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Optimal Sensor Hop Selection: Sensor Energy Minimization and Network Lifetime Maximization with Guaranteed System Performance

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Abstract— In this paper we consider state estimation carried over a sensor network. A fusion center forms a local multi-hop tree of sensors and gateways and fuses the data into a state estimate. It is shown that the optimal estimator over a sensor tree is given by a Kalman filter of certain structure. The number of hops that the sensors use to communicate data with the fusion center is optimized such that either the overall transmission energy is minimized or the network lifetime is maximized. In both cases the fusion center provides a specified level of estimation accuracy. Some heuristic algorithms are proposed which lead to suboptimal solutions in the energy minimization problem, while an algorithm that leads to the global optimal solution is proposed in the lifetime maximization problem. In both cases, the algorithms are shown to have low computational complexity. Examples are provided to demonstrate the theory and algorithms.
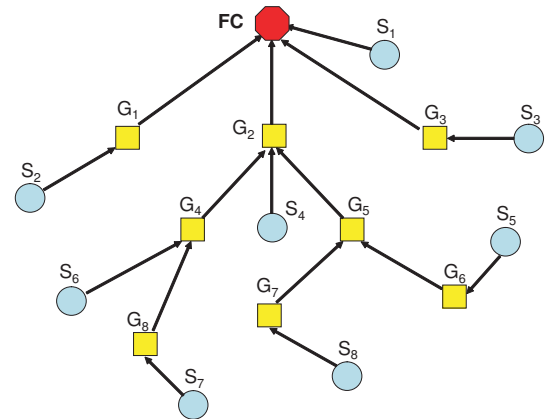
Fig. 1. Sensor Network with Gateways

I. Introduction

Wireless sensor networks have attracted much attention in the past few years and this area of research brings together researchers from computer science, communication, control, etc. [1]. A typical wireless sensor network consists of a large number of sensor nodes, some gateway nodes and some base stations. Sensor nodes are usually battery powered and have limited processing capabilities. They interact with the physical world and collect information of interest, e.g, temperature, humidity, pressure, air density, etc. Depending on the Media Access Control (MAC) and routing protocols, as well as the available resources (network bandwidth, node energy, etc.), the collected data are transmitted to their final destination, usually a fusion center, at appropriate times. Sensor networks have a wide range of applications, including environment and habitat monitoring, health care, home and office automation and traffic control [2].

Although tremendous progress has been made in the past few years in making sensor network an enabling technology, many challenging problems remain to be solved, e.g, network topology control and routing, collaborative signal collection and information processing, and synchronization [3]. In particular any practical design must fully consider the constraints posed by the limited processing capability and energy supply of each individual sensor.

We investigated such constraints in [4] by looking at LQG control over a wireless sensor network. We presented a sensor tree reconfiguration algorithm to meet a specified level of control performance in such a way that the total energy usage of the active sensor nodes is minimized. However, when a sensor node is not a leaf node, it not only needs to send a measurement data packet, but also needs to receive, aggregate and forward data packets from its child nodes. The fact that receiving a packet costs considerable amount of energy [2], together with the recently proposed Wireless HART protocol [5], motivates the current work. In addition to the set of sensor nodes $\mathcal{S} = \{S_1, \cdots, S_q\}$ considered in [4], we also assume a set of gateway nodes $\mathcal{G} = \{G_1, \cdots, G_p\}$ is available (Fig. 1). Gateway nodes are already very popular in Wireless HART applications. These gateway nodes act as relay nodes, i.e., they do not take any measurements but simply aggregate and forward any incoming data packets. The gateways form the backbones of the network and execute a known routing protocol.

The main contribution of this paper is that a collection of efficient algorithms are proposed to determine which sensor communicates to which gateway in such a way that either the total transmission energy of the sensors is minimized or the network lifetime is maximized. In both cases, a certain specified level of estimation accuracy at the fusion center is guaranteed. When the network path for the sensor data is optimized, the resulting local sensor topology has the structure of a tree for which the fusion center is the root. When the plant is given by a linear system, the optimal estimator is given by a Kalman filter with extra memory

due to the communication delays.

The rapid developments of wireless and sensor technologies enable drastic change of the architecture and embedded intelligence in these systems. The theory and design tools for these systems with spatially and temporally varying control demands are not well developed, but there are a lot of current research.

One way to deal with the problem of asynchronous generation of sensor data is to use event-triggered control instead of conventional time-triggered control [6], [7]. How to efficiently encode control information for event-triggered control over communication channels with severe bandwidth limitations is discussed in [8].

Kalman filtering under certain information constraints, such as decentralized implementation, has been extensively studied [9]. Implementations for which the computations are distributed among network nodes is considered in [10]–[12]. Kalman filtering over lossy networks is considered in [13], [14]. The interaction between Kalman filtering and how data is routed on a network seems to be less studied. Routing of data packets in networks are typically done based on the distance to the receiver node [15]. Some recent work addresses how to couple data routing with the sensing task using information theoretic measures [16]. An heuristic algorithm for event detection and actuator coordination is proposed in [17]. For control over wireless sensor networks, the experienced delays and packet losses are important parameters. Randomized routing protocols that gives probabilistic guarantees on delay and loss are proposed in [18], [19].

A robust control approach to control over multi-hop networks is discussed in [20]. A general cross-layer approach to control and data routing seems to be an open and rather difficult topic due to many practical constraints. Our approach is different in that we make the assumption that a tree-structured sensor topology with certain properties can be superimposed on the sensor network. The routing of individual packets is not considered, but instead paths are dynamically established between the sensor nodes and the fusion center.

The rest of the paper is organized as follows. The energy minimization and network lifetime maximization problems are formulated in Section II. After the optimal estimation for fixed number of hops between the sensors and the fusion center is obtained in Section III, algorithms are presented to solve the energy minimization and the network lifetime maximization problems in Section IV and V respectively. An example is given in Section VI to demonstrate the theory and algorithms developed in the paper. Concluding remarks and future work are given in Section VII in the end.

II. PROBLEM SET-UP

A. System and Sensor Network Models

Consider the following LTI system whose state dynamics is given by

$$x_k = Ax_{k-1} + w_{k-1}, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state, $w_k \in \mathbb{R}^n$ is the process noise which is white Gaussian, zero-mean and with covariance matrix $Q \in \mathbb{R}^{n \times n}, Q \geq 0$.

A wireless sensor network is used to measure the state in Eqn (1). The network consists of a fusion center, a set of gateways $\mathcal{G}$ and a set of sensor nodes $\mathcal{S}$. When S_i takes a measurement of the state in Eqn (1), it returns

$$y_k^i = H_i x_k + v_k^i, \tag{2}$$

where $y_k^i \in \mathbb{R}^{o_i}$ is the measurement, $v_k^i \in \mathbb{R}^{o_i}$ is the measurement noise which is white Gaussian, zero-mean and with covariance matrix $\Pi_i \in \mathbb{R}^{o_i \times o_i}, \Pi_i > 0$. The sensor measurements are sent to the fusion center via these gateways. The fusion center then processes the received measurements and computes the optimal state estimate.

We suppose that there is a non-zero single-hop communication delay, which is smaller than the sampling time of the process. All sensors are synchronized in time, so the data packet transmitted from S_i to the fusion center is delayed one sample when compared with its parent node. For example, in Fig. 1, G_3 is the parent node of S_3 and the measurement from S_3 to fusion center is delayed one step compared with S_1. We assume perfect data communications, i.e., we do not consider possible data packet drops introduced by the wireless links.

B. Sensor Energy Cost Model

We assume that the gateways are externally powered while the sensor nodes are battery powered. Sensors spend energy in many ways, i.e., sensing, idle listening, computing, packet transmission and reception, etc. [2]. By appropriately designing the MAC protocol (such as TDMA), packet transmission dominates the total energy usage. Define e_i as the energy cost for S_i sending a measurement packet to its parent node, which typically grows rapidly with the distance to its parent node [1]. Almost all transceivers in sensor nodes nowadays have an adjustable transmission power, so without loss of generality, we assume that S_i can send its measurements to a subset of the gateways or to fusion center directly by adjusting its transmission power.

C. Optimal Hop Selection Problem

Define the following quantities at the fusion center.

$$
\begin{aligned}
\hat{x}_k &\triangleq \mathbb{E}[x_k | \text{all measurements up to } k], \\
P_k &\triangleq \mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)'], \\
P_\infty &\triangleq \lim_{k \to \infty} P_k, \text{if the limit exists.}
\end{aligned}
$$

Denote τ_i as the hop number between S_i and fusion center, then the delay of a measurement from S_i to fusion center is $d_i = \tau_i - 1$. Without loss of generality, we assume the transmission energy $e_i(\tau_i)$ is decreasing in τ_i. For example, in Fig. 1, S_6 can send its measurements to fusion center, G_1, G_4 or G_8 with corresponding hop numbers $1, 2, 3$ or 4

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[1]An estimate of e_i can be be computed based on the considered wireless technology. A common model is that if the distance between S_i and the receiver is d_i, then $e_i = \beta_i + \alpha_i (d_i)^{n_i}$, where β_i represents the static part of the energy consumption and $\alpha_i (d_i)^{n_i}$ the dynamic part. The path loss exponent n_i is typically between 2 and 6.

and with decreasing transmission energy. On the other hand, the steady state error covariance P_∞ grows with increasing delays of the measurements. Hence there is an apparent tradeoff between τ_i and P_∞. Let us define the *network lifetime* to be the maximum time before any one of the sensors stops working due to insufficient energy supply, and we consider the following optimization problems

Problem 2.1: (Sensor Energy Minimization)

$$\min_{(\tau_1,\cdots,\tau_q)} \sum_{i=1}^{q} e_i(\tau_i)$$

subject to

$$P_\infty(\tau_1,\cdots,\tau_q) \le P_{\text{desired}}$$
$$1 \le \tau_i \le m_i, i = 1,\cdots,q.$$

Problem 2.2: (Network Lifetime Maximization)

$$\max_{(\tau_1,\cdots,\tau_q)} \min_i \frac{E_i}{e_i(\tau_i)}$$

subject to

$$P_\infty(\tau_1,\cdots,\tau_q) \le P_{\text{desired}}$$
$$1 \le \tau_i \le m_i, i = 1,\cdots,q,$$

where m_i is the upper bound of the maximum number of hops between S_i and the fusion center, and E_i is the initial energy level of S_i.

Intuitively, the total transmission energy of the sensors is minimized in Problem 2.1 and the network lifetime is maximized in Problem 2.2. The first problem is motivated from the case where the difference between sensor energy levels is small, and the second one is motivated from the case when the difference is large. For both problems, the variables that the objective function is optimized over are the hop numbers between S_i's and fusion center. We will present solutions to both problems in the next few sections.

III. OPTIMAL ESTIMATION FOR FIXED τ_i

In order to solve Problems 2.1 and 2.2, we need to evaluate $P_\infty(\tau_1,\cdots,\tau_q)$ given $(\tau_1,\cdots,\tau_q)$ so as to find the feasible set to the problems. Let S_{i_j} be a node that is j hops away from fusion center and for $X \in \mathbb{R}^{n\times n}, X \ge 0$, define

$$
\begin{aligned}
\tau_{\min} &\triangleq \min\{\tau_1,\cdots,\tau_q\}\\
\tau_{\max} &\triangleq \max\{\tau_1,\cdots,\tau_q\}\\
\Gamma_j &\triangleq [H_{1_j}; H_{2_j}; \cdots], j = \tau_{\min},\cdots,\tau_{\max}\\
C_i &\triangleq [\Gamma_1; \cdots; \Gamma_i], i = \tau_{\min},\cdots,\tau_{\max}\\
\Upsilon_j &\triangleq \text{diag}\{\Pi_{1_j}, \Pi_{2_j}, \cdots\}, j = \tau_{\min},\cdots,\tau_{\max}\\
R_i &\triangleq \text{diag}\{\Upsilon_1,\cdots,\Upsilon_i\}, i = \tau_{\min},\cdots,\tau_{\max}\\
h(X) &\triangleq AXA' + Q,\\
g_{C_i}(X) &\triangleq AXA' + Q - AXC_i'[C_iXC_i' + R_i]^{-1}C_iXA'.
\end{aligned}
$$

Further define $\mathcal{Y}_k^{k-i+1}$ as all measurements available at the fusion center for time $k-i+1$ at time k, $i = \tau_{\min},\cdots,\tau_{\max}$. For example, in Fig. 1, $\tau_{\min} = 1, \tau_{\max} = 4$ and

$$
\begin{aligned}
\mathcal{Y}_k^k &= \{y_k^1\},\\
\mathcal{Y}_k^{k-1} &= \{y_{k-1}^1, y_{k-1}^2, y_{k-1}^3, y_{k-1}^4\},\\
\mathcal{Y}_k^{k-2} &= \{y_{k-2}^1, y_{k-2}^2, y_{k-2}^3, y_{k-2}^4, y_{k-2}^6\}.
\end{aligned}
$$

With these definitions, we have the following theorem which shows how we can obtain the closed form solution to $P_\infty(\tau_1,\cdots,\tau_q)$.

Theorem 3.1: Given $(\tau_1,\cdots,\tau_q)$, $\hat{x}_k$ and P_k can be computed as

$$
\begin{aligned}
\hat{x}_k &= A^{\tau_{\min}-1}(\hat{x}_{k-\tau_{\min}+1})\\
P_k &= h^{\tau_{\min}-1}(P_{k-\tau_{\min}+1})
\end{aligned}
$$

and

$$(\hat{x}_{k-i+1}, P_{k-i+1}) = \mathbf{KF}(\hat{x}_{k-i}, P_{k-i}, \mathcal{Y}_k^{k-i+1}, C_i, Q, R_i)$$

where $i = \tau_{\min},\cdots,\tau_{\max}$ and $\mathbf{KF}$ denotes the standard Kalman filter. If the limits exists, P_∞ satisfies

$$P_\infty = h^{\tau_{\min}-1}(P_\infty^- - P_\infty^- C_{\tau_{\min}}' \Sigma^{-1} C_{\tau_{\min}} P_\infty^-) \quad (3)$$

where

$$
\begin{aligned}
\Sigma &= C_{\tau_{\min}} P_\infty^- C_{\tau_{\min}}' + R_{\tau_{\min}}\\
P_\infty^- &= g_{C_{\tau_{\min}+1}} \cdots \circ g_{C_{\tau_{\max}-1}}(\bar{P}_\infty^-)
\end{aligned}
$$

and $\bar{P}_\infty^-$ is the unique solution to $g_{C_{\tau_{\max}}}(\bar{P}_\infty^-) = \bar{P}_\infty^-$.
Proof: Similar to the proof for Theorem 4.1 in [4]. ∎

IV. OPTIMAL HOP SELECTION: MINIMIZING TOTAL ENERGY

In this section, we provide solution to Problem 2.1. When q and m_i's are small, we can find the global optimal solution to Problem 2.1 via the following algorithm.

Global Optimal Search Algorithm
1 : For each $i = 1,\cdots,q$
    • for each $\tau_i = 1,\cdots,m_i$
    if $P_\infty(\tau_1,\cdots,\tau_i) \le P_{\text{desired}}$,
    record $(\tau_1,\cdots,\tau_q)$ as well as $\sum_{i=1}^{q} e_i(\tau_i)$.
2 : Return $\arg\min \sum_{i=1}^{q} e_i(\tau_i)$.

Apparently, the *Global Optimal Search Algorithm* takes time $O(\prod_{i=1}^{q} m_i)$. For large $\prod_{i=1}^{q} m_i$, it then becomes very inefficient, therefore we propose some local search algorithms to approximate the optimal solution. Before we present some classical local search algorithms, we provide another efficient algorithm. It is also simpler to implement than most local search algorithms [21].

The algorithm we propose is *Greedy Efficiency Search Algorithm*. For simplicity, let us define $\mathcal{E}(\tau) = \sum_{i=1}^{q} e_i(\tau_i)$ as the total energy cost given τ. We also write $\mathcal{E}(\tau)$ as $\mathcal{E}(\tau_{-i}, \tau_i)$ when we look at the hop number of S_i.

Greedy Efficiency Search Algorithm
1 : $\tau_i := 1, i = 1,\cdots,q$
2 : For each i, if $P_\infty(\tau_{-i}, \tau_i + 1) \le P_{\text{desired}}$
    • compute $\Delta(i) = \frac{\Delta E_i}{\Delta P_i}$ where
    $\Delta E_i = \mathcal{E}(\tau_{-i}, \tau_i) - \mathcal{E}(\tau_{-i}, \tau_i + 1)$
    $\Delta P_i = P_\infty(\tau_{-i}, \tau_i + 1) - P_\infty(\tau_{-i}, \tau_i)$
3 : Let $s = \arg\max \Delta(i)$. Update $\tau_s := \tau_s + 1$.
4 : Repeat Step 2 until the incremental decrease of the total energy is within a certain threshold.

Remark 4.1: It is easy to verify that the solution from the *Greedy Efficiency Search Algorithm* always satisfies the accuracy constraint. And in every iteration, the total energy decreases. It is also easy to note that if at certain iteration, the global optimal solution is achieved, then the algorithm stops and returns that optimal solution.

We provide another two classical search algorithms, i.e., the *Randomized Greedy Search* and the *TABU Search*, and we compare the performances of the three algorithms in Section VI.

Define $\mathcal{N}(\tau)$ as the neighborhood solutions of τ. The size of $\mathcal{N}(\tau)$ determines the time complexity and the optimality of the solution. Apparently, we need to pick up $\mathcal{N}(\tau)$ of reasonable size. For instance, we define

$$\mathcal{N}(\tau) = \{\tau' : 1 \leq \tau_i' \leq m_i, |\tau_i' - \tau_i| \leq 1\}$$

as the neighborhood solutions for the example in Section VI. In the extreme case, if $\mathcal{N}(\tau) = \{\tau' : 1 \leq \tau_i' \leq m_i\}$, then the *Randomized Greedy Search algorithm* is the same as the *Global Optimal Search Algorithm*. The multiple seeds version of the *Randomized Greedy Search algorithm* runs by executing the one seed version a few times. Let $\mathcal{E}^*(t)$ denotes the optimal solution found at each time t. The the minimum of $\mathcal{E}^*(t)$ and the corresponding $\tau^*(t)$ is returned.

Randomized Greedy Search: One Seed
1 : τ^* is randomly determined. $\mathcal{E}^* := \mathcal{E}(\tau^*)$.
2 : While (stop criterion is not met)
   • generate $\mathcal{N}(\tau^*)$
   • for each $\tau \in \mathcal{N}(\tau^*)$
     - if $\mathcal{E}(\tau) < \mathcal{E}^*$
        $\tau^* := \tau, \mathcal{E}^* := \mathcal{E}(\tau)$

As we can see, *Randomized Greedy Search* algorithm uses a fixed structure of neighborhood solutions $\mathcal{N}(\tau^*)$ at each iteration. The *TABU Search* algorithm [22], [23], on the other hand, uses a dynamic structure of neighborhood solutions. It maintains a memory structure: once a potential solution is visited, it is marked as "taboo" and is inserted into a tabu list, $\mathcal{T}$, so that the algorithm does not visit that solution repeatedly. The length of $\mathcal{T}$, the size of $\mathcal{N}(\tau)$, as well as the initial solution affect the performance of the algorithm. There are various versions of *TABU Search* algorithms, and we consider one version below.

TABU Search
1 : Select an initial τ. $\tau^* := \tau, \mathcal{E}^* := \mathcal{E}(\tau^*)$. Set the iteration counter $t := 0$ and begin with $\mathcal{T}$ empty.
2 : Generate $\mathcal{N}(\tau)$.
   If $\mathcal{N}(\tau) - \mathcal{T}$ is empty, goto Step 4.
   Otherwise, set $t := t + 1$ and select
   $s_k(\tau) \in \mathcal{N}(\tau) - \mathcal{T}$ which has $\min \mathcal{E}(s_k(\tau))$.
3 : Let $\tau := s_k(\tau)$. If $\mathcal{E}(\tau) < \mathcal{E}^*$, let $\tau^* := \tau$.
4 : If a chosen number of iterations has elapsed either in total or since τ^* was last updated, or if $\mathcal{N}(\tau) - \mathcal{T} = \emptyset$ upon reaching this step directly from Step 2, stop; Otherwise, insert τ into $\mathcal{T}$ and delete the oldest entry in $\mathcal{T}$ if it is full. Return to Step 2.

V. OPTIMAL HOP SELECTION: MAXIMIZING LIFETIME

In this section, we study Problem 2.2. Unlike Problem 2.1, where the global optimal solution cannot be found efficiently in general, global optimal solution to Problem 2.2 can be solved very efficiently via the following algorithm.

Max Lifetime Search Algorithm
1 : $c := 1$
2 : For $i = 1, \cdots, q$
   • let $\tau_i[c] = \min\{\tau_i : e_i(\tau_i) \leq \frac{E_i}{c}\}$
   if $\tau_i[c] > m_i$, goto Step 4.
3 : If $P_\infty(\tau_1[c], \cdots, \tau_q[c]) \leq P_{\text{desired}}$
   • $\tau_i^* = \tau_i[c]$
   • $c := c + 1$; goto Step 2.
   else goto Step 4.
4 : Return τ^*.

Define l^* as the maximum network lifetime, i.e.,

$$l^* = \max_{(\tau_1, \cdots, \tau_q)} \min_i \frac{E_i}{e_i(\tau_i)}.$$

Theorem 5.1: The *Max Lifetime Search Algorithm* returns the optimal τ^* and $l^* = c - 1$ when the algorithm stops.
Proof: Let τ^* be the optimal solution corresponding to l^*. We divide the proof into two parts.

1) For any $c < l^*$,

$$P_\infty(\tau_1[c], \cdots, \tau_q[c]) \leq P_{\text{desired}},$$

and $\tau_i[c] \leq m_i$ for all $i = 1, \cdots, q$.
2) For any $c > l^*$, either

$$P_\infty(\tau_1[c], \cdots, \tau_q[c]) \nleq P_{\text{desired}}$$

or there exists at least one S_i such that $\tau_i[c] > m_i$.
Once these two parts are proved, the optimality of the algorithm follows as the algorithm stops exactly at $c = l^*+1$.

Proof for part 1): Since l^* is optimal, $\tau_i[l^*] \leq m_i$ for all $i = 1, \cdots, m_i$, and τ^* must be feasible, i.e.,

$$P_\infty(\tau_1[l^*], \cdots, \tau_q[l^*]) \leq P_{\text{desired}}.$$

Hence for any $c < l^*$,

$$
\begin{aligned}
\tau_i[c] &= \min\{\tau_i : e_i(\tau_i) \leq \frac{E_i}{c}\} \\
&\leq \min\{\tau_i : e_i(\tau_i) \leq \frac{E_i}{l^*}\} \\
&= \tau_i[l^*] \leq m_i.
\end{aligned}
$$

The first inequality holds as $e_i(\tau_i)$ is decreasing in τ_i. From Lemma 1.1 in Appendix ,

$$
\begin{aligned}
P_\infty(\tau_1[c], \cdots, \tau_q[c]) &\leq P_\infty(\tau_1[c+1], \cdots, \tau_q[c]) \\
&\vdots \\
&\leq P_\infty(\tau_1[l^*], \cdots, \tau_q[c]) \\
&\vdots \\
&\leq P_\infty(\tau_1[l^*], \cdots, \tau_q[l^*]) \\
&\leq P_{\text{desired}}
\end{aligned}
$$

Algorithms	τ_1	τ_2	τ_3	$\mathcal{E}(\tau)$	P_∞	γ
Global Optimal	8	1	1	9.58	0.1802	512
Greedy Search	1	1	8	10.04	0.1802	24
Greedy Efficiency	8	1	1	9.58	0.1802	26
TABU Search	1	1	8	10.04	0.1802	88

TABLE II

$P_{\text{DESIRED}} = 1$

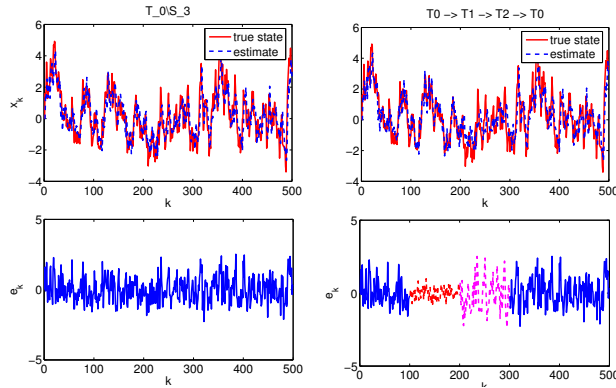| Algorithms	τ_1	τ_2	τ_3	$\mathcal{E}(\tau)$	P_∞	γ
Global Optimal	8	8	2	3.5	0.7419	512
Greedy Search	8	8	2	3.5	0.7419	136
Greedy Efficiency	8	2	8	4.12	0.7419	48
TABU Search	8	8	2	3.5	0.7419	182

Fig. 2. Dynamic Sensor Selection

which completes the proof for part 1).

Proof for Part 2): If $c > l^*$ and

$$P_\infty(\tau_1[c], \cdots, \tau_q[c]) \le P_{\text{desired}}$$

and $\tau_i[c] \le m_i$ for all S_i, then $\tau[c]$ is a feasible solution and $c > l^*$ violates the optimality assumption of l^*. ∎

VI. EXAMPLE

In this section, we consider an example to demonstrate the algorithms developed so far. As the *Max Lifetime Search Algorithm* returns the optimal solution, we focus on the algorithms for the energy minimization problem presented in Section IV.

We consider the following system and sensor models.

$$x_k = 0.9x_{k-1} + w_{k-1}$$
$$y_k^i = x_k + v_k^i, i = 1, 2, 3.$$

The following parameters are used throughout this section.

$$Q = 0.5, \Pi_1 = \Pi_2 = \Pi_3 = 0.5$$
$$e_1(\tau_1) = [5 \ 3.8 \ 2.6 \ 1.5 \ 1 \ 0.4 \ 0.1 \ 0.08]$$
$$e_2(\tau_2) = [5.0 \ 4 \ 2.8 \ 1.8 \ 1.2 \ 0.5 \ 0.15 \ 0.12]$$
$$e_3(\tau_3) = [4.5 \ 3.3 \ 2.1 \ 1.2 \ 0.5 \ 0.24 \ 0.05 \ 0.04]$$

Assume the following performance specification is received at the fusion center:

$$P_\infty \le 1, 1 \le k \le 100,$$
$$P_\infty \le 0.25, 101 \le k \le 200,$$
$$P_\infty \le 1.5, 201 \le k \le 300,$$
$$P_\infty \le 1, 301 \le k \le 500.$$

Define γ as *the number of feasible solutions that each algorithm visits* during its execution. Table I- III show the results when we run the different algorithms corresponding to different P_{desired}. Since in this example, $\prod_{i=1}^3 m_i = 512$, we are able to compute the global optimal solution via exhaustive search.

When $P_{\text{desired}} = 0.25$, the *Greedy Efficiency Search* is the best among all three algorithms which is also the same

as the global optimal solution, as shown in Table I. It also visits the least number of feasible solutions before returning the optimal solution. When $P_{\text{desired}} = 1$, both *Randomized Greedy Search* and *TABU Search* return the optimal solution and are better than *Greedy Efficiency Search*, but at the price of visiting much more feasible solutions. When we further increase P_{desired} to be 1.5, neither algorithm return the optimal solution and the *Greedy Efficiency Search* algorithm still visits the least number of feasible solutions.
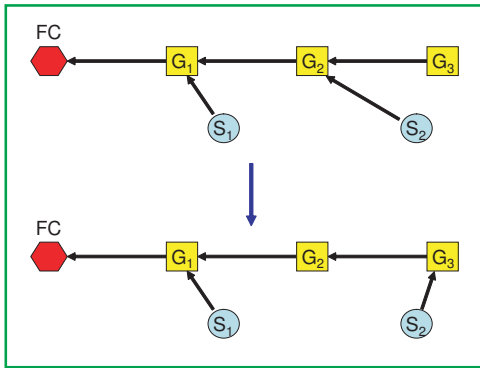
In practice, we can run the three algorithms and take the best solution of them. Fig. 2 shows the simulation result of estimating x_k using the estimation scheme presented in Theorem 3.1. The left hand figures demonstrate the result when a fixed topology is used ($\tau = [8 \ 8 \ 2]$) in which case, $P_\infty = 0.7419$ and a constant energy consumption of 3.5 units per time is required. The right hand figures show that the dynamic hop selection is used to adapt to performance specification. As we can see, during $101 \le k \le 200$, a new topology is used ($\tau = [8 \ 1 \ 1]$) and the energy consumption is increased to 9.58 units per time, however, P_∞ reduces to 0.1802 which satisfies the performance specification. Similarly P_∞ is required to be less than 1.5 when $201 \le k \le 300$, a different sensor topology ($\tau = [4 \ 8 \ 8]$) is adopted which only requires 1.66 units energy consumption per time.

By dynamically determine the sensor hop numbers, we can therefore minimize the sensor energy consumption as much as possible yet still guarantee a specified level of performance.

TABLE III

$P_{\text{DESIRED}} = 1.5$

Algorithms	τ_1	τ_2	τ_3	$\mathcal{E}(\tau)$	P_∞	γ
Global Optimal	8	8	4	1.4	1.3918	512
Greedy Search	4	8	8	1.66	1.3918	155
Greedy Efficiency	8	4	8	1.92	1.3918	54
TABU Search	4	8	8	1.66	1.3918	182

Fig. 3. $\tau_2 \rightarrow \tau_2 + 1$

VII. CONCLUSION AND FUTURE WORK

In this paper, we have considered the optimal sensor hop selection problem for state estimation over a wireless sensor network. Efficient algorithms are proposed to solve the energy minimization and network lifetime maximization problems. For both problems, a certain specified level of system performance is guaranteed.

There are a few extensions of the current work that we will pursue in the future which include closing the loop based on the estimation scheme; experimentally evaluate the algorithms developed in the paper; consider packet drops issues in the communication link which is often seen due to the nature of wireless communications.

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APPENDIX

Lemma 1.1: For any $1 \leq i \leq q$,

$$P_\infty([\tau_1, \cdots, \tau_i, \cdots, \tau_q]) \leq P_\infty([\tau_1, \cdots, \tau_i + 1, \cdots, \tau_q]).$$

Proof: We give a proof for the following case (Fig. 3). The extension to general case is straightforward. From Theorem 3.1,

$$P_\infty([\tau_1 = 2, \tau_2 = 3]) = h(\tilde{g}(\bar{P}_\infty^-))$$

where $\tilde{g}(X) = X - XH_2'[H_2XH_2' + \Pi_2]^{-1}H_2X$ and $\bar{P}_\infty^-$ satisfies $g_{[H_1;H_2]}(\bar{P}_\infty^-) = \bar{P}_\infty^-$. Similarly,

$$P_\infty([\tau_1 = 2, \tau_2 = 4]) = h(\tilde{g}(P_\infty^-))$$

where $P_\infty^- = g_{H_2}(\bar{P}_\infty^-)$. Therefore

$$\begin{aligned}
&P_\infty([\tau_1 = 2, \tau_2 = 4]) \\
=\ & h\Big(\tilde{g}\big(g_{H_2}(\bar{P}_\infty^-)\big)\Big) \\
\geq\ & h\Big(\tilde{g}\big(g_{[H_1;H_2]}(\bar{P}_\infty^-)\big)\Big) \\
=\ & h\big(\tilde{g}(\bar{P}_\infty^-)\big) \\
=\ & P_\infty([\tau_1 = 2, \tau_2 = 3])
\end{aligned}$$

where the inequality follows from Lemma 1.2.   ∎

Lemma 1.2: For any $X \geq Y \geq 0$, the following holds
1) $h(X) \geq h(Y)$ and $\tilde{g}(X) \geq \tilde{g}(Y)$.
2) $g_{[H_1;H_2]}(X) \leq g_{H_2}(X)$.

Proof: For proof of part 1), see [13]. For proof of part 2), see [4].   ∎

REFERENCES

[1] D. Culler, D. Estrin, and M. Srivastava, "Overview of wireless sensor networks," *IEEE Computer, Special Issue in Sensor Networks*, Aug 2004.

[2] N. P. M. (Ed.), *Sensor Networks and Configuration.* Springer, 2007.

[3] C.-Y. Chong and S. P. Kumar, "Sensor networks: Evolution, opportunities, and challenges," *Proceedings of the IEEE*, vol. 91, no. 8, Aug 2003.

[4] L. Shi, K. H. Johansson, and R. M. Murray, "Change sensor topology when needed: How to efficiently use system resources in control and estimation over wireless networks." Proceedings of the 46th IEEE Conference on Decision and Control, New Orleans, Dec 2007.

[5] http://www.hartcomm2.org.

[6] H. Kopetz, "Should responsive systems be event triggered or time triggered?" *IEICE Trans. on Information and Systems*, vol. E76-D(10), pp. 1525–1532, 1993.

[7] K. J. Åström and B. Bernhardsson, "Comparison of periodic and event based sampling for first-order stochastic systems," vol. J. In Preprints 14th World Congress of IFAC, Beijing, P.R. China, pp. 301–306.

[8] L. Bao, M. Skoglund, and K. H. Johansson, "Encoder–decoder design for event-triggered feedback control over bandlimited channels." American Control Conference, Minneapolis, Minnesota, USA, 2006.

[9] D. Siljak, *Large-Scale Dynamic Systems: Stability and Structure*. North-Holland, New York, 1978.

[10] P. Alriksson and A. Rantzer, "Distributed kalman filtering using weighted averaging." Proc. 17th International Symposium on Mathematical Theory of Networks and Systems, Kyoto, Japan, 2006.

[11] R. Olfati-Saber, "Distributed kalman filter with embedded consensus filters." Proceedings of the 44th IEEE Conference on Decision and Control and European Control Conference, 2005.

[12] D. Spanos, R. Olfati-Saber, and R. M. Murray, "Distributed kalman filtering in sensor networks with quantifiable performance." Proceedings of the 4th International Conference on Information Processing in Sensor Networks, 2005.

[13] B.Sinopoli, L.Schenato, M.Franceschetti, K.Poolla, M.Jordan, and S.Sastry, "Kalman filtering with intermittent observations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1453–1464, 2004.

[14] J. Hespanha, P. Naghshtabrizi, and Y. Xu, "Networked control systems: Analysis and design," *To appear in the Proc. of IEEE, Special Issue on Networked Control Systems*, 2007.

[15] D. P. Bertsekas and R. Gallager, *Data Networks*, 2nd ed. Prentice Hall, 1991.

[16] F. Z. Juan Liu and D. Petrovic, "Information-directed routing in ad hoc sensor networks," *IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS*, vol. 23, no. 4, April 2005.

[17] E. Ngai, M. Lyu, and J. Liu, "A real-time communication framework for wireless sensor-actuator networks." Aerospace Conference, 2006 IEEE, 2006.

[18] A. Bonivento, C. Fischione, and A. Sangiovanni-Vincentelli, "Randomized protocol stack for ubiquitous networks in indoor environment," *IEEE CCNC*, vol. 1, April 2006.

[19] W. Lai and I. C. Paschalidis, "Routing through noise and sleeping nodes in sensor networks: latency vs. energy trade-offs." Proceedings of the 45th IEEE Conference on Decision and Control, San Diego, CA, USA, Dec 2006, pp. 551 – 555.

[20] A. Panousopoulou, G. Nikolakopoulos, A. Tzes, and J. Lygeros, "Experimental evaluation of a mobile ad-hoc networked (manet)controlled system." Proc. 17th International Symposium on Mathematical Theory of Networks and Systems, Kyoto, Japan, 2006.

[21] C. Blum and A. Roli, "Metaheuristics in combinatorial optimization: Overview and conceptual comparison," *ACM Computing Surveys*, vol. 35, no. 3, pp. 268–308, September 2003.

[22] F. Glover, "Tabu search - part i," *ORSA Journal on Computing*, vol. 1, pp. 190–206, Summer 1989.

[23] ——, "Tabu search - part ii," *ORSA Journal on Computing*, vol. 2, pp. 4–32, Winter 1990.