Fault Detection and Mitigation in Kirchhoff Networks

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Abstract—In this letter, we study the problem of fault detection and mitigation in networks where the measurements satisfy Kirchhoff's voltage law. First, we characterise the class of faults appearing as an additive fault vector (injected by a malicious adversary or due to equipment failures) that can be detected by taking into account the topology of the network. Second, we consider the problem of estimating the fault vector via tools from compressive sensing. Moreover, we comment on the applicability of the developed methods to the case where the measurements satisfy Kirchhoff's current law. The proposed methods are validated via numerical examples with application to time synchronization networks.

I. Introduction

WHILE the distributed nature of networked systems ensures reliability and availability by removing single points of failure in the system, it provides a malicious agent with multiple points of entry into the system that may be used by an agent to eavesdrop on the data and/or corrupt the communicated/measured information between subsystems in the network. In addition, by introducing more components to the system we increase the probability of hardware failures and having faulty measurements. Due to these considerations recently the research community has started studying fault detection in networked systems, see [1] and references therein.

The first contribution of this letter is addressing the problem of detecting additive faults that corrupt the measurements in a network. Later, we propose a method to estimate these faults and counter-act their effects in networks in which the inter-agent measurements should follow Kirchhoff's voltage law (KVL), i.e., the measurements add to zero in each loop in the network [2]. An important example for the case where measurements satisfy KVL is the well-studied scenario of clock synchronization in networks [3]–[6]. We note that recently the problem of identifying faulty measurements through linear models has attracted much attention. For example, in [7] the authors propose fault identification via belief propagation in a network and have shown their method's superiority comparing with other state-of-the-art methods. The main differences between the scenario studied in [7] and our result are that, first, we do not make any assumptions on knowing the probability of occurrence of

a given fault or a set of faults. Second, in addition to identifying the faulty elements of the network, we are interested in the calculation of the fault values when the fault vector is sparse without making any further assumption on its sparsity. We use tools from graph theory, detection theory, and compressive sensing to address these problems. The only information that we assume is available is the measurements and the topology of the network. No knowledge of the parameters of the network, such as the inter node message propagation time in clock networks, or any knowledge on the faults is assumed. Later, we briefly comment on the applicability of similar results to networks with measurements satisfying Kirchhoff's current law (KCL), e.g., power or water flows in electrical and water networks, respectively. There is a vast body of literature on the centralized detection and isolation in power systems [8]. However, those results mainly assume a complete knowledge of the model of the system. The method introduced here is applicable to cases where the model is not known exactly but it is known to satisfy KVL.

The structure of this letter is the following. In Section II we introduce the necessary background and define the problems of interest. In Section III we address the problems described in Section II. In Section IV a numerical example is introduced where the measurements in a network are faulty and the goal is to mitigate the effect of such a fault by estimating the additive fault vector that has corrupted the measurements. Some concluding remarks come in the end.

II. Problem of Interest

Consider the network $\mathcal{N}$ isomorphic to the (directed) graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{i\}_{i=1}^{N}$, is its vertex set and $\mathcal{E}$ its edge set with $|\mathcal{E}| = m$. Each node i has a state value denoted by $x_i \in \mathbb{R}$. Moreover the edge $(i,j) \in \mathcal{E}$ if and only if node i measures the state of node j. In this letter, we use the standard definition of cycles in a graph, see [9] for more details. We assume that the direction of a cycle is the order in which the nodes are visited. We let $\mathcal{C}(\mathcal{G})$ denote the set of all cycles of $\mathcal{G}$, and $|\Lambda|$ be the number of edges in the cycle Λ. Moreover, we assume that $\mathcal{C}(\mathcal{G}) \neq \emptyset$. Note that in a directed graph, the cycle directions are independent of the direction of the individual edges composing the cycles, i.e., a cycle does not require that all edges point in the same direction. We primarily focus on the following setting in this letter.

Definition 1: For each $(i,j) \in \mathcal{E}$ node i measures $\hat{y}_{ij} = y_{ij} + f_{ij} + \epsilon_{ij}$. Moreover, $y_{ij} = x_j - x_i$, where ϵ_{ij} is the noise in the measurements and is modeled as a stochastic variable with zero mean, and f_{ij} is an unknown fault (typically deterministic) by which the measurements are corrupted. Furthermore, let $\mathcal{N}_i$ be the set of all the nodes j such that $(i,j) \in \mathcal{E}$. Define $\tilde{y}_i \in \mathbb{R}^{|\mathcal{N}_i|}$, as the vector obtained from stacking all the measurements $\hat{y}_{ij}, \forall j \in \mathcal{N}_i$. Also, let $\tilde{\mathbf{y}} = [\tilde{y}_1^\top, \cdots, \tilde{y}_N^\top]^\top$, and we assume that y_{ij} for all $(i,j) \in \mathcal{E}$ satisfy the Kirchhoff's Voltage Law (KVL), viz. $\sum_{(i,j) \in \Lambda} y_{ij} = 0, \forall \Lambda \in \mathcal{C}(\mathcal{G})$. Finally, let $\mathbf{f}$ be obtained from stacking all f_{ij}, for all $(i,j) \in \mathcal{E}$ where the
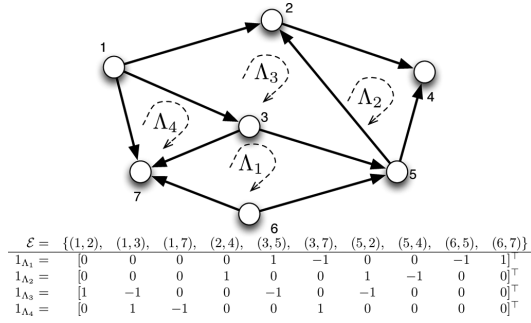
Fig. 1. A graph with $\ell = 4$ fundamental cycles and vectors.

k-th element of $\mathbf{f}$ corresponds to the possible fault in the measurement associated with the k-th entry of $\tilde{\mathbf{y}}$.

Before continuing we briefly comment on the *physical layer* and the *network layer* considered in this setting. The physical layer is the underlying system where certain physical constraints are enforced, e.g., it corresponds to the electrical network which enforces the Kirchhoff voltage law. Then, the network layer is the communication network, possibly an IT infrastructure, that determines what measurements are available and how they are interchanged among the nodes.

Definition 2: For $\Lambda \in \mathcal{C}(\mathcal{G})$, the cycle vector is the vector $1_\Lambda \in \{-1, 0, +1\}^m$, $1_\Lambda \in \mathbb{R}^m$ whose i-th entry is $+1$ if the i-th edge belongs to Λ and its orientation is consistent with the orientation of Λ, -1 if the i-th edge belongs to Λ and its orientation is opposite the orientation of Λ, and is 0 otherwise.

Definition 3: Let $\mathcal{L}$ be the set of all the cycle vectors of graph $\mathcal{G}$, i.e., $\mathcal{L} = \{1_\Lambda, \forall \Lambda \in \mathcal{C}(\mathcal{G})\}$. A set of fundamental cycle vectors $\mathcal{L}_f \subset \mathcal{L}$ is a subset of $\mathcal{L}$ that constitute a base for the subspace spanned by all the vectors in $\mathcal{L}$. The elements of $\mathcal{L}_f$ are called fundamental cycle vectors. Moreover, the cycle matrix $\mathbf{C}_\mathcal{L}$ of a directed graph $\mathcal{G}$ is the $m \times k$ matrix $\mathbf{C}_\mathcal{L} = [1_{\Lambda_1}, \dots, 1_{\Lambda_k}]$ where k is the cardinality of $\mathcal{L}$. The $m \times \ell$ matrix $\mathbf{C} \subset \mathbf{C}_\mathcal{L}$, with $\ell = |\mathcal{L}_f| = m - N + 1$, such that each column represents a fundamental cycle vector in $\mathcal{L}_f$, is called the fundamental cycle matrix $\mathbf{C} = [1_{\Lambda_1}, \dots, 1_{\Lambda_\ell}], \forall 1_{\Lambda_i} \in \mathcal{L}_f$.

Fig. 1 depicts a graph and its fundamental cycles. Note that $\mathbf{C}$ is not unique since it depends on the choice of the fundamental cycles vectors, and it is a full column rank matrix. Given a set of fundamental cycle vectors $\mathcal{L}_f$, we let $\mathcal{L}_f(\mathcal{G})$ denote the associated fundamental cycles $\mathcal{L}_f(\mathcal{G}) = \{\Lambda \in \mathcal{C}(\mathcal{G}) | 1_\Lambda \in \mathcal{L}_f\}$. For more information on how to calculate fundamental sets of cycles and computational complexity of doing so one may refer to [10]—for a planar graph the complexity of calculating the fundamental cycles is linear in the number of vertices. In general graphs, the complexity is not generically more than $O(n^2)$.

Consider the network in Definition 1 and centralized computations. Just knowing the measured values, the distribution of the noise in the measurements, the network graph, and the fact that the measurements satisfy KVL, the answers to the following problems are desired:

Problem 1: How can one deduce that for some (or all) $(i, j) \in \mathcal{E}$, f_{ij} is nonzero? In other words, how can one detect if the measurements are faulty?

Problem 2: How can an estimate for $\mathbf{f}$, termed $\mathbf{f}^\star$, be calculated to be used to compute $\tilde{\mathbf{y}}_c \triangleq \tilde{\mathbf{y}} - \mathbf{f}^\star$, in which, the effect of $\mathbf{f}$ is ameliorated?

III. MAIN RESULT

In this section, we first consider the noiseless case where $\epsilon_{ij} = 0$ for all i and j such that $(i, j) \in \mathcal{E}$. The following theorem addresses Problem 1 for this case and serves as a starting point for the noisy case where $\epsilon_{ij} \neq 0$ for all i and j such that $(i, j) \in \mathcal{E}$.

Theorem 1: Consider the network described in Definition 1. Additionally, assume $\epsilon_{ij} = 0$ for all i and j such that $(i, j) \in \mathcal{E}$. If

$$\|\mathbf{C}^\top \tilde{\mathbf{y}}\| > 0 \tag{1}$$

then $\mathbf{f} \neq 0$.

Proof: Note that $\mathbf{C}$ has full column rank, $\|\mathbf{C}^\top \tilde{\mathbf{y}}\| = \|\mathbf{C}^\top \mathbf{y} + \mathbf{C}^\top \mathbf{f}\|$, and $\|\mathbf{C}^\top \mathbf{y}\| = 0$. Thus, $\|\mathbf{C}^\top \tilde{\mathbf{y}}\| = \|\mathbf{C}^\top \mathbf{f}\|$. Assuming $\|\mathbf{C}^\top \tilde{\mathbf{y}}\| > 0$, then we have $\|\mathbf{C}^\top \mathbf{f}\| > 0$. Then it is immediate that $\mathbf{f} \neq 0$. ∎

Note that (1) can be used as a test to determine if the measurements are faulty for the noiseless case, i.e., if the inequality holds then the measurements are faulty.

In what comes next we consider the noisy case where $\epsilon_{ij} \neq 0$ for all i and j such that $(i, j) \in \mathcal{E}$. So

$$\tilde{\mathbf{y}} = \bar{\mathbf{y}} + \bar{\epsilon} \tag{2}$$

where the k-th entries of $\bar{\mathbf{y}}$ and $\bar{\epsilon}$ correspond to the exact measurement and the noise in the measurement associated with the k-th entry of $\tilde{\mathbf{y}}$ where $\mathbf{f} = 0$, respectively. Here $\bar{\epsilon}$ is a variable with zero mean and $\Sigma_{\bar{\epsilon}} = \text{diag}(\bar{\sigma}_1, \dots, \bar{\sigma}_m)$ as its covariance matrix. For the case where the measurements are not faulty, we have $\mathbf{r} \triangleq \mathbf{C}^\top \tilde{\mathbf{y}} = \mathbf{C}^\top(\bar{\mathbf{y}} + \bar{\epsilon}) = \mathbf{C}^\top \bar{\epsilon}$. It is obvious that $\mathbf{r}$ is again a zero mean Gaussian variable with $\Sigma_{\mathbf{r}} = \mathbf{C}^\top \Sigma_{\bar{\epsilon}} \mathbf{C}$ as its covariance. Moreover, we know that $\mathbf{C}^\top \Sigma_{\bar{\epsilon}} \mathbf{C}$ is nonsingular and in fact its minimum eigenvalue is equal to or greater than $\min(\bar{\sigma}_1, \dots, \bar{\sigma}_m)$ (This is a direct consequence of Theorem 2.3 of [11].). Consider the quadratic cost function $J(\mathbf{r}) = \mathbf{r}^\top \Sigma_{\mathbf{r}}^{-1} \mathbf{r}$. In the absence of fault, i.e. $\mathbf{f} = 0$, the quadratic form $\mathbf{r}^\top \Sigma_{\mathbf{r}}^{-1} \mathbf{r}$ has a chi-squared distribution with ℓ degrees of freedom. Hence, standard binary hypothesis tests can determine if the measurements are corrupted. We omit the discussion about them for the sake of brevity. The reader may refer to [12] for more information on such tests.

For the rest of this section we mainly focus on Problem 2. Specifically we focus on the problem of mitigating the effect of the fault on the measurements. Hence, we study the possibility of reconstructing the vector $\mathbf{f}$. We address this problem under the following assumption.

Assumption 1: A maximum of s sensor measurements are faulty, i.e., $\mathbf{f}$ has at most s non-zero entries. In other words, $\mathbf{f}$ is s-sparse and s is unknown to the system administrator.

Remark 1: Assumption 1 is a realistic one for small values of s because in the context of a networked system it is not the case that all the sensors report faulty measurements at the same time. Or in case that the faults are due to a malicious agent it can be assumed that the agent has limited resources, hence, only capable of corrupting s measurements.

Now, consider (2) where the measurements are corrupted by $\mathbf{f}$: $\tilde{\mathbf{y}} = \bar{\mathbf{y}} + \mathbf{f} + \bar{\epsilon}$. Multiplying both sides by $\mathbf{C}^\top$ and setting $\tilde{\epsilon} = \mathbf{C}^\top \bar{\epsilon}$ we have

$$\mathbf{r} = \mathbf{C}^\top(\mathbf{f} + \bar{\epsilon}) = \mathbf{C}^\top \mathbf{f} + \tilde{\epsilon}. \tag{3}$$

Note that $\tilde{\epsilon} \sim N(0, \Sigma_{\tilde{\epsilon}})$, $\Sigma_{\tilde{\epsilon}} = \mathbf{C}^{\top} \Sigma_{\bar{\epsilon}} \mathbf{C}$. Normalizing (3) we have

$$\Sigma_{\tilde{\epsilon}}^{-1/2} \mathbf{r} = \Sigma_{\tilde{\epsilon}}^{-1/2} \mathbf{C}^{\top} \mathbf{f} + \Sigma_{\tilde{\epsilon}}^{-1/2} \tilde{\epsilon}$$
$$\bar{\mathbf{r}} = \tilde{\mathbf{C}} \mathbf{f} + \delta, \qquad (4)$$

where $\bar{\mathbf{r}} \triangleq \Sigma_{\tilde{\epsilon}}^{-1/2} \mathbf{r}$, $\tilde{\mathbf{C}} = \Sigma_{\tilde{\epsilon}}^{-1/2} \mathbf{C}^{\top}$, $\delta \triangleq \Sigma_{\tilde{\epsilon}}^{-1/2} \tilde{\epsilon}$, and $\delta \sim N(0, I)$. Before continuing discussing the reconstruction method based on compressive sensing theory we comment on why reconstructing an $\mathbf{f}$ using the pseudoinverse of $\tilde{\mathbf{C}}$, $\tilde{\mathbf{C}}^{\dagger}$, might not be desirable. For affine systems such as (4), the pseudoinverse may be used to construct the solution of minimum Euclidean norm among all solutions. That is $\mathbf{f}_p = \tilde{\mathbf{C}}^{\dagger} \bar{\mathbf{r}}$ satisfies $\|\mathbf{f}_p\|_2 \le \|\mathbf{f}\|_2$, for all solutions $\mathbf{f}$ to (4) with $\delta = 0$. This raises two issues. First, $\mathbf{f}_p$ is generically dense (Hence, violating Assumption 1.) and one cannot identify which measurements are likely to be faulty by identifying the nonzero entries of $\mathbf{f}_p$. Second, if the magnitude of the real fault vector $\mathbf{f}$ is very large, the error in the reconstructed $\mathbf{f}_p$ will be generically large. Now we return to reconstructing $\mathbf{f}$ using the methods introduced in [13]. Since the entries of δ are i.i.d. Gaussian random variables, one can solve the following convex optimization problem, the *Dantzig selector*, to reconstruct $\mathbf{f}$

$$\min_{\tilde{\mathbf{f}}} \|\tilde{\mathbf{f}}\|_1 \quad \text{s.t.} \quad \|\tilde{\mathbf{C}}^{\top} \rho\|_{\infty} \triangleq \sup_{1 \le i \le m} \left| (\tilde{\mathbf{C}}^{\top} \rho)_i \right| \le c_i \lambda, \qquad (5)$$

where $\rho = \tilde{\mathbf{C}} \tilde{\mathbf{f}} - \bar{\mathbf{r}}$, $(\tilde{\mathbf{C}}^{\top} \rho)_i$ is the i-th entry of $\tilde{\mathbf{C}}^{\top} \rho$, c_i is the Euclidean norm of the i-th column of $\tilde{\mathbf{C}}$, and λ is some positive scalar. The solution to (5) is $\mathbf{f}^{\star}$ that can be used to mitigate the effect of the fault by using $\tilde{\mathbf{y}}_c \triangleq \tilde{\mathbf{y}} - \mathbf{f}^{*}$ instead of $\tilde{\mathbf{y}}$. Moreover, one can use all the nonzero entries of $\mathbf{f}^{\star}$ to identify which sensor measurements are likely to be faulty. Algorithm 1 is proposed to identify the faulty edges in the network.

Algorithm 1 Identification for t Rounds

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for i from 1 to t {t is an integer indicating the number of identification rounds.} **do**

    Collect measurements $\tilde{\mathbf{y}}$;

    Calculate $\bar{\mathbf{r}}$ as in (4);

    Calculate $\mathbf{f}^{\star}$ by solving (5);

    for Entries of $\mathbf{f}^{\star} \ge \varepsilon$ **do**

        Identify the corresponding edges as faulty; {$0 \le \varepsilon$}

    end for

end for

Moreover, we propose a mechanism to minimize the number of faulty measurements:

Definition 4 (Edge Healing): An edge (i, j) that is detected to be faulty is healed when f_{ij} is guaranteed to be zero.

We are assuming that each node has as many sensors as the number of its neighbours (equivalently its degree). Thus, an edge incident at a node can be healed by installing better sensing equipments to replace the faulty hardware or using encryption in the context of fault injection by a malicious agent [14]. Hence, healing an edge involves replacing/modifying only the sensor in
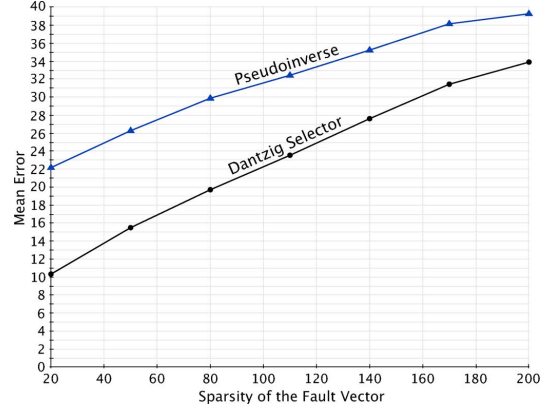

Fig. 2. The mean error between the reconstructed fault vector and the real fault vector for different sparsities of the fault vector using two different methods ($\|\mathbf{f} - \mathbf{f}^{\star}\|$ and $\|\mathbf{f} - \mathbf{f}_p\|$).

the node that corresponds to the edge being healed, not all the sensors in the node. Hence, at each round of identification applying Algorithm 1 all the identified faulty edges are healed. In next result we identify the types of faults that cannot be either detected or reconstructed using the methods introduced earlier.

Corollary: Consider the network described in Definition 1. Assuming that for a set of measurements, $(i, j) \in \mathcal{E}_f \subseteq \mathcal{E}$, each measurement $\hat{y}_{ij}$ is corrupted by some nonzero values f_{ij}. The existence of $\mathbf{f}$ cannot be detected using (1) and it cannot be reconstructed using (5) if and only if

$$\mathbf{C}^{\top} \mathbf{f} = 0. \qquad (6)$$

This corollary, is a direct consequence of the fact that if (6) is satisfied the obtained measurements correspond to a physically feasible network, albeit, different from the network that is being considered.

IV. NUMERICAL EXAMPLES

In this section we consider the problem of detecting the situations where the measurements based on the time-of-arrival (TOA) stamps measured at individual sensors from a limited number of wireless signals transmitted by certain *neighbour* nodes in the network are corrupted. First we describe the measurements taken in the network. The simplest approach for pairwise clock synchronization is to compute the estimate $\hat{y}_{ij} = ((\hat{t}_{ji}^f - \hat{x}_i^a) - (\hat{t}_{ij}^b - \hat{x}_j^b))/2) = ((t_{ji}^a - x_i^a) - (t_{ij}^b - x_j^b))/2) + \epsilon_{ij} = y_{ij} + \epsilon_{ij}$, where node i broadcasts a packet a to node j along with $\hat{x}_i^a$ (the unknown time of transmission for message a measured in node i's internal time frame). Node j returns a packet b to node i along with $\hat{t}_{ji}^a$, the measured travel time of packet a to j, and $\hat{x}_j^b$. Node i then measures $\hat{t}_{ij}^b$ and computes $\hat{y}_{ij}$ where the real relative clock bias at i is $y_{ij} = x_j - x_i$. In the rest of this section we consider the scenario where the time difference measurements to achieve synchronization in a randomly generated network with 100 nodes and 435 edges are faulty (`Matlab` codes can be found at http://eemensch.tumblr.com/post/17650877823/kvlsim.). Specifically, it is considered that measurements in the range of 0 and 20 are corrupted by faults uniformly picked from $[-10, 10]$. However, note that none of these information are used to reconstruct the fault vector, moreover, it is assumed that the measurements noise is a zero mean
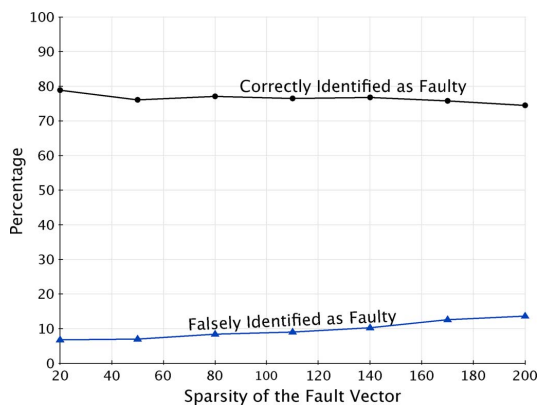
Fig. 3. The percentage of correctly identified nonzero entries of $\mathbf{f}$ using the Dantzig selector, and the percentage of falsely identified nonzero entries of $\mathbf{f}$ using the Dantzig selector.
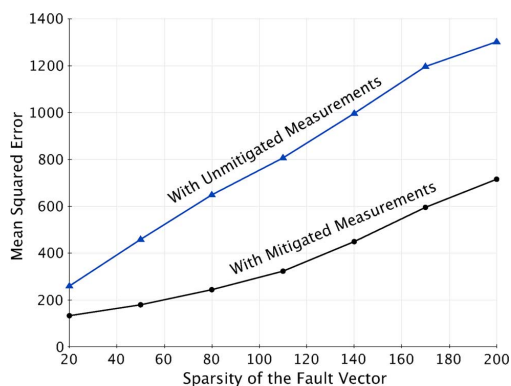

Fig. 4. The mean squared error between the exact measurements and the measured values for both the cases where the effect of the fault vector is mitigated and not after applying the method described in [5].

Gaussian variable with variance of one. For this example 7 different cases are considered where the sparsity of the fault vectors (number of non-zero entries of the fault vector) varies from 20 to 200 in steps of 30. The mean error between the reconstructed fault vector and the real fault vector for each of these cases is depicted in Fig. 2 both when the reconstruction is done using the pseudoinverse matrix and the Dantzig selector with $\lambda = 3.486$, see [13] for more discussion on λ. In Fig. 3 the percentage of correctly and the percentage of falsely identified nonzero entries of $\mathbf{f}$ using Algorithm 1 for each of the aforementioned cases is depicted. In the next scenario, we exhibit the benefit of reconstructing the fault vectors and mitigating the effect of them via calculating $\tilde{\mathbf{y}}_c$. We have repeated the simulations for 50 times. We use the method described in [5] to enforce the cycle constraints to improve the measurements using the raw measurements, $\tilde{\mathbf{y}}$, and measurements values corrected by deducting the estimated fault vector from them, $\tilde{\mathbf{y}}_c$. The mean square error between the resulting values and the real measurements are depicted in Fig. 4. In the next scenario we apply Algorithm 1 to detect the faulty measurements and consequently healing those faulty edges. For $s = 200$ and $t = 2$ on average 94% of the faulty edges are healed and the source of the faulty measurements in these edges will be eliminated. Simultaneously, in the same scenario, 12.7% edges are falsely identified as faulty. We conclude this section by commenting on the fact that choosing different $\mathbf{C}$ does not affect the performance of the proposed al-

gorithms. The reason is to the fact that $\mathbf{C}$ is always well-conditioned. And while different choices of $\mathbf{C}$ might result in slight numerical improvements, they will not have any decisive effect on the output of the algorithm.

V. Conclusion

In this letter, we considered the problem of detecting if a set of measurements in a network are faulty. We focused on those types of measurements that satisfy KVL and used tools from graph theory to detect if the measurements are corrupted without assuming any knowledge about the parameters of the network. Later, we used tools from compressive sensing to reconstruct the fault vector corrupting the measurements (either due to a malicious agent or hardware faults) to mitigate the effect of such faults. Moreover, we proposed an edge healing method to reinforce certain vulnerable edges in the network. We showed the applicability of the proposed methods in this letter through numerical examples. We note that when accurate *a priori* statistical characterisation of the faults are available application of the method proposed in [7] possibly yields better results.

We conclude this letter by commenting on how to address problems similar to Problems 1 and 2 in a network where measurements satisfy Kirchhoff's Current Law (KCL). In such a scenario, instead of $\mathbf{C}$ we use $N-1$ columns of the node-to-edge incidence matrix of $\mathcal{G}$, denoted by $\mathbf{N} \in \mathbb{R}^{m \times N-1}$. It is further assumed that all the flows, e.g., currents or water flows, along the edges are known. Noting that $\mathbf{N}$ is a full column matrix, the same ideas as described earlier in this letter can be applied to address problems similar to Problems 1 and 2.

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