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Brief paper

Scheduling networked state estimators based on Value of Information ☆

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ABSTRACT

This paper proposes a novel method for information management of a networked system for the purpose of efficient state estimation. Data shared between entities is transmitted over a common communication network. Based on the notion of Value of Information (VoI), we design a prioritizing scheduler in which systems determine the benefit of sharing their real-time data to improve state estimation accuracy. The system with the highest VoI is granted a time slot to provide its data. By using a rollout strategy, feasible algorithms are developed for computing the VoI-based priorities. For decoupled subsystems, performance certificates of the VoI-based strategy are derived. An automotive case study evaluates the proposed approach.

1. Introduction

Networked control systems (NCS) can be seen as a compound of self-contained sensors, controllers, data aggregators, and supervisors exchanging data to achieve a joint objective. NCS commonly have two characteristics: (i) real-time data and decision making are distributed, (ii) data shared among entities is transmitted over a common network. If the communication bandwidth becomes sparse, then a key challenges is to develop scheduling algorithms that make use of the resource efficiently while ensuring the successful delivery of important data packets for keeping state estimates consistent among the entities. What makes such design difficult is the concurrent satisfaction of real-time requirements that vary among the subsystems over time. This fact imposes the necessity of a close interaction of state estimation and communication while disqualifying the usage of purely static solutions such as time-trigged schedulers.

Prioritizing data packets based on their impact on the state estimation accuracy is a promising approach to address the aforementioned challenge. In this work, we design a novel prioritizing scheduler for distributed state estimation, in which priorities are based on the Value of Information (VoI) related to estimation accuracy.

The idea of using the system state or measurements to determine channel access for NCS has been prevalent for some time now (Otanez, Moyne, & Tilbury, 2002; Walsh & Ye, 2001; Yook, Tilbury, & Soparkar, 2002). In Walsh and Ye (2001), the Try-Once-Discard (TOD) protocol uses Maximum Error First for prioritization. The focus of the literature on TOD including the aforementioned works has been predominantly on proving stability of NCS. Though significant performance improvements are indicated in these works, there has been no certification that TOD always outperforms its time-triggered counterpart to the best knowledge of the authors. Using VoI gives us a systematic method for: (i) determining priorities similarly as for TOD and (ii) certification for the case of decoupled systems. The concept of VoI is well-known in information analysis where it is defined as the price a decision maker is willing to pay for taking the information into account (Howard, 1966). It is extensively applied in diverse areas such as information economics (Arrow, 1984; Bikchandani, Riley, & Hirshleifer, 2013), data fusion (Antunes, Heemels, Hespanha, & Silvestre, 2012; Gupta, Chung, Hassibi, & Murray, 2006; Krause & Guestrin, 2009) and sensor sampling (Antunes & Heemels, 2014; Soleymani, Hirche, & Baras, 2016).

Our main contribution is the development of a dynamic priority scheme for scheduling real-time data over a shared network for state estimation. The novel feature is the introduction of a variant of VoI for calculating priorities. In Howard's original VoI formulation, it is presumed that the actual target information, of which the VoI is computed, is not known beforehand (Howard, 1966). Contrary to the literature on VoI, our approach takes real-time data into account when computing its VoI. Tractable expressions for the VoI are derived by using a rollout strategy

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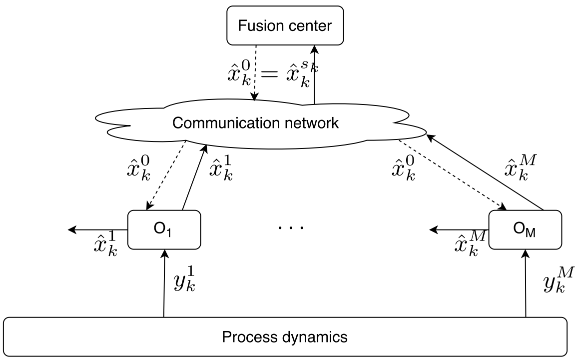

Fig. 1. NCS of M systems sharing a communication network.

similarly to Antunes and Heemels (2014). This strategy assumes that the future sensor schedule is predetermined by a baseline heuristic (Bertsekas, 1995). For decoupled systems, we show that the proposed scheduler is identical with the optimal centralized decision rule based on the rollout strategy. For a time-triggered scheduler as baseline, we can ensure that our approach outperforms the time-triggered scheduler with regard to its estimation accuracy. An automotive case study finally illustrates the benefits of the VoI scheduler. Parts of this paper have been presented in Molin, Esen, and Johansson (2016), Molin, Ramesh, Esen, and Johansson (2015a). The main innovation of this work is the design of the VoI-based scheduler for the coupled case. This paper is organized as follows. Section 2 defines the NCS model. Section 3 develops the VoI-based scheduling scheme, which is divided into two parts analyzing the coupled and the decoupled system class. Section 4 gives a comparative study on a platooning use case.

2. Networked estimation model

An overview of the NCS is given in Fig. 1. We consider a set of M estimators, indexed by j, $1 \leq j \leq M$ that observe a common process. The process state evolves by

$$\begin{aligned} x_{k+1} &= Ax_k + w_k, \\ y_k^j &= C^j x_k + v_k^j \end{aligned} \tag{1}$$

with state $x_k \in \mathbb{R}^n$, measurement $y_k^j \in \mathbb{R}^{m_j}$, system matrices $A \in \mathbb{R}^{n\times n}$, $C^j \in \mathbb{R}^{m_j \times n}$ and statistical assumptions: $x_0 \sim \mathcal{N}(\bar{x}_0, R_0)$, $R_0 \in \mathbb{R}^{n\times n}$, $w_k \sim \mathcal{N}(0, R_w)$, $R_w \in \mathbb{R}^{n\times n}$, $v_k^j \sim \mathcal{N}(0, R_{v,j})$, $R_{v,j} \in \mathbb{R}^{m_j \times m_j}$. The initial state x_0, the process noise w_k and the measurement noise v_k^j are assumed to be mutually independent. The sensor fusion architecture consists of local state estimators, the fusion center and the data management that coordinates the traffic between subsystems and the infrastructure. In this paper, we model the uplink channel as a limited resource, which allows the transmission of at most one data packet at each time step k, i.e. only one system can transmit its state estimate to the fusion center at a time. The downlink channel broadcasts the fused state estimate to all systems at each time step. For simplifying the analysis, we assume that the estimate $\hat{x}_k^j$ has fused all data available at time k. The state estimation procedure of system j follows the subsequent steps.

(1) Local state prediction
(2) Local state update with measurement y_k^j
(3) Determining priority according to VoI
(4) Data transmission
(5) Local state update with broadcasted estimate

As the infrastructure broadcasts the state estimate at each time k, any system has at least as much information as the infrastructure. The fused estimate $\hat{x}_k^0$ at the fusion center and the local estimate $\hat{x}_k^j$ transmitted at time k from system j are identical, as the infrastructure will not provide any new data to be fused. Let $s_k \in \{1, \dots, M\} \cup \{\varnothing\} = \mathcal{I}$ be the system transmitting at time k, where $s_k = \varnothing$ denotes that no system transmits. Then, we have $\hat{x}_k^0 = \hat{x}_k^{s_k}$. Denote $\hat{x}_k^j$ and P_k^j the state estimate and the covariance at the end of the data fusion procedure. The local state estimate for system j can then be written as

$$\tilde{x}_{k|k-1}^j = A\hat{x}_{k-1}^j, \tag{2}$$

$$\tilde{x}_k^j = \tilde{x}_{k|k-1}^j + K_k^j(y_k^j - C^j\tilde{x}_{k|k-1}^j), \tag{3}$$

where we define $\tilde{x}_{0|-1}^j = \bar{x}_0$. Given the index s_k of the transmitting system at time k, we fuse the local estimate $\tilde{x}_k^j$ with $\tilde{x}_k^{s_k}$ by using the best linear unbiased estimator (BLUE) according to Bar-Shalom, Willett, and Tian (2011), Li, Zhu, Wang, and Han (2003), i.e.,

$$\hat{x}_k^j = \begin{cases} \tilde{x}_k^j + K_k^{s_k j}(\tilde{x}_k^{s_k} - \tilde{x}_k^j) & j \neq s_k, \\ \tilde{x}_k^j & j = s_k. \end{cases} \tag{4}$$

To keep the computation of the covariance matrices tractable, we introduce the following assumption.

Assumption 1. The random variables s_ℓ, $\ell < k$, are assumed to be sample-path independent.

The above assumption allows us to resort to first- and second-order statistics. Assumption 1 can be justified as signaling data s_k contributes only little to the state estimate compared to the broadcast estimate $\hat{x}_k^{s_k}$, see Anantharam and Verdu (1996) and Molin, Sandberg, and Johansson (2015b). Considering Assumption 1, the filter gains K_k^j and K_k^{ij}, where $i = s_k$ can be computed as in Anderson and Moore (2012), Bar-Shalom et al. (2011) by

$$K_k^j = \tilde{P}_{k|k-1}^j C^j (C^j \tilde{P}_{k|k-1}^j C^{j\mathsf{T}} + R_{v,j})^{-1},$$
$$K_k^{ij} = (\tilde{P}_k^j - \tilde{P}_k^{ij})(\tilde{P}_k^j + \tilde{P}_k^i - \tilde{P}_k^{ji} - \tilde{P}_k^{ij})^{-1}.$$

The calculation of the covariances $\tilde{P}_{k|k-1}^j$, $\tilde{P}_k^i$ and the cross-correlations $\tilde{P}_k^{ji}$, $\tilde{P}_k^{ij}$ can be found in the Appendix.

3. VoI-based scheduling design

The design of the scheduling variable s_k that determines the information flow is the subject of this section.

3.1. Synthesis of priorities

The priority-based scheduler aims at the minimization of the expected value of the weighted squared error

$$J = \sum_{k=0}^{N-1} \vec{e}_k^{\mathsf{T}} \Gamma_k \vec{e}_k \tag{5}$$

over horizon N with $\Gamma_k \geq 0$ and $\vec{e}_k^{\mathsf{T}} = [\, e_k^{1\mathsf{T}} \; \cdots \; e_k^{M\mathsf{T}} \,]$, $e_k^j = x_k - \hat{x}_k^j$. The development of the scheduler is based on two complementary notions aiming at minimizing (5): VoI and rollout algorithms. The priorities are based on the VoI given the data at system j. The VoI is defined as the benefit in terms of decreasing J by taking data of system j at time k into account. The computation of the VoI in this setting is a difficult task, as scheduling rules must be determined for the entire horizon that lead to complex cost-to-go functions. Moreover, dynamic programming fails due to the distributed nature of the scheduler. This motivates us to use the rollout strategy (Bertsekas, 1995) as an approximation technique that presumes a baseline heuristic for the future schedule.

Definition 2. A *baseline heuristic* is a static scheduling sequence $\{\bar{s}_0, \ldots, \bar{s}_{N-1}\}$, $\bar{s}_k \in \mathcal{I}$, over horizon N.

The VoI of system j is defined as the difference of the cost-to-go when no estimate is sent at time k and the cost-to-go when sending $\tilde{x}_k^j$. In both cases, a baseline heuristic is used in future steps. We refer to this assumption as the rollout strategy. The cost-to-go function is computed based on the data I_k^j at system j defined as

$$I_{k+1}^j = \{I_k^j, y_{k+1}^j, \hat{x}_k^{s_k}, s_k\} \tag{6}$$

with $I_0^j = \{y_k^j\}$. The decision rule for data scheduling of the distributed estimators is defined as

$$s_k = \arg\max_{1\le j\le M} \mathrm{VoI}_k^j, \tag{7}$$

$$\mathrm{VoI}_k^j = \mathbf{E}[J|I_k^j, s_k = \varnothing] - \mathbf{E}[J|I_k^j, s_k = j]. \tag{8}$$

where VoI_k^j denotes the priority of system j at time k. In case, there are multiple subsystems with the same VoI, an arbitrary subsystem is selected.

3.2. Scheduling coupled estimators

This section develops a feasible method for approximating the VoI. First, we reformulate (8) into

$$\mathrm{VoI}_k^j = \sum_{\ell=k}^{N-1} \mathrm{tr}\Big[\Gamma_\ell\big(\underbrace{\mathbf{E}[\vec{e}_\ell\vec{e}_\ell^{\mathsf{T}}|I_k^j, s_k=\varnothing]}_{\approx Q_\ell^{j,0}} - \underbrace{\mathbf{E}[\vec{e}_\ell\vec{e}_\ell^{\mathsf{T}}|I_k^j, s_k=j]}_{\approx Q_\ell^{j,1}}\big)\Big],$$

where $Q_\ell^{j,0}$ and $Q_\ell^{j,1}$ denote approximations of the error covariances that arise from two assumptions imposed next. Let $\Delta_k = \tilde{e}_k - \vec{e}_k$ with $\tilde{e}_k = [(\tilde{e}_k^1)^{\mathsf{T}}, \ldots, (\tilde{e}_k^M)^{\mathsf{T}}]^{\mathsf{T}}$, $\tilde{e}_k^j = x_k - \tilde{x}_k^j$ and $\Delta_k^j = \tilde{e}_k^j - e_k^j$. Then,

$$\begin{aligned}
&\mathbf{E}[\vec{e}_k\vec{e}_k^{\mathsf{T}}|I_k^j, s_k = \varnothing] = \mathbf{E}[\tilde{e}_k\tilde{e}_k^{\mathsf{T}}|I_k^j] \\
&= \mathbf{E}[\vec{e}_k\vec{e}_k^{\mathsf{T}}|I_k^j, s_k = j] + \mathbf{E}[\vec{e}_k\Delta_k^{\mathsf{T}}|I_k^j, s_k = j] \\
&+ \mathbf{E}[\Delta_k\vec{e}_k^{\mathsf{T}}|I_k^j, s_k = j] + \mathbf{E}[\Delta_k\Delta_k^{\mathsf{T}}|I_k^j, s_k = j]
\end{aligned} \tag{9}$$

To compute this term, we introduce two assumptions.

Assumption 3. Let $s_k = j$. For $i \neq j$, assume that

$$\Delta_k^i \approx \hat{\Delta}_{j,k}^i = \mathbf{E}[\Delta_k^i|I_k^j]. \tag{10}$$

Assumption 4. Assume that

$$\mathbf{E}[\vec{e}_\ell\vec{e}_\ell^{\mathsf{T}}|I_k^j, s_k = j] \approx \mathbf{E}[\vec{e}_\ell\vec{e}_\ell^{\mathsf{T}}|s_k = j],$$
$$\mathbf{E}[\vec{e}_\ell\hat{\Delta}_k^{\mathsf{T}}|I_k^j, s_k = j] \approx \mathbf{E}[\vec{e}_\ell\hat{\Delta}_k^{\mathsf{T}}|s_k = j].$$

Regarding Assumption 3, we have $\Delta_k^{s_k} = 0$. To facilitate computations, we can take the simple estimate $\hat{\Delta}_k^i = A\hat{x}_{k-1}^0 - \tilde{x}_k^j$, as done in Section 4. Assumption 4 holds true in case of a Kalman filter in the LQG context, but not necessarily for any BLUE, see Anderson and Moore (2012). Considering Assumptions 3 and 4 for $Q_k^{j,0}$, we can write

$$\begin{aligned}
Q_k^{j,0} &= \mathbf{E}[\vec{e}_k\vec{e}_k^{\mathsf{T}}|s_k = j] + \mathbf{E}[\vec{e}_k\hat{\Delta}_{j,k}^{\mathsf{T}}|s_k = j] \\
&+ \mathbf{E}[\hat{\Delta}_{j,k}\vec{e}_k^{\mathsf{T}}|s_k = j] + \hat{\Delta}_{j,k}\hat{\Delta}_{j,k}^{\mathsf{T}} = h_k(\vec{P}_{k-1}, j) + \hat{\Delta}_{j,k}\hat{\Delta}_{j,k}^{\mathsf{T}}
\end{aligned}$$

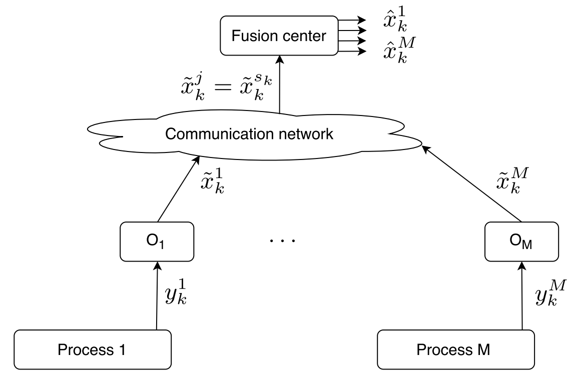

Fig. 2. Remote estimation of decoupled systems.

with the covariance matrix $\vec{P}_k = \mathbf{E}[\vec{e}_k\vec{e}_k^{\mathsf{T}}]$ where the ijth block matrix entry $[\vec{P}_k]_{ij} = P_k^{ij}$, $i \neq j$ is the cross-correlation of e_k^i and e_k^j and $[\vec{P}_k]_{jj} = P_k^j$ the covariance matrix of e_k^j, and $h_k(\cdot)$ is defined in (A.1). Equality holds due to the orthogonality principle for BLUE (Kay, 2013). Applying (A.1) onto $Q_k^{j,0}$ and onto $Q_k^{j,1}$ and by taking into account Assumption 4, we obtain the recursion

$$\begin{aligned}
Q_\ell^{j,0} &= h_\ell(Q_{\ell-1}^{j,0}, \bar{s}_\ell) \\
Q_\ell^{j,1} &= h_\ell(Q_{\ell-1}^{j,1}, \bar{s}_\ell)
\end{aligned} \quad k < \ell \le N-1, \tag{11}$$

with $Q_k^{j,1} = h_k(\vec{P}_{k-1}, j)$. Eventually, this gives us an approximation of VoI_k^j that can be expressed as

$$\widehat{\mathrm{VoI}}_k^j = \sum_{\ell=k}^{N-1} \mathrm{tr}\Big[\Gamma_\ell\big(Q_\ell^{j,0} - Q_\ell^{j,1}\big)\Big]. \tag{12}$$

3.3. Scheduling decoupled estimators

This section analyzes the important special case, in which M decoupled systems need to be monitored over the shared network, see Fig. 2. In this system class, we establish a link between the VoI-based approach and the optimal law minimizing the mean of J in (5). This highlights the benefits of the rollout strategy, as we can compute the VoI explicitly and it allows us to obtain performance guarantees related to the baseline schedule. What makes this system class amenable to analysis is the decoupling of each local estimate that constitutes to be the minimum mean-square estimator (MMSE) eliminating the need for further assumptions.

3.3.1. Decoupled systems model

Consider M isolated systems as depicted in Fig. 2. Related to (1) with partitioned state $x_k = [x_k^1, \ldots, x_k^M] x_k^j \in \mathbb{R}^{n_j}$, and cost (5), the system is restricted to

$$\begin{aligned}
&A = \mathrm{diag}[A^1, \ldots, A^M] \; R_w = \mathrm{diag}[R_w^1, \ldots, R_w^M], \\
&\Gamma_k = \mathrm{diag}[\Gamma_k^1, \ldots, \Gamma_k^M] \; C^j = [0, \ldots, 0, \tilde{C}^j, 0, \ldots, 0].
\end{aligned} \tag{13}$$

With slight abuse of notation, we assume that $\hat{x}_k^j \in \mathbb{R}^{n_j}$ is the local state estimate of x_k^j. Then, the cost J in (5) is

$$J = \sum_{j=1}^{M}\sum_{k=0}^{N-1} {e_k^j}^{\mathsf{T}}\Gamma_k^j e_k^j \tag{14}$$

with $e_k^j = x_k^j - \hat{x}_k^j$. At the local observer j, the Kalman estimate is computed by the recursion

$$\tilde{x}_k^j = \tilde{x}_{k|k-1}^j + K_k^j(y_k^j - \tilde{C}^j\tilde{x}_{k|k-1}^j) \tag{15}$$

$$P^j_k = (I - K^j_k C^j) P^j_{k|k-1} \tag{16}$$

$$\tilde{x}^j_{k+1|k} = A^j \tilde{x}^j_{k|k-1} \tag{17}$$

$$P^j_{k+1|k} = A^j P^j_k (A^j)^{\mathsf{T}} + R_{w,j} \tag{18}$$

where $K^j_k = P^j_{k|k-1}(\tilde{C}^j)^{\mathsf{T}}(\tilde{C}^j P^j_{k|k-1}(\tilde{C}^j)^{\mathsf{T}} + R_{v,j})^{-1}$ and $\tilde{x}_{0|-1} = \bar{x}_0$, $P^j_{0|-1} = R^j_0$. To circumvent Assumption 1, we apply a result from Ramesh, Sandberg, and Johansson (2013) by restricting the admissible VoI^j_k's to be symmetric in the discrepancy of local and remote estimate. Then, Ramesh et al. (2013) shows that the MMSE at the fusion center for system j is

$$\hat{x}^j_k = \begin{cases} \tilde{x}^j_k & s_k = j \\ \hat{x}^j_{k|k-1} & s_k \neq j \end{cases} \tag{19}$$

where $\hat{x}^j_{0|-1} = \bar{x}_0$, $\hat{x}^j_{k|k-1} = A^j \hat{x}^j_{k-1}$ and define $e^j_{k|k-1} = x^j_k - \hat{x}^j_{k|k-1}$. We shall see in the next section that the derived VoI^j_k is indeed satisfying the above restriction.

3.3.2. Computation of priorities

Since the systems are decoupled, we omit the broadcast information $\hat{x}^j_k$, i.e. the information available for computing priorities is $\tilde{I}^j_{k+1} = \{\tilde{I}^j_k, y^j_{k+1}, s_k\}$. As data from system j is not beneficial for state estimation in the other systems $i \neq j$, we obtain a simplified expression for (8).

$$\mathrm{VoI}^j_k = \sum_{\ell=k}^{N-1} \left[\mathbf{E}[{e^j_\ell}^{\mathsf{T}} \Gamma^j_\ell e^j_\ell | \tilde{I}^j_k, s_k \neq j] - \mathbf{E}[{e^j_\ell}^{\mathsf{T}} \Gamma^j_\ell e^j_\ell | \tilde{I}^j_k, s_k = j] \right]. \tag{20}$$

Note that we implicitly assume that the baseline $\{\bar{s}_{k+1}, \ldots, \bar{s}_{N-1}\}$ takes over in the future for system j. For the running cost, the first term can be written as

$$\mathbf{E}[{e^j_k}^{\mathsf{T}} \Gamma^j_k e^j_k | \tilde{I}^j_k, s_k \neq j] = \mathrm{tr}\left[\Gamma^j_k \mathbf{E}[e^j_{k|k-1} {e^j_{k|k-1}}^{\mathsf{T}} | \tilde{I}^j_k] \right].$$

Define $e_{j,k} = x^j_k - \tilde{x}^j_k$ and $\tilde{e}_{j,k} = \tilde{x}^j_k - \hat{x}^j_{k|k-1}$. Then,

$$\mathbf{E}[e^j_{k|k-1} {e^j_{k|k-1}}^{\mathsf{T}} | \tilde{I}^j_k] = \mathbf{E}[e_{j,k} e^{\mathsf{T}}_{j,k} | \tilde{I}^j_k] + \tilde{e}_{j,k} \tilde{e}^{\mathsf{T}}_{j,k}. \tag{21}$$

Equality holds as the Kalman estimate $\tilde{x}^j_k$ is identical to the conditional mean of x^j_k given $\tilde{I}^j_k$ and the fact that $\tilde{e}_{j,k}$ is measurable with respect to $\tilde{I}^j_k$ given by

$$\tilde{e}_{j,k} = \sum_{n=\tau_{j,k}+1}^{k} (A^j)^{k-n} K^j_n \tilde{y}^j_n, \tag{22}$$

$$\tau_{j,k} = \max\{\ell \mid s_\ell = j \wedge \ell < k\}. \tag{23}$$

where $\tilde{y}^j_n = y^j_n - \tilde{C}^j \tilde{x}^j_{n|n-1}$ and $\tau_{j,k}$ is the last transmission time. For the term ${e^j_k}^{\mathsf{T}} \Gamma^j_\ell e^j_k$ of the second expectation in (20), we have

$$\begin{aligned} &\mathbf{E}[{e^j_k}^{\mathsf{T}} \Gamma^j_k e^j_k | \tilde{I}^j_k, s_k = j] \\ &= \mathrm{tr}\left[\Gamma^j_k \mathbf{E}[(x^j_k - \tilde{x}^j_k)(x^j_k - \tilde{x}^j_k)^{\mathsf{T}} | \tilde{I}^j_k] \right] = \mathrm{tr}\left[\Gamma^j_k P^j_k \right], \end{aligned}$$

where the last equality arises from the invariance of the error covariance matrix of the Kalman filter with regard to its own observations (Anderson & Moore, 2012). In the remainder of this section, we are concerned with the future terms at $k+1, \ldots, N-1$ of the cost-to-go function in (20). Because of (19), the evolution of the observer estimate $\hat{x}^j_\ell$ will be independent of previous scheduling choices, once a transmission is performed at future time ℓ. Define the first transmission time after k of the baseline schedule as

$$\bar{\tau}_{j,k} = \min\{\ell \,|\, \bar{s}_k = j \wedge k < \ell \leq N-1\}, \tag{24}$$

where we define $\bar{\tau}_{j,k} = N - 1$ if no transmission will occur in the future based on the baseline schedule. Based on our previous statement, we only need to consider cost terms until $\bar{\tau}_{j,k}$ in our VoI calculations, as the predicted estimate at the observer, $\hat{x}^j_\ell$, $\bar{\tau}_{j,k} \leq \ell \leq N-1$, will be the same for the case $s_k \neq j$ and for the case $s_k = j$. Therefore, the VoI can be computed as

$$\mathrm{VoI}^j_k = \mathrm{tr}[\sum_{\ell=k}^{\bar{\tau}_{j,k}-1} \Gamma^j_\ell \tilde{Q}^j_\ell], \tag{25}$$

where $\tilde{Q}^j_\ell$ can be computed recursively by

$$\begin{aligned} \tilde{Q}^j_k &= \tilde{e}_{j,k} \tilde{e}^{\mathsf{T}}_{j,k}, \\ \tilde{Q}^j_{\ell+1} &= A^j \tilde{Q}^j_\ell {A^j}^{\mathsf{T}} \end{aligned} \tag{26}$$

for $k \leq \ell < \bar{\tau}_{j,k}$ with $\tilde{e}_{j,k}$ be given by (22).

3.3.3. Performance guarantee

Our goal of this section is to give a performance guarantee that relates the cost of the VoI strategy to the cost of the baseline schedule. For that reason, we introduce a centralized decision rule for s_k that chooses the control system according to the complete information $I_k = \{I_{1,k}, \ldots, I_{M,k}\}$. Though this scheme violates the imposed restrictions in the information structure to compute priorities, it enables us to bridge the gap between the cost J in (14) resulting from a baseline schedule and the VoI-based priority assignment.

Definition 5. Suppose a given deterministic baseline schedule $\{\bar{s}_1, \ldots, \bar{s}_{N-1}\}$. The optimal centralized rollout strategy for finding s_k at time k is defined as

$$J^* = \min_{s_k} \mathbf{E}[\sum_{j=1}^{M} \sum_{\ell=k}^{N-1} {e^j_\ell}^{\mathsf{T}} \Gamma^j_\ell e^j_\ell | I_k, s_{k+1} = \bar{s}_{k+1}, \ldots], \tag{27}$$

where s_k is a function of the global information I_k.

Using Definition 5, we obtain the subsequent result.

Lemma 6. *Let the system be defined as in (13) with estimators (15)–(18), (19), and the cost as in (14). Then, the VoI-based scheduler (7) leads to the same minimal cost J^* as the optimal centralized rollout strategy.*

Proof. As the systems are decoupled and we are using a deterministic baseline strategy, measurements $i \neq j$ are independent of variables appearing in system j.

Therefore, the cost given in (27) decomposes into

$$\sum_{j=1}^{M} \mathbf{E}[\sum_{\ell=k}^{N-1} {e^j_\ell}^{\mathsf{T}} \Gamma^j_\ell e^j_\ell | \tilde{I}^j_k].$$

This implies that each system j can evaluate its cost $\mathbf{E}[\sum_{\ell=k}^{N-1} {e^j_\ell}^{\mathsf{T}} \Gamma^j_\ell e^j_\ell | \tilde{I}^j_k]$ independently for the case either measurements $\{y^j_{\tau_{j,k}+1}, \ldots, y^j_k\}$ are used ($s_k = j$) or are not used ($s_k \neq j$) for state estimation at the controller. By taking the state estimation update that yields the greatest benefit reflected by the difference of these cost terms, we obtain the optimal decision rule minimizing the cost (14). This rule coincides with the VoI-based priority assignment defined in (20). As the event in which the VoIs are identical among different systems occurs with probability zero, we can conclude our proof.

Using Lemma 6, a performance guarantee for the VoI-based strategy is obtained in the subsequent theorem whose proof can be found in the appendix.

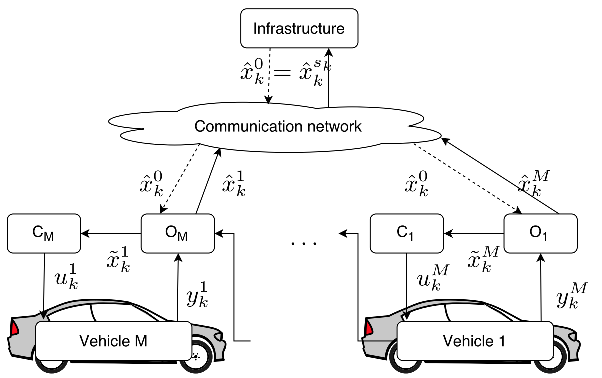

Fig. 3. Platooning case study.

Theorem 7. *Assume the system as in Lemma 6 and let* $\{\bar{s}_0, \ldots, \bar{s}_{N-1}\}$ *be a baseline heuristic with cost* $\bar{J}$. *Then,* $\bar{J}$ *is an upper bound for the cost resulting from the priority assignment based on* VoI_k^j *defined in (25) using the rollout strategy with baseline heuristic* $\{\bar{s}_0, \ldots, \bar{s}_{N-1}\}$.

3.4. Computational complexity

This section discusses the numerical effort to compute VoI priorities. The numerically intensive part for the coupled case is given by the computation of the covariance matrices $Q_\ell^{j,0}$ and $Q_\ell^{j,1}$. In the time-variant case with time-dependent predictor $h_k(\cdot)$, the numerical complexity is $\mathcal{O}((Mn)^3N)$, see Golub and Van Loan (2012). For the time-invariant case, the complexity reduces to $\mathcal{O}((Mn)^3)$ by pre-computing the matrices of the predictor map. Hence by using a receding horizon approach together with a stationary estimator, the numerical complexity can be reduced significantly, when N is large. It follows from (25)–(26) for the decoupled case that VoI_k^j is a quadratic function in $\tilde{e}_{j,k}$. What makes the method numerically attractive is the fact that we only need $k - \bar{\tau}_{j,k}$ prediction steps to compute the VoI reducing the complexity to $\mathcal{O}((n_j)^2\bar{\tau}_j^{\max})$, (Golub & Van Loan, 2012), where $\bar{\tau}_j^{\max}$ is the maximum inter-sampling interval of the baseline schedule. This implies that the approach can deal with a large horizon N.

4. Numerical case study

One of the elementary operations for automated driving is distance keeping, that is addressed in the following platooning use case of a group of M vehicles coordinated by the infrastructure. The architecture, see Fig. 3, is an extension of the estimation architecture defined in Section 2, in which subsystems employ the local state estimate for control purposes. We compare our VoI-based approach with time-triggered scheduling, in which vehicles send their estimate successively at a pre-given order.

The dynamic vehicle model for the longitudinal control has been adopted from Molin et al. (2016). Consider a sampling time of $T_S = 100\,\mathrm{ms}$ and a platoon of $M = 6$ vehicles with $j = 1$ as lead. The model is defined as

$$v_k^1 = a_0 v_k^1 + b_0 u_k^1 w_k^1, \tag{28}$$

$$d_k^j = d_k^j + \lambda(v_k^{j-1} - v_k^j) + b_1(u_k^{j-1} - u_k^j) + w_k^{j,d}, \tag{29}$$

$$v_k^j = a_0 v_k^j + b_0 u_k^j + w_k^{j,v}, \quad j = 2, \ldots, 6, \tag{30}$$

in which u_k^1 controls the vehicle to a desired reference velocity v_k^{ref} while u_k^j is designed to keep a desired reference distance d_k^{ref} to the preceding vehicle. The vehicle parameters are $a_0 = 0.98$ and $\lambda = 0.1$, $b_0 = 1.0 \times 10^{-3}$, $b_1 = 0.05 \times 10^{-3}$. Each vehicle implements an integral state that accumulates the reference error $v_k^1 - v_k^{\mathrm{ref}}$ for the lead vehicle and $d_k^j - d_k^{\mathrm{ref}}$ for the followers. Suppose a standard deviation for the process noise $\sigma_{v_k^j} = 0.05$, $\sigma_{v_k^{\mathrm{ref}}} = 0.05$ and $\sigma_{d_k^j} = 0.001$, $\sigma_{t_k^j} = 0.001$. Each vehicle j measures its velocity with standard deviation $\sigma_v = 0.01$, while the followers $j > 1$ measure also distances to its preceding vehicle with $\sigma_d = 0.01$. An LQR controller is employed whose linear gains are taken from Molin et al. (2016), which also yields the values for weighting matrix Γ_k. The controller and VoI-based scheduler are operated in a receding horizon fashion. Hence, the time-invariant control gain obtained for $k = 0$ is used, while VoI_k^j is determined using a periodic base-line schedule.

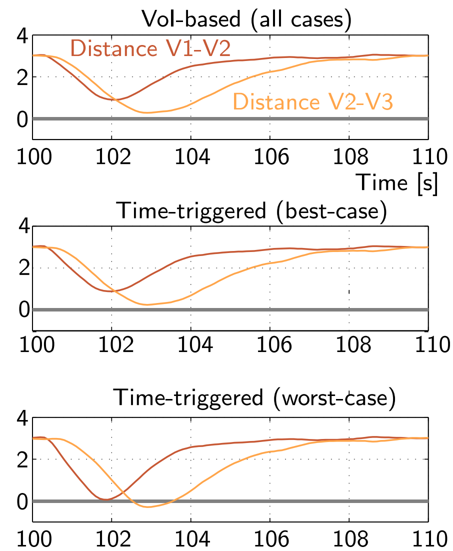

Fig. 4. Inter-vehicle distances.

Fig. 4 illustrates the behavior of a platoon when the lead vehicle brakes at time $100\,\mathrm{s} \pm 1\,\mathrm{s}$, implying that the reference velocity v_k^{ref} drops from 14 to $5\,\mathrm{m/s}$. The timing behavior is illustrated in the table above of Fig. 4. The VoI-based scheduler detects the emergency brake in all trials causing a timely broadcast of vehicle 1. For the time-triggered scheduler, the same behavior is only observed, if the braking event coincides with the transmission slot of V_1 (best-case), while in the worst-case a delay of 5 time slots occurs until the braking event is broadcast. By running Monte Carlo simulations with 10 000 trials, none of the sample paths led to a vehicle collision for the VoI-based approach, while collisions occurred in 19.7% of the cases for time-triggered scheduling.

5. Summary

In this paper, we have developed a methodology for data scheduling of a system of networked estimators that employs the Value of Information to determine the urgency to transmit data over the shared communication network. The benefits of our approach have been approved theoretically for decoupled remote estimators by yielding a performance certificate. The numerical case study have unveiled significant improvement of our proposed strategy with regard to reactivity to unpredicted events.

Appendix A. Calculation of error (cross-) covariances

The error evolution at different stages is given by

$$\tilde{e}^j_{k|k-1} = Ae^j_{k-1} + w_k \qquad \text{(after prediction)}$$
$$\tilde{e}^j_k = (I - K^j_k C^j)\tilde{e}^j_{k|k-1} - K^j_k v^j_k, \qquad \text{(after } y^j_k \text{ update)}$$
$$e^j_k = (I - K^{mj}_k)\tilde{e}^j_k + K^{mj}_k \tilde{e}^m_k \qquad \text{(after fusion)}$$

assuming system $m \neq j$ broadcast $\tilde{x}^m_k$ while $e^m_k = \tilde{e}^m_k$ for $j = m$. Based on Assumption 1 and assuming that system m is transmitting at time k, the error covariances and cross-correlations have the relations

$$\tilde{P}^j_{k|k-1} = \mathbf{E}[\tilde{e}^j_{k|k-1}(\tilde{e}^j_{k|k-1})^\top] = AP^j_{k-1}A^\top + R_w$$
$$\tilde{P}^{ij}_{k|k-1} = \mathbf{E}[\tilde{e}^i_{k|k-1}(\tilde{e}^j_{k|k-1})^\top] = AP^{ij}_{k-1}A^\top + R_w$$
$$\tilde{P}^j_{0|-1} = R_0, \quad \tilde{P}^{ij}_{0|-1} = R_0.$$
$$\tilde{P}^j_k = \mathbf{E}[\tilde{e}^j_k(\tilde{e}^j_k)^\top] = (I - K^j_k C^j)\tilde{P}^j_{k|k-1}(I - K^j_k C^j)^\top + K^j_k R_{v,j}(K^j_k)^\top$$
$$\tilde{P}^{ij}_k = \mathbf{E}[\tilde{e}^i_{k|k-1}\tilde{e}^{j\,\top}_{k|k-1}] = (I - K^i_k C^i)\tilde{P}^{ij}_{k|k-1}(I - K^j_k C^j)^\top.$$
$$P^j_k = \mathbf{E}[e^j_k e^{j\,\top}_k] = (I - K^{mj})\tilde{P}^j_k(I - K^{mj}_k)^\top + (I - K^{mj}_k) \times \tilde{P}^{jm}_k K^{mj\,\top} + K^{mj}_k \tilde{P}^{mj}_k (I - K^{mj}_k)^\top + K^{mj}_k \tilde{P}^m_k K^{mj\,\top}_k,$$
$$P^{ij}_k = \mathbf{E}[e^i_k e^{j\,\top}_k] = (I - K^{mi}_k)\tilde{P}^{ij}_k(I - K^{mj})^\top + (I - K^{mi}_k) \times \tilde{P}^{im}_k K^{mj\,\top}_k + K^{mi}_k \tilde{P}^{mj}_k (I - K^{mj})^\top + K^{mi}_k \tilde{P}^m_k K^{mj\,\top}_k.$$

By concatenating the above update equations, we can determine the recursive update map h_k for $\vec{P}_k$ defined as

$$\vec{P}_{k+1} = h_k(\vec{P}_k, m). \tag{A.1}$$

Appendix B. Proof of Theorem 7

Proof. Let $y_k = \{y^1_{k+1}, \dots, y^M_{k+1}\}$. Then, the centralized information structure follows the recursion

$$I_{k+1} = \{I_k, y_k, s_k\}. \tag{B.1}$$

Let $s_k = \pi^{\mathrm{RO}}_k(I_k)$ be the rollout strategy based on the heuristic $\{\bar{s}_0, \dots, \bar{s}_{N-1}\}$, $0 \leq k \leq N-1$. Let the running cost at time k be defined as

$$c_k(I_k, s_k) = \mathbf{E}[\sum_{j=1}^{M} e^{j\,\top}_k \Gamma^j_k e^j_k | I_k, s_k]. \tag{B.2}$$

Let $J^{\mathrm{RO}}_k(I_k)$, $\bar{J}_k(I_k)$ be the cost-to-go of the rollout strategy and of the heuristic at time k. We prove inductively that $J^{\mathrm{RO}}_k(I_k) \leq \bar{J}_k(I_k)$ at each time k. For $k = N$, we have $J^{\mathrm{RO}}_N(I_N) = \bar{J}_N(I_N) = 0$ as there is no terminal cost in J, see (14). Assume that $J^{\mathrm{RO}}_{k+1}(I_{k+1}) \leq \bar{J}_{k+1}(I_{k+1})$ for all I_{k+1}. Then, we have from (27)

$$\begin{aligned} &J^{\mathrm{RO}}_k(I_k) \\ &= \mathbf{E}[c_k(I_k, \pi^{\mathrm{RO}}_k(I_k)) + J^{\mathrm{RO}}_{k+1}(\{I_k, y_{k+1}, \pi^{\mathrm{RO}}_k(I_k)\})|I_k] \\ &\leq \mathbf{E}[c_k(I_k, \pi^{\mathrm{RO}}_k(I_k)) + \bar{J}_{k+1}(\{I_k, y_{k+1}, \pi^{\mathrm{RO}}_k(I_k)\})|I_k] \\ &\leq \mathbf{E}[c_k(I_k, \bar{s}_k) + \bar{J}_{k+1}(\{I_k, y_{k+1}, \bar{s}_k\})|I_k] = \bar{J}_k(I_k) \end{aligned}$$

The first and the last equalities state the definition of the cost-to-go of the rollout and baseline scheduler, respectively. The first inequality is due to the induction hypothesis, while the second arises from the fact that $\pi^{\mathrm{RO}}_k(I_k)$ solves (27). This completes the induction.

By Lemma 6, the centralized rule $\pi_k(I_k)$ is identical to using scheduling with the VoI-based priority assignment given by (20). With this, we can conclude the proof.

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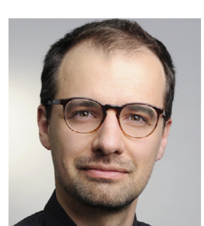

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