha_0\gamma_j + B_0 u_{\rm G} \\ \hat{y}_{\rm L} &= C_0\hat{x} \end{cases} \tag{23}$$

with the zero initial condition $\hat{x}(0) = 0$. As a result, the closed-loop system can be realized with the entire system composed of (2) and (13).

Finally, we show the following theorem to satisfy the criteria on glocal controller design.

Theorem 2: Set the parameters of the global and local models as (21) and the initial values of the states in $\mathcal{K}_{\rm G}, \mathcal{K}_{\rm L}$, and $\mathcal{C}$ to be zero. If (7) and (11) are satisfied with $K_{\rm L}$ and $K_{\rm G}$, then the entire system with (13) is stable and $f(\epsilon_{\rm L}, \epsilon_{\rm G}) = (\epsilon_{\rm G} + 1)\epsilon_{\rm L}$ satisfies (14).

Proof: The proof is omitted due to the page limit. □

Based on this theorem, the solution to Problem 1 is given as follows: Design $\mathcal{K}_{\rm G}$ and $\mathcal{K}_{\rm L}$ such that the conditions in Theorem 2 are satisfied. Construct a compensator as (23). Then the closed-loop system satisfies a performance specification (14). Fig. 2 shows a signal-flow diagram of the equivalent systems (2) and (20) with the controllers and the compensator.

IV. NUMERICAL SIMULATION

In this section, we show the effectiveness of the proposed control strategy through numerical simulation for IEEJ EAST 30-machine power system model, which is a benchmark model of the bulk power system in the eastern half of Japan. The model consists of 107 buses, 30 generators, 31 loads, and transmission network connecting them as shown in Fig. 3. The loads are modeled as constant impedance loads. Each generator model consists of a synchronous machine described as a Park model [16], turbine, and exciter with an automatic voltage regulator (AVR). The models of the turbine and the exciter with AVR are described in [17]. Let us consider a linearlized state space equation of the power system around an equilibrium point, which can be written by (2). The control objective here is to enhance damping performance of generators' velocity.

Let us suppose that a fault happens in the area surrounded by the blue line in Fig. 3. The effect of the fault is modeled

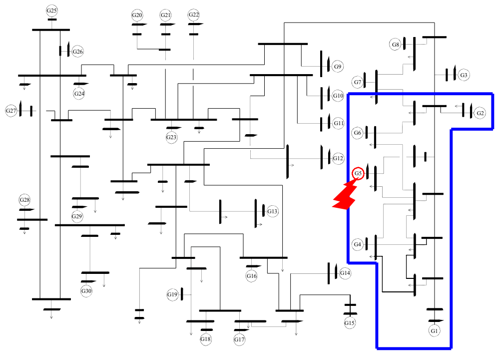

Fig. 3. IEEJ EAST 30-machine power system. It is supposed that a fault happens in the area surrounded by the blue line.

by an impulsive disturbance on the angle of the generator indicated by the red one in Fig. 3. To mitigate the influence caused by the fault, we consider applying our method to the power system regarding the system as a homogeneous network system, where we set the set of the generators in the area as a subsystem. While the global controller $K_{\rm G}$ in (13) uniformly actuates the AVR's of all generators, the local controller $K_{\rm L}$ in (13) actuates to the AVR's of the generators in the area. We set the performance criterion for the local controller $\epsilon_{\rm L} = 1$ in (7).

We plot the responses of all generator frequencies in Fig. 4. The red lines correspond to the case without control and we see that the free responses without any control have large oscillation. The green lines correspond to the case of the proposed glocal control (13) where the performance criterion for the global control is given as $\epsilon_{\rm G} = 5 \cdot 10^6$ in (11). We can see from Fig. 4 that the green trajectories after $t = 5$ show similar behavior as compared to the red ones. This implies that non-coherent behavior of the case without control can be mitigated by the local controller. However, coherent frequency deviation remains with slow convergence, since the global controller is designed with an extremely low gain as complying with a conservative condition derived from the small gain theorem. The blue lines in Fig. 4 correspond to the case of the glocal control (13) where we design $K_{\rm G}$ with $\epsilon_{\rm G} = 11$. Even though the robust stability is not theoretically guaranteed, the coherent frequency deviation, depicted by the green lines, is significantly damped by increasing the gain of the global controller. Note that the non-coherent dynamics is not excited by the global controller.

V. CONCLUSIONS

We have proposed a glocal control design method for network systems. This design method is based on a new representation of the system, called *glocal model*, which is an integrated system of a truncated subsystem model and an aggregated model. Owing to the cascade structure of the glocal model, we can independently design global and local controllers and improve the system performance. Simulation results have been illustrated using the IEEJ EAST 30-machine power system.

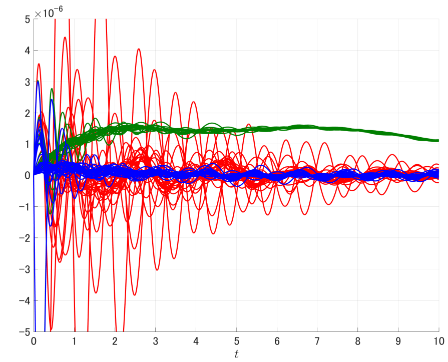

Fig. 4. Responses of all generators' frequencies, $t \in [0, 10]$. Red, green, and blue are the cases of no-control, the proposed glocal control (13) with $\epsilon_{\rm G} = 5 \cdot 10^6$, and the proposed glocal control with $\epsilon_{\rm G} = 11$, respectively.

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