Design of State-Based Schedulers for a Network of Control Loops

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***Abstract*—For a closed-loop system with a contention-based multiple access network on its sensor link, the medium access controller (MAC) may discard some packets when the traffic on the link is high. We use a local state-based scheduler to select a few critical data packets to send to the MAC. In this paper, we analyze the impact of such a scheduler on the closed-loop system in the presence of traffic, and show that there is a dual effect with state-based scheduling. In general, this makes the optimal scheduler and controller hard to find. However, by removing past controls from the scheduling criterion, we find that certainty equivalence holds. This condition is related to the classical result of Bar-Shalom and Tse, and it leads to the design of an innovations-based scheduler with a certainty equivalent controller. However, this controller is not an equivalent design for the optimal controller, in the sense of Witsenhausen. The computation of the estimate can be simplified by introducing a symmetry constraint on the scheduler. Based on these findings, we propose a dual predictor architecture for the closed-loop system, which ensures separation between scheduler, observer and controller. We present an example of this architecture, which illustrates a network-aware event-triggering mechanism.**

***Index Terms*—Event-based systems, networked control systems, state-based schedulers.**

I. Introduction

CONSIDER a network of control systems, where the communication between the individual sensors and controllers of different control loops occurs through a shared network, as shown in Fig. 1. This is an important scenario, in the context of wireless networked control systems (NCS), for industrial and process control [1]. A medium access control layer is required in the sensor's protocol stack to arbitrate access to the shared network. To focus on the implications of a medium access controller (MAC) on the sensor link, we assume that the communication between the controllers and the corresponding actuators occurs over a point-to-point network, not a shared network. This is a common architecture in practice [2], [3]. The MAC can implement a contention-free or a contention-based multiple access method, both of which have their own challenges [4]. A contention-free multiple access method requires a dynamic scheduler to prevent poor channel utilization, and such a scheduler is hard to construct and implement on an interference-constrained shared network [5], [6]. Contention-based methods have proven popular in standards such as IEEE 802.15.4 [7], as they facilitate an easy deployment on sensor nodes. However, such methods result in *random access*, which could significantly deteriorate the performance of a closed-loop system [8]. Thus, the design of a MAC for networked control systems is a challenging problem, and calls for innovative solutions [9].

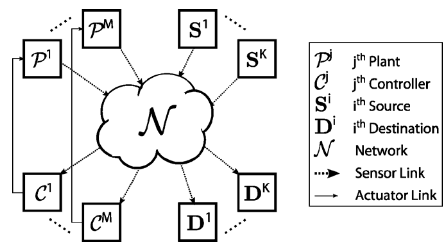

Fig. 1. Network of M control loops, with each loop consisting of a plant $\mathcal{P}^j$ and a controller $\mathcal{C}^j$ for $j \in \{1, \ldots, M\}$. The loops share access to a common medium on the sensor link, along with K other communication flows from generic source-destination pairs. The controllers and actuators communicate over dedicated networks, not shared links.

In this paper, we explore the design of a *state-aware* contention-based MAC, as opposed to an *agnostic* contention-based MAC. The state-aware MAC is capable of influencing the randomness of channel access in favour of the state of the plant in the closed-loop system. However, directly using the state of the plant to determine an access probability may result in a MAC that is difficult to implement and analyze [10]. Instead, we use the state of the plant to select packets to send to the MAC, motivated by an understanding of the two roles played by a MAC: Any random access method works by resolving contention between simultaneous channel access requests, thus spreading traffic that arrives in bursts. The carrier sense multiple access with collision avoidance (CSMA/CA) method does this by assigning a random back-off to packets that attempt to access a busy channel, thus spreading the traffic over a longer interval of time. Similarly, the p-persistent CSMA method does this by probabilistically limiting access to the channel and permitting a number of retransmissions if the channel is busy [11]. However, all of these methods permit only a finite number of retransmissions, beyond which the packet is discarded. We appropriate this latter role of discarding packets to a local *state-based scheduler*, which sends fewer, but more important packets to the MAC for transmission across the network.

A similar strategy has previously been proposed from the more general perspective of reducing network traffic [3]. When applied to the newly posed NCS problem [12], this approach has driven the design of event-based sampling systems [13],

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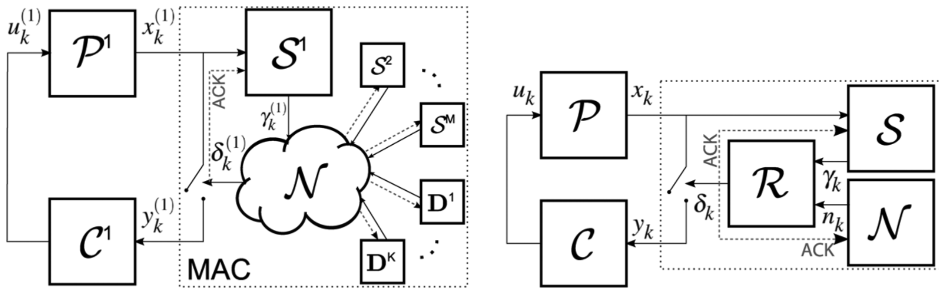

(a) (b)

Fig. 2. Plant ($\mathcal{P}^1$), state-based scheduler ($\mathcal{S}^1$) and controller ($\mathcal{C}^1$) share the network ($\mathcal{N}$) with $M-1$ other closed-loop systems with state-based schedulers ($\mathcal{S}^j, j \in \{2,\ldots,M\}$), and K generic devices ($\mathbf{D}^i, i \in \{1,\ldots,K\}$), in (a). A model, from the perspective of a single closed-loop system in the network, is depicted in (b). (a) A multiple access (MA) scenario for NCSs. (b) The MA model for each closed-loop system.

[14], which have been shown to outperform periodically sampled systems under certain conditions [15]–[17]. More recently, such systems have been analyzed for estimation over networks [18], [19], but the extensions to the control setting remain incomplete. We approach the NCS problem from a different perspective, but one that leads to a *network-aware* design of event triggering methods.

There are two main contributions in this paper. The first contribution is an analysis of the impact of having a state-based scheduler in the closed-loop. Primarily, a state-based scheduler permits the information available at the controller to be altered with the plant state. This information is not entirely random, like in the case of packet losses due to a noisy channel [20], [21], and it can result in a sharply asymmetrical estimation error, unlike in the case of encoder design over limited data rate channels [22], [23]. It seems reasonable to ask if we can use the controller to move the plant state across the threshold and force a transmission. If this is possible, the controller plays two roles: the first one being to control the plant, and the second one being to control the information available at the next time step. This relates to the classical concept of a dual effect, as described in [24] and [25]. The answer to this question determines the ease of optimal controller design, as the certainty equivalence principle would not hold if there is a dual effect [26]. We examine our system and find that there is a dual effect with a state-based scheduler in the closed-loop, and the certainty equivalence principle does not hold. Hence, the optimal state-based scheduler, estimator and controller designs are coupled. A restriction on the input arguments to the state-based scheduler, such that these arguments are no longer a function of the past control actions, renders the setup free of a dual effect, and enables the certainty equivalence principle to hold. These results can be seen as an interpretation, within the state-based scheduler setup, of the classical work on information patterns [27], dual effect, certainty equivalence and separation by Witsenhausen [28], Bar-Shalom and Tse [26], and on adaptive control by Feldbaum [24], Åström and Wittenmark [25], and many others [29].

The second contribution of this paper is on the *dual predictor architecture*, which is our proposed solution to the state-based scheduler design problem. In this architecture, the state-based scheduler thresholds the squared difference of the innovation contained in the latest measurement to the estimator across the network. This results in an optimal certainty equivalent controller, and a simple observer which generates the minimum mean-squared error (MMSE) estimate. Tuning parameters in the state-based scheduler in this architecture based on the current network traffic can result in a scheduling law that guarantees a probabilistic performance. This is not easy to show, in general, as the performance analysis of a closed-loop system with a state-based scheduler in a multiple access network is a difficult problem [16], [17]. However, we illustrate the guaranteed performance using simulations, and thus claim that the state-based scheduler we propose results in a network-aware event-triggering mechanism.

The rest of the paper is organized as follows. In Section II, we present the problem formulation. In Section III, we derive theoretical results for the case when full state information is available, with and without exogenous network traffic. In Section IV, we present the dual predictor architecture. We look at an extension to output-based systems in Section V. We present a counterexample to validate our results on the dual effect, along with other examples that illustrates our notion of network-aware event-triggering, in Section VI. Providing performance guarantees remains a difficult problem, as we indicate under future work, along with the conclusions, in Section VII.

II. Preliminaries

We present the problem setup and a few important definitions, along with a review of the classical concepts of dual effect and certainty equivalence in this section.

A. Problem Formulation

We consider a network of M control loops, as shown in Fig. 2(a). Each control loop, for $j \in \{1,\ldots,M\}$, consists of a plant $\mathcal{P}^j$, a state-based scheduler $\mathcal{S}^j$ and a controller $\mathcal{C}^j$. The loops share access to a common medium on the sensor link. A closed-loop system in this network can be modeled as shown in Fig. 2(b), with the index j dropped for simplicity. The block $\mathcal{N}$ represents the network as seen by this loop, and the block $\mathcal{R}$ denotes the contention resolution mechanism (CRM), which determines access to the network. Each of the blocks in Fig. 2(b) is explained below.

Plant: The plant $\mathcal{P}$ has state dynamics given by

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and w_k is independent and identically distributed (i.i.d.) zero-mean Gaussian with covariance matrix R_w. The initial state x_0 is zero-mean Gaussian with covariance matrix R_0.

State-Based Scheduler: There is a local scheduler $\mathcal{S}$, situated in the sensor node, between the plant and the controller, which decides if the state is to be sent across the network or not. The scheduler output is denoted γ_k, where $\gamma_k \in \{0, 1\}$. It takes a value 1 when the state x_k is scheduled to be sent and 0 otherwise. The scheduling criterion is denoted by the policy f, which is defined on the information pattern of the scheduler $\mathbb{I}_k^S$, and is given by

$$\gamma_k = f_k(\boldsymbol{u}_0^{k-1}, \omega_k^s) \tag{2}$$

where f_k is not a constant function of $\boldsymbol{u}_0^{k-1}$, i.e., $\exists\ \boldsymbol{u}_1, \boldsymbol{u}_2$ such that $f(\boldsymbol{u}_1, \cdot) \neq f(\boldsymbol{u}_2, \cdot)$. The scheduling policy f_k is also a function of $\omega_k^s \in \Omega_k^s$, and Ω_k^s is the σ-algebra generated by the information set at the scheduler, given by $\mathbb{I}_k^S = \{\boldsymbol{x}_0^k, \boldsymbol{y}_0^{k-1}, \boldsymbol{\gamma}_0^{k-1}, \boldsymbol{\delta}_0^{k-1}\}$. Here, we use bold font to denote a sequence of variables such as $\boldsymbol{a}_t^T = \{a_t, a_{t+1}, \ldots, a_T\}$. Note that an explicit acknowledgement (ACK) of a successful transmission is required for δ_k to be available to the scheduler. The scheduler output γ_k is now a function of the state, as suggested by the epithet "state-based scheduler."

Network: The network $\mathcal{N}$ generates exogenous traffic, as is indicated by $n_k \in \{0, 1\}$. It takes a value 1 when the network traffic attempts to access the channel, and 0 otherwise. The network traffic is considered to be stochastic, as it could be generated by another control loop, or by any other communicating node in the network. Thus, n_k is a binary random variable, which is not required to be i.i.d. We say that there is no exogenous network traffic if $n_k \equiv 0$, for all k.

CRM: The CRM block $\mathcal{R}$ resolves contention between multiple simultaneous channel access requests, i.e., when $\gamma_k = 1$ and $n_k = 1$. If the CRM resolves the contention in favor of our control loop, $\delta_k = 1$, and otherwise 0. The CRM can be modeled as the MAC channel response $\mathcal{R}$, with MAC output δ_k given by

$$\delta_k = \mathcal{R}(\gamma_k, n_k)\ . \tag{3}$$

For brevity, we also define $\bar{\delta}_k = 1 - \delta_k$, which takes a value 1 when the packet is not transmitted. The MAC channel response $\mathcal{R}$ is modeled as a discrete memoryless channel at the sampling time scale, requiring the CRM to resolve contention with respect to this packet before the next sampling instant. This translates to a limitation on the sampling rates supported by the model.

Measurement: The measurement across the network is denoted y_k. It is a nonlinear function of the state x_k, and is given by

$$y_k = \delta_k x_k = \begin{cases} x_k & \delta_k = 1, \\ \emptyset & \delta_k = 0, \end{cases} \tag{4}$$

where $\emptyset$ indicates an erasure. A successful transmission results in the full state being sent to the controller. However, even non-transmissions convey information as the scheduler output δ_k can be treated as a noisy and coarsely quantized measurement of the state x_k.

Controller: The control law g denotes an admissible policy for the finite horizon N defined on the information pattern of the controller, $\mathbb{I}_k^C$, and is given by

$$u_k = g_k(\omega_k^c) \tag{5}$$

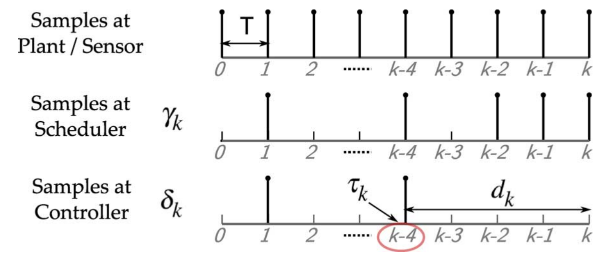

Fig. 3. Illustration of the delay since the last received packet (d_k) and the index of the last received packet (τ_k).

where $\omega_k^c \in \Omega_k^c$, and Ω_k^c is the σ-algebra generated by the information pattern $\mathbb{I}_k^C = \{\boldsymbol{y}_0^k, \boldsymbol{\delta}_0^k, \boldsymbol{u}_0^{k-1}\}$. The objective function, defined over a horizon N is given by

$$J(f, g) = \mathbb{E}\left[x_N^T Q_0 x_N + \sum_{s=0}^{N-1}(x_s^T Q_1 x_s + u_s^T Q_2 u_s)\right] \tag{6}$$

where Q_0 and Q_1 are positive semi-definite weighting matrices and Q_2 is positive definite.

In the rest of the paper, we address the following questions.

1) For a NCS with a state-based scheduler, what is the optimal control policy which minimizes the cost J in (6)?
2) Can we find a simple, but suboptimal, closed-loop system architecture for the given NCS?

To answer the first question, we need to examine whether the system exhibits a dual effect. This also requires us to check if we can find an equivalent system, in the sense of Witsenhausen, for which certainty equivalence holds. The second question requires us to identify restrictions on the scheduling policy f, which can ensure separation of the scheduler, controller and observer.

Definitions and Properties

We present a few definitions and properties that are used in the rest of the paper.

Definition 2.1 (Uncontrolled Process): An auxiliary uncontrolled process ($\bar{\mathcal{P}}$) can be defined for any closed-loop system, by removing the effect of the applied control signals from the state. The resulting uncontrolled state is denoted $\bar{x}_k$, and given by

$$\bar{x}_k = A^k x_0 + \sum_{\ell=1}^{k} A^{\ell-1} w_{k-\ell}\ . \tag{7}$$

Last Received Packet Index: The time index of the last received packet is denoted τ_k at time k (illustrated in Fig. 3), and for $-1 \le \tau_k \le k$, it is given by

$$\tau_k = \max\{t : \delta_t = 1, \quad \text{for} \quad -1 \le t \le k\ , \delta_{-1} = 1\}\ . \tag{8}$$

An iterative relationship for τ_k can be found as

$$\tau_k = \bar{\delta}_k \tau_{k-1} + \delta_k k, \quad \tau_{-1} = -1\ . \tag{9}$$

If a packet arrives at current time k, the last received packet index $\tau_k = k$, but if there is no packet at time k, then the last received packet index is the same as the last received packet index from time $k-1$, i.e., $\tau_k = \tau_{k-1}$. This implies that $\tau_k \in \{-1, \ldots, k\}$.

Dual Effect: Note that the control u_k might affect the future state uncertainty, in addition to its direct effect on the state. This is called the dual effect of control [24], and is discussed for state-based schedulers in Section III-A.

Definition 2.2 (No Dual Effect [26]): A control signal is said to have no dual effect of order $r \geq 2$, if

$$\mathbb{E}[M_{k,i}^r | \mathbb{I}_k^C] = \mathbb{E}[M_{k,i}^r | x_0, \boldsymbol{w}_0^{\tau_k}, \boldsymbol{n}_0^k] \quad (10)$$

where $M_{k,i}^r = (x_{k,i} - \mathbb{E}[x_{k,i}|\mathbb{I}_k^C])^r$ is the rth central moment of the ith component of the state $x_{k,i}$ conditioned on $\mathbb{I}_k^C$ and τ_k is the time index of the last received measurement at time k.

Note that $M_{k,i}^r$ in (10) must specifically not be a function of the past control policies $\boldsymbol{g}_0^{k-1}$ for the control signal to have no dual effect of order r. In other words, if there is no dual effect, the expected future uncertainty is not affected by the controls $\boldsymbol{u}_0^{k-1}$. In the presence of a dual effect, the optimal control laws are hard to find [25].

Certainty Equivalence: There are two closely related terms: a certainty equivalent controller and the certainty equivalence principle. We define both these terms with respect to the deterministic optimal controller, with full state information, for the above problem setup [26], [30]. These properties are discussed for state-based schedulers in Section III-C.

Definition 2.3 (Certainty Equivalent Controller): A certainty equivalent controller uses the deterministic optimal controller, with the state x_k replaced by the estimate $\hat{x}_{k|k} = \mathbb{E}[x_k|\mathbb{I}_k^C]$, as an *ad hoc* control procedure.

Sometimes, there is no loss in optimality in using a certainty equivalent controller. Then, we say that the certainty equivalence principle holds.

Definition 2.4 (Certainty Equivalence Principle): The certainty equivalence principle holds if the closed-loop optimal controller has the same form as the deterministic optimal controller with the state x_k replaced by the estimate $\hat{x}_{k|k}$.

Correlated Network Noise: We state a property of feedback systems with state-based schedulers that share a contention-based multiple access network. Even if the initial states and disturbances of all the plants in the network are independent, the contention-based MAC introduces a correlation between the traffic sources, as noted in [16] and [17].

Lemma 2.1: For a closed-loop system defined by (1)–(5), the exogenous network traffic indicated by n_k is correlated to the state of the plant x_k.

Proof: The MAC output δ_{k-1} is a function of the state x_{k-1} and the indicator of network traffic n_{k-1}, from (2) and (3). The control signal u_{k-1} is a function of the MAC output δ_{k-1} from (5), and is applied through feedback to the plant. Thus, x_k and γ_k are correlated to δ_{k-1}. Similarly, the network traffic from other closed-loop systems (and its indicator n_k) is correlated to δ_{k-1}, and consequently, x_k. ■

III. Optimal Controller Design

We present the main results of this paper in this section. We first analyze the effects of a state-based scheduler on a control loop with no exogenous network traffic, i.e., $n_k \equiv 0$. As a consequence of this, the MAC output is equal to the scheduler output, i.e., $\delta_k = \gamma_k$. We show that there is a dual effect of the control signal, and that the scheduling policy must be restricted from using the past control inputs for the certainty equivalence principle to hold. We illustrate this for a second-order system with a state-based scheduler in Fig. 4, and show that the controller is not oblivious to the scheduler boundaries. We extend our results to the case with exogenous network traffic.

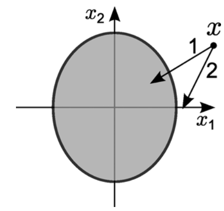

Fig. 4. For states $x \in \mathbb{R}^2$, we define a state-based scheduling policy f that generates an event when the state lies outside the shaded region. The resulting estimation error within the shaded region is nonzero, whereas it is zero outside this region. The dual role of the controller, shown in Theorem 3.1, arises from the incentive to move some states along path 2 (and remain outside of the shaded region), as compared to path 1 (this only reduces the variance of the state).

A. Dual Effect With State-Based Scheduling

We observe that the estimation error is a function of the applied controls, and that it does not satisfy the condition for no dual effect in (10). Thus, we have the following result.

Theorem 3.1: For the closed-loop system defined by (1)–(5), with no exogenous network traffic, i.e., $n_k \equiv 0$, the control signal has a dual effect of order $r = 2$.

Proof: We examine the estimation error, and show that it is not equivalent to the estimation error generated by the uncontrolled process $\bar{\mathcal{P}}$ (from Definition 2.1) in place of $\mathcal{P}$. Thus, we prove that the estimation error covariance is a function of the applied controls $\boldsymbol{u}_0^{k-1}$.

From (4), we know that a successful transmission results in the full state being sent to the controller, whereas a non-transmission conveys only a single bit of information (δ_k is binary) about the state to the controller. Thus, the estimate, $\hat{x}_{k|k} \triangleq \mathbb{E}[x_k|\mathbb{I}_k^C]$, is given by

$$\hat{x}_{k|k} = \delta_k x_k + \bar{\delta}_k \mathbb{E}[x_k|\mathbb{I}_k^C, \delta_k = 0] \,.$$

The variable δ_k cannot be removed from the above expression due to the asymmetry in the resolution of the received information with and without a transmission. The scheduler outcome, and consequently δ_k, are influenced by the applied control inputs $\boldsymbol{u}_0^{k-1}$ in a state-based scheduler such as (2). The estimation error, defined as $\tilde{x}_{k|k} \triangleq x_k - \mathbb{E}[x_k|\mathbb{I}_k^C]$, is given by

$$\tilde{x}_{k|k} = (x_k - \mathbb{E}[x_k|\mathbb{I}_k^C, \delta_k = 0]) \cdot \bar{\delta}_k \quad (11)$$

and thus depends on δ_k. The estimation error when there is no transmission is defined as $\tilde{x}_{k|k}^0 \triangleq x_k - \mathbb{E}[x_k|\mathbb{I}_k^C, \delta_k = 0]$, and is given by

$$\begin{aligned}\tilde{x}_{k|k}^0 &= A^k x_0 + \sum_{\ell=1}^{k} A^{\ell-1}(Bu_{k-\ell} + w_{k-\ell}) \\ &\quad - \mathbb{E}\left[A^k x_0 + \sum_{\ell=1}^{k} A^{\ell-1}(Bu_{k-\ell} + w_{k-\ell})|\mathbb{I}_k^C, \delta_k = 0\right] \\ &= \bar{x}_k - \mathbb{E}[\bar{x}_k|\mathbb{I}_k^C, \delta_k = 0]\end{aligned}$$

where $\bar{x}_k$ is the state of the uncontrolled process (see Definition 2.1). As shown above, the additive terms containing the past applied controls can indeed be removed with knowledge of the applied controls at the estimator. However, δ_k and $\bar{\delta}_k$, the second factor of the product in (11), remain a function of the applied controls, and cannot be generated by the uncontrolled process alone.

Thus, the estimation error is always dependent on the applied controls and this distinguishes the current problem from other related problems, such as in [22], [23]. The error covariance, $P_{k|k} \triangleq \mathbb{E}[\tilde{x}_{k|k}\tilde{x}_{k|k}^T|\mathbb{I}_k^C]$, is given by

$$P_{k|k} = \bar{\delta}_k \cdot \left(\mathbb{E}[\tilde{x}_{k|k}\tilde{x}_{k|k}^T|\mathbb{I}_k^C, \delta_k = 0]\right) . \tag{12}$$

The covariance $P_{k|k}$ is zero if the scheduling criterion in (2) is fulfilled, and nonzero otherwise. Through δ_k, $P_{k|k}$ is a function of the past controls. Hence, $P_{k|k}$ does not satisfy the condition (10) required to have no dual effect. Thus, the system (1)–(5) exhibits a dual effect of order $r = 2$. ■

In this setup, there is an incentive for the control policy to modify the estimation error along with controlling the plant, as illustrated in Fig. 4. Thus, the controller might choose to keep the state out of the shaded region to improve the estimation error for future time steps, even if this results in an increased variance of the state.

B. Witsenhausen Equivalence

Suppose that every state-based scheduler f, from (2), can be transformed into an innovations-based scheduler $\tilde{f}$, such as

$$\gamma_k = \tilde{f}_k(\tilde{\omega}_k^s) \tag{13}$$

where, $\tilde{\omega}_k^s \in \tilde{\Omega}_k^s$, $\tilde{\Omega}_k^s$ is the σ-algebra generated by the information pattern $\tilde{\mathbb{I}}_k^S = \{x_0, \boldsymbol{w}_0^{k-1}\}$, and the policy $\tilde{f}_k$ is defined in advance, and can be realized without any knowledge of the control policies used in the system. The output of such a scheduler is only a function of the innovations, and not a function of the applied controls $\boldsymbol{u}_0^{k-1}$. This distinguishes an innovations-based scheduler from a state-based scheduler (2). The innovations-based scheduler does not result in a dual effect of the control signal, as we show below. Even so, we cannot replace a state-based scheduler (2) in a closed-loop system with an innovations-based scheduler (13), unless it results in an equivalent control design. We now examine the question of equivalent designs, following Witsenhausen [28].

Definition 3.1: An equivalent design, in the sense of Witsenhausen, g_{eq} for the optimal controller g^*, which minimizes the cost criterion (6) for the system defined by (1)–(5), satisfies the equivalence relation given by

$$\boldsymbol{u}^* = \Upsilon(\boldsymbol{\omega}, g^*) = \Upsilon(\boldsymbol{\omega}, g_{\text{eq}}) \tag{14}$$

where Υ is obtained by recursive substitution for the control signals in the system equations with the respective control policy and the primitive random variables $\boldsymbol{\omega}_k = [x_0, \boldsymbol{w}_0^{k-1}]$.

For brevity, we adopt the following notation. Let $\{\mathcal{P}, f_1, g_1\}$ denote a system with a plant $\mathcal{P}$ given by (1), with f_1 as the given scheduler and g_1 as the optimal controller for the cost in (6). We now note the following result.

Theorem 3.2: Let there be no exogenous network traffic, i.e., $n_k \equiv 0$. For any state-based scheduler f and innovations-based scheduler $\tilde{f}$ that result in the same schedules, the corresponding optimal designs, g^* and $\tilde{g}$, respectively, are not equivalent in the sense of Witsenhausen.

Proof: Definition 3.1 requires the control signals obtained using the policies g^* and $\tilde{g}$ to be equal. In this proof, we find the optimal control policies for $\tilde{g}$ and g^*, and show that they do not result in the same control signals.

For the optimal control policy, which minimizes the quadratic cost J in (6), to be certainty equivalent, we need to find a solution to the Bellman equation [30], which is a one-step minimization of the form

$$V_k = \min_{u_k} \mathbb{E}[x_k^T Q_1 x_k + u_k^T Q_2 u_k + V_{k+1}|\mathbb{I}_k^C] . \tag{15}$$

In general, without defining a structure for the estimator, the solution to the functional is given in the form of

$$V_k = \mathbb{E}\left[x_k^T S_k x_k|\mathbb{I}_k^C\right] + s_k \tag{16}$$

where S_k is a positive semi-definite matrix and both S_k and s_k are not functions of the applied control signals $\boldsymbol{u}_0^{k-1}$, see [26]. We now prove that a solution of this form can be found for $\{\mathcal{P}, \tilde{f}, \tilde{g}\}$, but not for $\{\mathcal{P}, f, g^*\}$.

First consider the system $\{\mathcal{P}, \tilde{f}, \tilde{g}\}$. We denote the state and control signals of this system as $\tilde{x}_k$ and $\tilde{u}_k$. At time N, the functional has a trivial solution with $S_N = Q_0$ and $s_N = 0$. This solution can be propagated backwards, in the absence of a dual effect. To show this, we use the principle of induction, and assume that a solution of the form (16) holds at time $k+1$. Then, at time k, we have

$$\begin{aligned} V_k &= \min_{u_k} \mathbb{E}[\tilde{x}_k^T Q_1 \tilde{x}_k + \tilde{u}_k^T Q_2 \tilde{u}_k + \tilde{x}_{k+1}^T S_{k+1}\tilde{x}_{k+1} + s_{k+1}|\mathbb{I}_k^C] \\ &= \min_{u_k} \mathbb{E}[\tilde{x}_k^T (Q_1 + A^T S_{k+1} A)\tilde{x}_k|\mathbb{I}_k^C] + \operatorname{tr}\{S_{k+1}R_w\} \\ &\quad + \mathbb{E}[s_{k+1}|\mathbb{I}_k^C] + \tilde{u}_k^T(Q_2 + B^T S_{k+1} B)\tilde{u}_k \\ &\quad + \hat{\tilde{x}}_{k|k}^T A^T S_{k+1} B \tilde{u}_k + \tilde{u}_k^T B^T S_{k+1} A \hat{\tilde{x}}_{k|k} \end{aligned}$$

where $\hat{\tilde{x}}_{k|k} \triangleq \mathbb{E}[\tilde{x}_k|\mathbb{I}_k^C]$. The optimal control is found to be

$$\tilde{u}_k = -L_k \hat{\tilde{x}}_{k|k} \ , \ L_k = (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A . \tag{17}$$

Substituting the expression for $\tilde{u}_k$ into V_k gives us a solution of the form in (16), with

$$\begin{aligned} S_k &= Q_1 + A^T S_{k+1} A \\ &\quad - A^T S_{k+1} B (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A \\ s_k &= \mathbb{E}[s_{k+1}|\mathbb{I}_k^C] + \operatorname{tr}\{S_{k+1} R_w\} \\ &\quad + \operatorname{tr}\{A^T S_{k+1} B (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A P_{k|k}\} \end{aligned} \tag{18}$$

where the matrix S_k is positive semi-definite and not a function of the applied controls $\tilde{\boldsymbol{u}}_0^{k-1}$. The scalar s_k is not a function of the applied controls $\tilde{\boldsymbol{u}}_0^{k-1}$ if and only if $P_{k|k}$ has no dual effect [26]. From the expression for the error covariance $P_{k|k}$ (12), it is clear that a scheduling criterion that is not a function of the past control actions, such as (13), results in no dual effect. Under this condition, s_k is not a function of the applied controls $\tilde{\boldsymbol{u}}_0^{k-1}$ and the proof by induction is complete. Since the optimal control signal (17) is a function of only the estimate $\hat{\tilde{x}}_{k|k}$, the certainty equivalence principle holds.

Now, consider the system $\{\mathcal{P}, f, g^*\}$ with state x_k and control u_k^*. Solving the backward recursion as we did above, we find that V_N and V_{N-1} have a solution of the form (16), with $S_N = Q_0$ and $s_N = 0$, and S_{N-1} and s_{N-1} given by (18) with $k = N-1$. However, V_{N-2} results in a different minimization problem for this system because of the dual effect in $\{\mathcal{P}, f, g^*\}$, as indicated next. The optimal control signal u_{N-2}^* can be obtained by solving $\frac{\partial V_{N-2}}{\partial u_{N-2}^*}$ as

$$\frac{\partial}{\partial u_{N-2}^*}\left(\operatorname{tr}\{(Q_2 + B^T S_N B)\mathbb{K}_{N-2} \cdot \mathbb{E}[P_{N-1|N-1}|\mathbb{I}_{N-2}^C]\}\right) + 2u_{N-2}^{*T}(Q_2 + B^T S_{N-1}B) + 2\hat{x}_{N-2|N-2}^T A^T S_{N-1} B = 0$$

where we set $\mathbb{K}_{N-2}$ as

$$(Q_2 + B^T S_{N-1} B)^{-1} A^T S_N B (Q_2 + B^T S_N B)^{-1} B^T S_N A.$$

Multiplying the above expression with $(Q_2 + B^T S_{N-1}B)^{-1}$ from the right and using (17) to denote $u_{N-2}^{CE} = -L_{N-2}\hat{x}_{N-2|N-2}$, we obtain the simpler equation

$$\frac{\partial}{\partial u_{N-2}^*}\left(\operatorname{tr}\{\mathbb{K}_{N-2}\mathbb{E}[P_{N-1|N-1}|\mathbb{I}_{N-2}^C]\}\right) + 2(u_{N-2}^{*T} - u_{N-2}^{CE,T}) = 0\,. \quad (19)$$

The first term in (19), related to the estimation error covariance $P_{N-1|N-1}$, is not equal to zero as implied by the dual effect property from Theorem 3.1. Due to this term, the above minimization problem is not linear, and thus, the solutions u_{N-2}^{CE} and u_{N-2}^* are not equal. Since u_{N-2}^{CE} has the same form as $\tilde{u}_{N-2}$, we also note that $\tilde{u}_{N-2}$ and u_{N-2}^* have very different forms. From this point on, the cost-to-go for the optimal control policy g^* does not have a solution of the form given by (16). Hence, the control signals $\{\tilde{\boldsymbol{u}}\}_0^{N-3}$ and $\{\boldsymbol{u}^*\}_0^{N-3}$ will not be equal. Now, the joint distribution of all system variables could be quite different for schedulers $\tilde{f}$ and f. Thus, the described transformation of the scheduling criterion does not result in an equivalent design. ■

The above theorem provides us a motivation for using a state-based scheduler, despite the inherent difficulties associated with the closed-loop design. Due to the dual effect, the optimal control action takes on two roles. One, to control the plant, and the other, to probe the plant state which could result in an improved estimate [25]. The innovations-based scheduler results in a simpler closed-loop design, as shown in (17)–(18). However, a probing action cannot be implemented in any controller in this setup due to the lack of a dual effect. Thus, the resulting control actions for the closed-loop systems with the state-based and innovations-based schedulers are not the same.

C. Conditions for Certainty Equivalence

From the previous discussions, it is clear that a scheduling criterion independent of the past control actions, such as the innovations-based scheduler, results in no dual effect. This result is presented next.

Corollary 3.3: For the closed-loop system defined by (1)–(5), with no exogenous network traffic, i.e., $n_k \equiv 0$, the optimal controller, with respect to the cost in (6), is certainty equivalent if and only if the scheduling decisions are not a function of the applied control actions, such as in (13).

Proof: In the proof of Theorem 3.2, it is clear from (17) that the optimal control policy $\tilde{g}$ for the system $\{\mathcal{P}, \tilde{f}, \tilde{g}\}$ is certainty equivalent.

To show the necessity of this condition for certainty equivalence, we need to show that if the optimal control signal has the form in (17) at time k, then the scheduling policy is not a function of the controls for $n < k$, for all k. Accordingly, assume that the optimal control signal is given by (17) for $k = N-1, \ldots, n+1$. Then, the optimal cost-to-go is of the form in (16), at time $n+1$ and $s_{n+1} = \sum_{k=n+1}^{N-1} \mathbb{E}[\operatorname{tr}\{A^T S_{k+1} B (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A P_{k|k} + S_{k+1} R_w\}|\mathbb{I}_k^C]$, when written out explicitly. We know that the optimal control signal u_n is obtained by minimizing (15) at time n. This control signal will have the form in (17) for all $Q_2 > 0$ only if s_{n+1} is independent of u_n, or if the estimation error covariances $P_{k|k}$, for $k = \{n+1, \ldots, N-1\}$, are not a function of u_n. From the result in Theorem 3.1, this is only possible when the scheduling policy is not a function of u_n. Since this is true for $n = 0, \ldots, N-1$, the scheduling policy must not be a function of $\boldsymbol{u}_0^{k-1}$. ■

Corollary 3.3 provides us with a restriction on the scheduler to ensure certainty equivalence. Note that the resulting design is not equivalent to the optimal design, as shown in Theorem 3.2.

D. Effect of State-Based Schedulers With Exogenous Network Traffic

In this subsection, we analyze the effects of a state-based scheduler on the control loop in the *presence* of exogenous network traffic. Thus, we have $n_k \not\equiv 0$ and a channel output given by (3). Recall from Lemma 2.1, that the network traffic indicator n_k is correlated to the state of the plant x_k. The certainty equivalence principle need not hold for plants where the measurement noise is correlated to the process noise [26]. To focus on the effect of state-based schedulers on the closed-loop system, the results presented in the previous subsection did not include exogenous network traffic. Now, we derive some of the above results for the system in the presence of exogenous network traffic.

Lemma 3.4: For the closed-loop system defined by (1)–(5), the control signal has a dual effect of order $r = 2$.

Proof: The MAC output δ_k (3) is clearly still a function of the applied controls, through the state-based scheduler outcome. Thus, the estimation error covariance $P_{k|k}$, in (12), remains a function of the applied controls $\boldsymbol{u}_0^{k-1}$. Since $P_{k|k}$ does not satisfy the condition (10) required to have no dual effect, we see that the system (1)–(5) exhibits a dual effect of order $r = 2$. ■

With the above result, Theorem 3.2 can be easily extended to include the case with exogenous network traffic. However, it is not as straightforward to extend Corollary 3.3. When the measurement noise is correlated to the process noise, certainty equivalence need not hold. To see why, recall the proof of Theorem 3.2, where we derive a solution of the form $V_k = \mathbb{E}[x_k^T S_k x_k|\mathbb{I}_k^C] + s_k$ for the Bellman (15). Now, if w_k is correlated to the variables in the information set $\mathbb{I}_k^C$, specifically $\boldsymbol{n}_0^k$, the minimization with respect to u_k in (17) must include the term $\operatorname{tr}\{S_{k+1}R_w\}$. Then, the optimal controller will not have the form shown in (17), and certainty equivalence will not hold.

We need to prove that w_k is independent of $\boldsymbol{n}_0^k$ for the certainty equivalence principle to hold, which we do below.

Corollary 3.5: For the closed-loop system defined by (1)–(5), the optimal controller, with respect to the cost criterion (6), is certainty equivalent if the exogenous network traffic indicator n_k is independent of the process noise w_k, and, if the scheduling decisions are not a function of the applied controls, i.e., if

$$\gamma_k = \breve{f}_k(\breve{\omega}_k^s) \tag{20}$$

where $\breve{\omega}_k^s \in \breve{\Omega}_k^s$, and $\breve{\Omega}_k^s$ is the σ-algebra generated by the information set $\breve{\mathbb{I}}_k^S = \{x_0, \boldsymbol{w}_0^{k-1}, \boldsymbol{n}_0^{k-1}\}$.

Proof: Note that n_k is only correlated to $\boldsymbol{\delta}_0^k$ and thus, to the signals $\boldsymbol{w}_0^{k-1}$, from Lemma 2.1. As the process noise is i.i.d, n_k is independent with respect to w_k. A scheduler of the form (20) is not a function of the applied controls, and thus, certainty equivalence holds. ∎

IV. Closed-Loop System Architecture

In this section, we find that symmetric scheduling policies simplify the observer design. We propose a dual predictor architecture for the closed-loop system, which results in a separation of the scheduler, observer and controller designs.

A. Observer Design

Due to the nonlinearity of the problem, the estimate in general can be hard to compute. However, the estimation error is reset to zero with every transmission, as we send the full state. Consider one such reset instance, a time k such that $\delta_k = 1$. The state is sent across the network, $y_k = x_k$, so the estimate $\hat{x}_{k|k} = x_k$. A suitable control signal u_k is found and applied to the plant, which results in the next state x_{k+1}. Now, the scheduler can generate one of two outcomes. We consider each case, and find an expression for the estimate below:

1) $\delta_{k+1} = 0$: We need an estimate of w_k. We use the scheduler output as a coarse quantized measurement to generate this, as follows:

$$\begin{aligned}\hat{x}_{k+1|k+1} &= \mathbb{E}[x_{k+1}|\mathbb{I}_{k+1}^C, \delta_{k+1} = 0] \\ &= Ax_k + Bu_k + \mathbb{E}[w_k|\delta_{k+1}(\grave{f}(w_k)) = 0]\end{aligned} \tag{21}$$

where

$$\begin{aligned}&\mathbb{E}[w_k|\delta_{k+1}(\grave{f}(w_k)) = 0] \\ &\quad= \sum_{\gamma\in\{0,1\}} \mathbb{E}[w_k|\grave{f}(w_k) = \gamma, \delta_{k+1} = 0] \\ &\qquad \cdot \mathbb{P}(\gamma_{k+1} = \gamma|\delta_{k+1} = 0)\end{aligned}$$

and $\grave{f}(w_k) \equiv f(Ax_k + Bu_k + w_k|x_k, u_k)$.

2) $\delta_{k+1} = 1$: The estimation error is zero as $\hat{x}_{k+1|k+1} = x_{k+1}$.

The transformation to $\grave{f}$ in (21), is not intended to remove the dual effect, but merely serves to remove the known variables from the expression. The dual effect has influenced the packet's transmission, i.e., the value of δ_{k+1}. To understand this expression clearly, we look at the next time instant. Now a signal u_{k+1} is generated, and applied to the plant. We note that $x_{k+2} = A^2x_k + ABu_k + Bu_{k+1} + Aw_k + w_{k+1}$. The state x_{k+2} is either sent to the controller or not depending on the scheduler outcome δ_{k+2}. Again, we look at both cases, and derive an expression for the estimate:

a') $\delta_{k+2} = 0$: We now need to estimate $Aw_k + w_{k+1}$, as the rest is completely known from x_{k+2}. We use both scheduler outputs δ_{k+1} and δ_{k+2} to generate an estimate of the unknown variables as

$$\hat{x}_{k+2|k+2} = A^2x_k + ABu_k + Bu_{k+1} + \bar{e}_k$$

where $\bar{e}_k = \mathbb{E}[Aw_k + w_{k+1} \mid \delta_{k+1}(\grave{f}(w_k)) = 0, \delta_{k+2}(\grave{f}(Aw_k + w_{k+1})) = 0]$.

b') $\delta_{k+2} = 1$: Again, $\hat{x}_{k+2|k+2} = x_{k+2}$.

This process can be continued recursively through a non-transmission burst, until finally a measurement is received and the estimation error is reset to zero. Thus, the observer computes the estimate at any time k as

$$\hat{x}_{k|k} = \begin{cases} x_k, & \delta_k = 1, \\ A^{k-\tau_k}x_{\tau_k} + \sum_{s=1}^{k-\tau_k} A^{s-1}Bu_{k-s} & \\ & \delta_k = 0, \\ +\mathbb{E}\left[\sum_{s=1}^{k-\tau_k} A^{s-1}w_{k-s}|\delta(\grave{f}_k), \ldots, \delta(\grave{f}_{\tau_k+1}) = 0\right], & \end{cases} \tag{22}$$

where τ_k is the time index of the last received measurement at time k, as defined in (8), and the argument to the function $\grave{f}_t$ is given by the term $\sum_{s=1}^{t-\tau_t} A^{s-1}w_{t-s}$.

B. State-Based Scheduler Design: Symmetric Schedulers

The computation of the term $\mathbb{E}[\sum_{s=1}^{k-\tau_k} A^{s-1}w_{k-s}|\delta(\grave{f}_k), \ldots, \delta(\grave{f}_{\tau_k+1}) = 0]$ makes the estimate (22) hard to evaluate, because the quantized noise is not Gaussian. As a suboptimal approach, consider the scheduling criterion given by any symmetric map $f^{sym}(r) = f^{sym}(-r)$ with

$$\gamma_k = f^{sym}\left(\sum_{s=1}^{k-\tau_{k-1}} A^{s-1}w_{k-s}\right). \tag{23}$$

Since τ_k is not defined without the MAC output δ_k in (8), we replace it with τ_{k-1}, which is also a measure of the non-transmission burst. Choosing the scheduler in this manner results in a zero mean estimate from the quantized noise when there is no transmission. Now, the estimate (22) is easy to compute and the observer can be designed without knowledge of the scheduling policy. Also, a certainty equivalent control can be applied. This observation is summarized below, and is used to design the scheduler presented in Section IV-C.

Proposition 4.1: For the closed-loop system defined by (1)–(6), the use of the symmetric scheduling policy (23) implies that certainty equivalence holds, and it also results in separation in design between the estimator and scheduler.

C. Dual Predictor Architecture

In this section, we examine closed-loop design of the complete system, including scheduler, observer and controller. From the results of Lemma 3.4 and Proposition 4.1, it is clear that the scheduler, observer and controller designs are coupled, in general. It is not possible to design the optimal scheduling policy

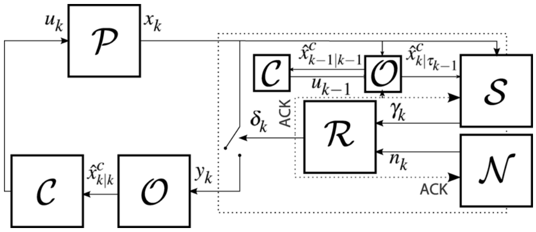

Fig. 5. State-based dual predictor architecture: the innovations to the observer serve as input to the scheduler. The resulting setup is certainty equivalent. The observer is simple, and computes the MMSE estimate.

independently and combine it with a certainty equivalent controller and optimal observer to get the overall optimal closed-loop system. At the same time, solving for the jointly optimal scheduler, observer and controller is a hard problem.

Thus, we propose an architecture, shown in Fig. 5, for a design of the state-based scheduler, and the corresponding optimal controller and observer. There are two estimators in this architecture, and hence, we call it a dual predictor architecture [31]. This architecture has been referred to previously in the context of mobile networks [32]. The scheduler, observer and controller blocks are described below.

1) Scheduler ($\mathcal{S}$): The scheduler output γ_k is given by

$$\gamma_k = f(x_k, \hat{x}_{k|\tau_{k-1}}) = \begin{cases} 1, & |x_k - \hat{x}_{k|\tau_{k-1}}|^2 > \epsilon \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

where $\epsilon > 0$ is a given threshold and $\hat{x}_{k|\tau_{k-1}}$ is the estimate at the controller at time k if the current packet is not scheduled for transmission. To realize such a scheduling policy, the observer must be replicated within the scheduler, and for the observer to be able to subtract the applied control, the controller must also be replicated within the scheduler. An explicit ACK is required to realize this information pattern, as indicated in Fig. 5.

2) Observer ($\mathcal{O}$): The input to the observer is $y_k = \delta_k x_k$. The observer generates the estimate $\hat{x}_{k|k}$ as given by

$$\hat{x}_{k|k} = \bar{\delta}_k \hat{x}_{k|\tau_k} + \delta_k x_k \ . \tag{25}$$

Recall that $\bar{\delta}_k = 1 - \delta_k$ takes a value 1 when the packet is not transmitted. In such a case, the estimate is given by $\hat{x}_{k|\tau_k}$, a model-based prediction from the last received data packet at time τ_k. This estimate is given by

$$\hat{x}_{k|\tau_k} = A\hat{x}_{k-1|k-1} + Bu_{k-1} \ . \tag{26}$$

3) Controller ($\mathcal{C}$): The controller generates the signal u_k based on the estimate alone, as given by

$$u_k = -L_k \hat{x}_{k|k} \tag{27}$$

where L_k is defined in (17).

Note that the scheduling criterion described in (24) can be rewritten as

$$|x_k - \hat{x}_{k|\tau_{k-1}}|^2 = |A\tilde{x}_{k-1|k-1} + w_{k-1}|^2 = |\tilde{x}_{k|\tau_{k-1}}|^2 \ .$$

Here, we use $\hat{x}_{k|\tau_{k-1}}$ as τ_k is not defined without δ_k. The criterion $|\tilde{x}_{k|\tau_{k-1}}|^2 \leq \epsilon$ captures the per-sample variance of the estimation error, when no transmission is scheduled. Taking expectations on both sides, we get $\mathrm{tr}\{P_{k|\tau_{k-1}}\} \leq \epsilon$. The scheduler attempts to threshold the variance of the estimation error, but this cannot be guaranteed in a network with multiple traffic sources. Also, note that the scheduling policy is a symmetric function of its arguments, as in Proposition 4.1. We now state the main result of this section.

Theorem 4.2: For the closed-loop system given by the plant (1), the state-based dual predictor (24)–(27), and the cost criterion (6), it holds that

i) The estimate (25) minimizes the mean-squared estimation error.
ii) The control signal does not have a dual effect.
iii) The certainty equivalence principle holds and the optimal control law is given by (27).
iv) The LQG cost is given by

$$J_{\mathrm{DP}} = \hat{x}_0^T S_0 \hat{x}_0 + \mathrm{tr}\{S_0 P_0\} + \sum_{n=0}^{N-1} \mathrm{tr}\{S_{n+1} R_w\} + \sum_{n=0}^{N-1} \mathrm{tr}\{(L_n^T (Q_2 + B^T S_{n+1} B) L_n) P_{n|n}\} \tag{28}$$

where $P_{k|k}$ is the error covariance of the estimate at the observer, with $S_N = Q_0$ and S_k obtained by backward iteration of (18).

Proof: We know that

$$A^{k-\tau_k} x_{\tau_k} + \sum_{n=1}^{k-\tau_k} A^{n-1} B u_{k-n} = A\hat{x}_{k-1|k-1} + Bu_{k-1} \ ,$$

$$\mathbb{E}\left[\sum_{\ell=1}^{k-\tau_k} A^{\ell-1} w_{k-\ell} | \boldsymbol{\delta}_{\tau_k+1}^k = 0\right] = 0$$

where the last expression results from the use of a symmetric scheduling policy. Substituting for these terms in the expression for the estimate in 22, we get

$$\mathbb{E}[x_k | \mathbb{I}_k^C] = \begin{cases} x_k, & \delta_k = 1 \\ A\hat{x}_{k-1|k-1} + Bu_{k-1}, & \delta_k = 0. \end{cases}$$

Thus, the estimate in (25) is the MMSE estimate [33].

The error covariance at the estimator is given by (12), where, from (24) and (3), it is clear that the scheduler outcome γ_k and the MAC output δ_k do not depend on the applied controls $\boldsymbol{u}_0^{k-1}$. Thus, the error covariance satisfies the definition in (10), and the control signal in this architecture does not have a dual effect.

From the above conclusion, note that the scheduling policy in (24) is of the form (20). Thus, from Corollary 3.5, we know that the optimal controller for this setup is certainty equivalent. Then, the optimal control signal is given by (17), which has the same form as the controller in this architecture (27). The expression for the control cost remains the same as in the case with partial state information, and is given by (28). ■

Thus, the dual predictor architecture results in a suboptimal but simplified closed-loop system.

V. Extensions and Discussions

In this section, we extend the above results to an output-based system. We also identify the existence of a dual effect when the cost function penalizes network usage and when the transmission, with a state-based scheduler, occurs over limited data-rate channels. Finally, we discuss the dual effect property that we have encountered in this problem with respect to other NCS architectures.

A. Measurement-Based Scheduler

We now consider a system without full state information, but with co-located measurements. We show that by placing an optimal observer, a Kalman Filter (KF) at the sensor, to estimate the state of the linear plant, and basing the scheduler decisions on this estimate, instead of on the measurement, we are able to establish the same conclusions as before.

Consider a linear plant with a state z_k, and a measurement m_k given by

$$z_{k+1} = Az_k + Bu_k + w_{z,k}\ , \quad m_k = Cz_k + v_{z,k} \tag{29}$$

where $w_{z,k}$ is i.i.d. zero-mean Gaussian with covariance matrix $R_{w,z}$. The initial state z_0 is zero-mean Gaussian with covariance matrix $R_{z,0}$. Also, the measurement $m \in \mathbb{R}^m$ and the matrix $C \in \mathbb{R}^{n\times m}$. The measurement noise $v_{z,k}$ is a zero mean i.i.d Gaussian process with covariance matrix $R_{v,z} \in \mathbb{R}^{m\times m}$, and it is independent of $w_{z,k}$.

We can place a KF at the sensor node, which receives every measurement m_k from the sensor and updates its estimate ($\hat{z}^s_{k|k}$) as

$$\hat{z}^s_{k|k} = A\hat{z}^s_{k-1|k-1} + Bu_{k-1} + K_{f,k}e_k \tag{30}$$

where $K_{f,k}$ denotes the KF gain and e_k denotes the innovation in the measurement. The innovation is Gaussian with zero-mean and covariance $R_{e,k}$. The error covariances for the predicted estimate and the filtered estimate are denoted P^s_k and $P^s_{k|k}$, respectively. These terms are given by

$$\begin{aligned}
e_k &= m_k - C(A\hat{z}^s_{k-1|k-1} + Bu_{k-1})\ ,\\
K_{f,k} &= P^s_k C^T R^{-1}_{e,k}\ , R_{e,k} = CP^s_k C^T + R_{v,z}\ ,\\
P^s_k &= AP^s_{k-1|k-1}A^T + R_{w,z}\ , P^s_{k|k} = P^s_k - K_{f,k}R_{e,k}K^T_{f,k}\ .
\end{aligned}$$

If we use the estimate to define a new state, such that $x_k \triangleq \hat{z}^s_{k|k}$, we have a linear plant disturbed by i.i.d Gaussian process noise $w_k = K_{f,k}e_k$. Thus, we have reestablished the problem setup from Section II-A, and the results from before can be applied to this plant. Note that the scheduler is now defined with respect to the estimate $\hat{z}^s_{k|k}$ and not the measurements m_k. However, the scheduler output remains a function of the state and the measurement, through the estimate.

B. Penalizing Network Usage

We have shown, in the proofs of Theorem 3.1 and Theorem 3.2, that the applied controls play a significant role in a state-based scheduler and cannot be removed from the scheduler inputs to create an equivalent setup without a dual effect. However, the minimizing solution to a cost criterion can render the effect of the applied controls redundant. To see an example of this, consider the problem of finding the jointly optimal scheduler-controller pair for the classical LQG cost criterion in (6). Since there is no penalty on using the network, the optimal scheduler policy is to transmit all the time. Now, the structure of the closed-loop system does not resemble the one presented in Theorem 3.1, and consequently, that result does not hold. In this scenario, there is no incentive for the controller to influence the transmissions and the jointly optimal scheduler-controller pair $(f^{\mathbb{1}}, g^{\mathbb{1}})$ is given by

$$f^{\mathbb{1}} : \delta^{\mathbb{1}}_k = 1 \quad \forall\, k\ , \quad g^{\mathbb{1}} : u^{\mathbb{1}}_k = -L_k x_k \quad \forall\, k \tag{31}$$

where L_k is given in (17). Note that in the rest of this paper, we do not consider finding the jointly optimal scheduler-controller pair, as the use of a contention-based MAC does not permit us to choose the schedule sequence.

Now, consider a cost criterion which penalizes the use of the network, such as

$$J_\Lambda = \min_{\mathbf{u}_0^{N-1}, \boldsymbol{\delta}_0^{N-1}} \mathbb{E}\left[x_N^T Q_0 x_N + \sum_{s=0}^{N-1}\left(x_s^T Q_1 x_s + u_s^T Q_2 u_s\right) + \sum_{s=0}^{N-1}\Lambda\delta_s\right] \tag{32}$$

where Q_0, Q_1 and Q_2 are positive definite weighting matrices and $\Lambda > 0$ is the cost of using the network. The optimal state-based scheduling policy chooses a schedule in relation to the penalty Λ, such that the average network use, i.e., $\mathbb{E}[\delta_k]$, decreases as Λ increases. Thus, we state the following result.

Proposition 5.1: For the closed-loop system defined by (1)–(5), with no exogenous network traffic, the control signals derived from the jointly optimal scheduler-controller pair, which minimize the cost criterion in (32), exhibit a dual effect of order $r = 2$.

Proof: It is easy to show that the scheduler-controller pair $(f^{\mathbb{1}}, g^{\mathbb{1}})$ does not minimize the cost in (32). Now, the scheduler uses the policy in (2) to select packets to send across the network. Thus, the closed-loop system has the same structure as in Theorem 3.1, and there is a dual effect of order $r = 2$ for any control signal in this setup. ∎

Proposition 5.1 provides the controller an incentive to modify the transmission outcome. As a result, the optimal scheduler and controller designs in this problem are coupled. Using the results of Lemma 3.4, the above results can be extended to include the effect of exogenous network traffic.

C. Using A Rate-Constrained Channel

Our proof of the dual effect in Theorem 3.1 relies on the asymmetry in the resolution of the received information; the full state is sent with a transmission and only a single-bit quantized encoding of the state is sent when there is no transmission. However, data channels are generally rate-constrained, and a full state can never be sent. If the encoder-decoder pair on the sensor link uses R bits of information the estimation error at the controller can be written as $\tilde{x}_{k|k} = \delta_k \cdot (x_k - \mathbb{E}[x_k|\mathbb{I}^C_k, \delta_k = 1]) + \bar{\delta}_k \cdot (x_k - \mathbb{E}[x_k|\mathbb{I}^C_k, \delta_k = 0])$, in place of (11). Note that δ_k, and consequently the applied controls, cannot be removed from the above expression, unless the estimation error with and without a transmission result in the same expression, i.e., $x_k - \mathbb{E}[x_k|\mathbb{I}^C_k, \delta_k = 1] = x_k - \mathbb{E}[x_k|\mathbb{I}^C_k, \delta_k = 0]$ for $R > 1$. Hence, there is a dual effect with a state-based scheduler, even when using a rate-constrained channel for transmission.

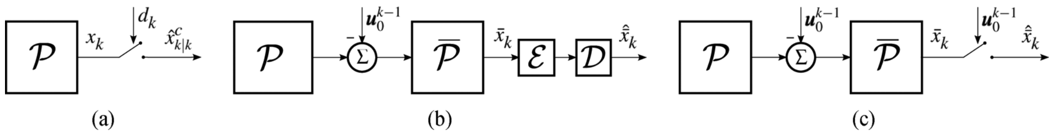

Fig. 6. Estimate is not influenced by the applied controls in (a) and (b), with knowledge of the applied controls. In contrast, the applied controls cannot be removed from the decision process in (c). (a) Packet losses (d_k). (b) Encoder–decoder ($\mathcal{E} - \mathcal{D}$) design. (c) State-based Scheduler.

D. Relation to Other NCS Architectures

The dual effect and certainty equivalence properties have been noted previously in other NCS problems. We discuss these occurrences and the connections to our problem setup below.

1) Packet Drops Over A Lossy Network: Packet drops in a lossy network are not influenced by the applied controls [Fig. 6(a)]. Hence, certainty equivalence holds, when there are packet drops on the sensor link [21]. However, when there are packet drops on the actuator link, separation holds only if there is an ACK of packets received or lost [20].

2) Importance of Side Information: In any NCS problem, the classical information pattern must be reconstructed for the certainty equivalence principle to hold [34]. This may require one or more side information channels to convey ACKs of received packets back to the transmitters [20], [35].

3) Encoder Design Over Limited Data Rate Channels: This problem differs from our setup in the sense that the encoder output is the only measurement available across the channel, and this potentially contains the same number of information bits, see Fig. 6(b). In [22], the applied controls are shown to not influence the estimation error.

4) Event-Based Systems: The results we have encountered in this paper show that the applied controls can push the state across the scheduler threshold, and influence the transmission outcome, as illustrated in Fig. 4. This is a consequence of the unequal information in the measurement y_k, with and without a transmission, see Fig. 6(c). A similar problem with a cost function such as (32), has been dealt with in [36], [37]. They use a transformation similar to the one presented for the encoder design problem in [23]. There are, however, subtleties in defining an equivalence class for a state-based scheduler: using an equivalent scheduler need not result in an equivalent design, as shown in Theorem 3.2.

VI. EXAMPLES

We present three examples in this section. The first example describes the problem setup, and illustrates the motivation for the problem. The second example illustrates the results of Theorem 3.1 and Theorem 3.2, which identify the dual role of the applied controls towards the information available to the controller. A counterexample is also presented, in which we identify some controllers which exploit this dual role. The third example illustrates the dual predictor architecture and provides an example of network-aware event triggering.

A. An Example of A Multiple Access NCS

This example illustrates the role of a state-based scheduler in our problem formulation in Section II-A, where a number of closed-loop systems share a contention-based multiple access network on the sensor link. We use a p-persistent CSMA protocol in the MAC. The observer and controller are chosen for simplicity of design, not as optimizers of any cost. We look at the performance of this network of control loops, with and without the state-based scheduler.

We consider a heterogenous network of 20 scalar plants, indexed by $j \in \{1, \ldots, 20\}$. There are three different types of plants, $\mathcal{P}^{[T1]}$, $\mathcal{P}^{[T2]}$ and $\mathcal{P}^{[T3]}$, given by

$$x_{k+1}^{(j)} = a^{[i]} x_k^{(j)} + u_k^{(j)} + w_k^{(j)} \tag{33}$$

where $a^{[i]} \in \{1, 0.75, 0.5\}$, and $R_w^{[i]} \in \{1, 1.5, 2\}$, for the plant $\mathcal{P}^{[Ti]}$. The systems numbered $j \in \{1, \ldots, 6\}$ are of type $\mathcal{P}^{[T1]}$, $j \in \{7, \ldots, 13\}$ are of type $\mathcal{P}^{[T2]}$ and $j \in \{14, \ldots, 20\}$ are of type $\mathcal{P}^{[T3]}$. The plants are sampled with different periods given by $T^{[i]} \in \{10, 20, 25\}$, for the different types of plants, respectively. The state-based scheduler uses the criterion $x_k^{(j)^2} > \epsilon^{(j)}$. A p-persistent MAC, with synchronized slots, which permits three retransmissions is used. The persistence probability is given by $p_\alpha^{(r)}$, where r denotes the retransmission index, and $p_\alpha^{(r)} \in \{1, 0.75, 0.5\}$ for $r \in \{1, \ldots, 3\}$. The LQG criterion in (6), with $N = 10$ and $Q_0 = Q_1 = Q_2 = 1$ is used to design a certainty equivalent controller (17) as an ad hoc policy, not an optimal one, as we know from Corollary 3.5. The observer calculates a simple estimate as given by (25)–(26).

We look at the performance of a closed-loop system in this network without a state-based scheduler, i.e., when $\epsilon^{(j)} = 0$ for all j. The cost of controlling the plants in the contention-based network is denoted $J_{\mathrm{CN}}^{[i]}$, and the values are listed in Table I. We compare these values to the costs obtained with a state-based scheduler in the closed-loop system, denoted $J_{\mathrm{SS}}^{[i]}$, when $\epsilon^{(j)} = 2.5$. There is a marked improvement with a state-based scheduler in the closed-loop. Fig. 7 depicts the state and the control signal for the first plant in this network, when a state-based scheduler is used. The above improvement is obtained due to fewer collisions in the contention-based MAC. The nonzero scheduling threshold reduces the traffic in the network, and increases the probability of a successful transmission for all the plants in the network.

B. Two-Step Horizon Example

We now look at a simple example to see the computational difficulties in identifying optimal estimates and controls for a system with a state-based scheduler in the closed-loop. We also show that for a scheduler such as $\tilde{f}$ in Section III-B, which renders the control signal free of a dual effect, the entire plant is altered, so the equivalence construction does not work. Finally, as a proof of the dual effect associated with a scheduler such as f in (2), we present a counterexample, obtained through simulations.

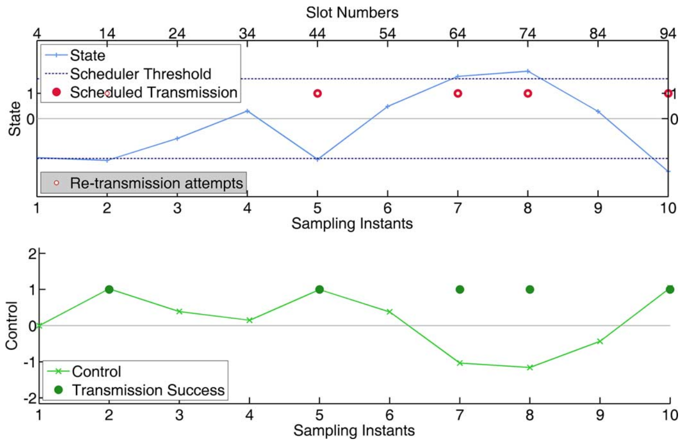

Fig. 7. State and the control signal with the channel use pattern: the red dots denote transmission requests, the white inner dots denote MAC retransmission attempts, and the green dots denote transmission success. Note that the requested bound on the state, which is marked with a dotted line, is sometimes exceeded due to network traffic. Also, the control signal corresponds closely to the state only when there is a successful transmission.

TABLE I
COMPARISON OF CONTROL COSTS WITH (J_{SS}) AND WITHOUT (J_{CN}) A STATE-BASED SCHEDULER IN THE CLOSED-LOOP

Plant Type	$\mathcal{P}^{[T1]}$	$\mathcal{P}^{[T2]}$	$\mathcal{P}^{[T3]}$
J_{CN}	45.3074	10.0028	6.1213
J_{SS}	23.5785	8.3489	5.3803

Consider a scalar plant, given by $x_{k+1} = ax_k + bu_k + w_k$, with $R_0 = R_w = 1$. The scheduling law is given by

$$\delta_k = \begin{cases} 1, & x_k \geq 0.5, \\ 0, & \text{otherwise.} \end{cases} \tag{34}$$

Our aim is to find both the optimal controller, with dual effect, and the certainty equivalent controller for the innovations-based scheduler and show that these result in different control actions for the same scheduling sequence. The controllers are designed to minimize the LQG cost (6), for a horizon of two steps, i.e., $N = 2$, and with $Q_0, Q_1, Q_2 > 0$. We first derive the optimal controller with dual effect. Then, for the same schedule, we define the certainty equivalent controller, assuming that an innovations-based scheduler of the form $\tilde{f}$ in (13) has been designed. We compare the resulting control actions, and comment on the differences.

1) Estimator: The estimates $\hat{x}_{0|0}$ and $\hat{x}_{1|1}$ are obtained using (22). The estimation error covariances $P_{0|0}$ and $P_{1|1}$ are presented in the Appendix. Since the estimation error is non-Gaussian, we need to derive the probability density functions of the estimation errors at each time instant. This makes the computation of the estimation errors and the error covariances hard.

2) Optimal Controller: To solve for the optimal control signals, we use V_1 and V_0 from (15). The complete derivations of V_1 and V_0 are presented in [38]. We find the control signal u_1 that minimizes V_1, and get

$$u_1 = -\frac{abQ_0}{Q_2 + b^2 Q_0}\hat{x}_{1|1}\,. \tag{35}$$

Then, to find u_0, we take a partial derivative of the expression for V_0 with respect to u_0 and get

$$\frac{\partial V_0}{\partial u_0} = 2u_0(Q_2 + b^2 S_1) + 2\hat{x}_{0|0}abS_1 + \frac{a^2 Q_0^2 b^2}{Q_2 + b^2 Q_0} \cdot \frac{\partial}{\partial u_0}\left(\mathbb{E}[P_{1|1}|\mathbb{I}_0^C]\right) = 0\,. \tag{36}$$

The optimal u_0 can be obtained by substituting for $P_{1|1}$ and solving the resulting equation.

3) Certainty Equivalent Controller: For the same scheduler outcomes δ_0, δ_1 obtained through an innovations-based scheduler which has no dual effect, the certainty equivalent controller gives us the control signals

$$\begin{aligned} u_1 &= -\frac{AbQ_0}{Q_2 + b^2 Q_0}\hat{x}_{1|1}\,, \\ u_0 &= -\frac{AbS_1}{Q_2 + b^2 S_1}\hat{x}_{0|0}\,. \end{aligned} \tag{37}$$

Note that the u_1 is found by minimizing V_1, which results in the same expression as for the optimal controller (35). However, when there is no dual effect, the last term in (36) vanishes, and u_0 for the certainty equivalent controller is obtained by solving

$$2u_0(Q_2 + b^2 S_1) + 2\hat{x}_{0|0}abS_1 = 0\,. \tag{38}$$

4) Discussion: A comparison of the control signals for the certainty equivalent controller (37) with u_1 and u_0 obtained in (35) and (36), shows that the signal u_1 remains the same. However, u_0 is different, and displays a dual effect in the optimal controller. From (38), it is clear that the additional term in (36) alters the solution for the optimal controller. This observation can be explained as follows. In a controller with a dual effect, the control signal can be chosen to probe the plant state in order to improve the quality of the estimate. However, there is no motive in improving the estimate in a one-step optimization process. Thus, u_1 is the same for both controllers. When the optimization is performed over two steps, a probing effect in the first step can improve the estimate and the corresponding control applied in the next step. Thus, u_0 is different for the optimal controller.

5) Counterexample: To illustrate further the existence of the dual effect in the state-based scheduler setup discussed above, we consider an explicit numerical example with parameters $a = 2$, $b = 1$, $R_0 = 1$ and $R_w = 100$ for the linear plant, and a cost function with $Q_0 = 100$, $Q_1 = 1$ and $Q_2 = 1$. Finding a u_0 that solves (36) is hard. Instead, we evaluate the cost of using a certainty equivalent controller with a state-based scheduler such as in (34) and compare it with alternative controllers $u_0 = -L_0\hat{x}_{0|0}$, which use a different value for the control gain L_0. We choose a range of values for L_0 centered around the certainty equivalent control gain L_0^{CE}, and plot the control costs obtained against the control gain in Fig. 8, for different values of the scheduler threshold. The certainty equivalent control cost is marked by a dotted line in all the plots, while the cost of using an alternate controller is plotted with a solid line. For each of the scheduling thresholds shown in Fig. 8, there exists a range of values of $L_0 \neq L_0^{CE}$ for which the resulting control cost is lesser than the certainty equivalent control cost. This validates the discussion preceding the counterexample, and

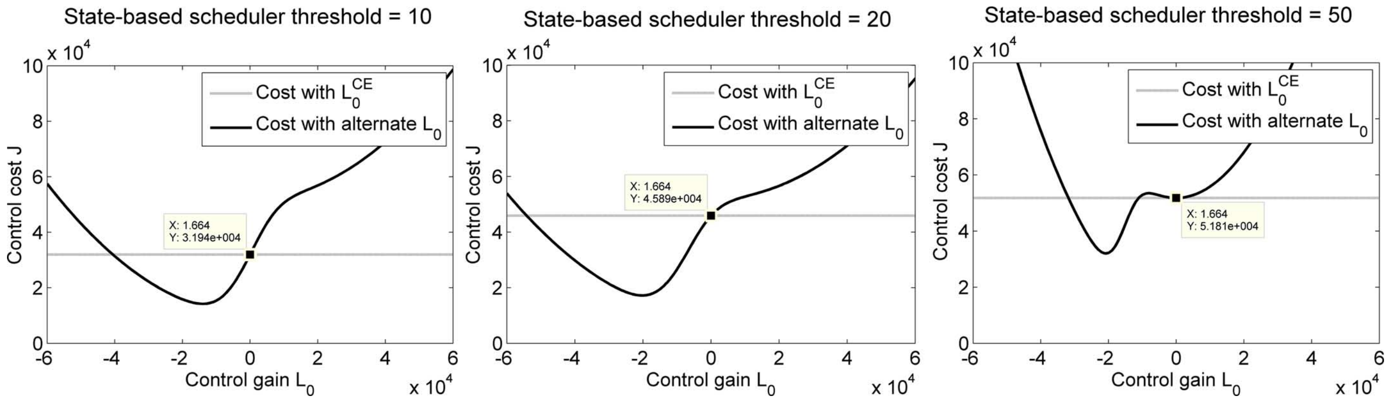

Fig. 8. A comparison of the control costs obtained using the certainty equivalent controller (with gain $L_0^{CE} = 1.664$, shown in dotted lines) and some alternative controllers (with gain L_0, shown in solid lines), for scheduler threshold $\epsilon = \{10, 20, 50\}$. Clearly, there are values of $L_0 \neq L_0^{CE}$ that result in a lower cost. This can be explained by the dual effect in the control signal, as shown in Theorem 3.1.

provides an example of controllers which utilize the probing incentive to improve upon the certainty equivalent controller. The improvement in cost reduces for larger thresholds, which might be explained by noting that the probing incentive, for a small threshold, is not accompanied by a high penalty in cost. As the threshold grows larger, the probing incentive might not be beneficial to exploit, in terms of the cost.

Another counterexample may be found in Curry's work on dual effect with nonlinear measurements [39]. He examines a system with a nonlinearity in the measurement, which may be interpreted as a simple state-based scheduling policy, and illustrates that the optimal controller for a two-step horizon cost differs from the certainty equivalent controller due to the dual effect.

This example shows that there is a dual effect, and even the same schedule can result in a different control sequence for a system without a dual effect. Thus, an equivalent construction for the scheduler does not result in an equivalent system.

C. *Example of the Dual Predictor Architecture*

In this example, we present the dual predictor architecture applied to a shared network. We tune the threshold of the state-based scheduling law to probabilistically guarantee an achievable control performance, given the traffic over the network. We use a homogenous network in this example to simplify the comparison of control cost versus the scheduling threshold.

We consider a shared network of 20 scalar plants, indexed by $j \in \{1, \ldots, 20\}$ and given by (33), where $a^{(j)} = 1$ and $R_w^{(j)} = 1$ for all j. The plants are sampled with a period given by $T = 10$. The innovations-based scheduler uses a similar criterion to (24), where ϵ is the threshold of the scheduler. A p-persistent MAC, with synchronized slots, which permits three retransmissions, is used. The persistence probability is given by $p_\alpha^{(r)}$, where r denotes the retransmission index and $p_\alpha^{(r)} \in \{1, 0.75, 0.5\}$ for $r \in \{1, 2, 3\}$. The LQG criterion in (6), with $N = 10$ and $Q_0 = Q_1 = Q_2 = 1$ is used to design the optimal certainty equivalent controller (17). The observer calculates the MMSE estimate given by (25)–(26).

The effect of varying ϵ on the control cost is shown in Fig. 9. For high values of ϵ, the network is under-utilized, and almost all the transmissions are successful. However, the control cost is high as the number of transmissions is low. As we decrease ϵ, the control cost initially decreases due to increased use of the network. However, for very low values of ϵ, the network is over-utilized and this results in collisions. Thus, the control cost increases again, due to dropped packets. It is interesting to note that the cost function is quite flat. Thus, it is not important, in practice, to use the optimal scheduling threshold ϵ.

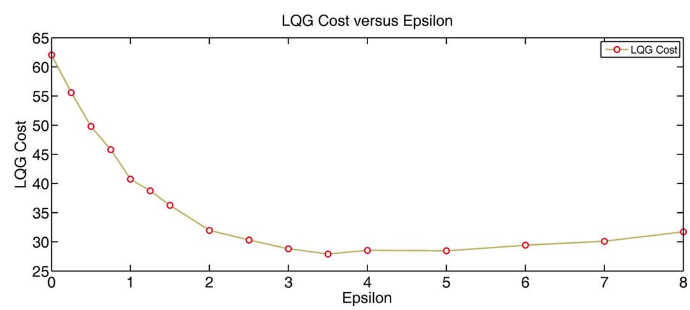

Fig. 9. The control cost $J_{\rm DP}$ versus the scheduler threshold ϵ. For low thresholds, the high traffic in the network causes collisions, and a high $J_{\rm DP}$. High values of ϵ result in an under-utilized network, and a high $J_{\rm DP}$ due to insufficient transmissions.

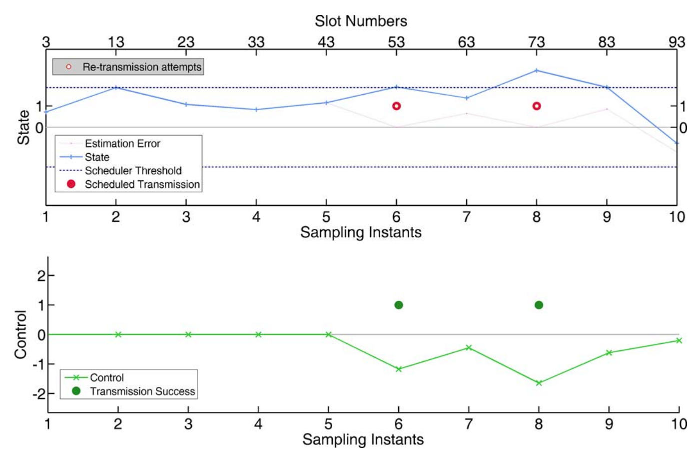

Fig. 10. The estimation error, state and control signal with the channel use pattern. Note that the requested bound on the predicted estimation error, which is marked with a dotted line, is rarely exceeded. Also, the control signal corresponds closely to the state only when there is a successful transmission.

Fig. 10 depicts the state and control signal of the first plant obtained from our simulation, for the best value of ϵ picked from the above plot. Note that the estimation error is bounded, with a probability of 0.94, by the scheduling threshold, for the value $\epsilon^{(1)} = 3.5$, and the resulting control cost is $J_{\rm DP} = 27.9235$.

VII. Conclusions and Future Work

This paper investigated the effects of a state-based scheduler on the design of a closed loop system. We showed that the optimal controller for a NCS with a state-based (or measurement-based) scheduler is, in general, difficult to find. This is due to the dual effect of the control signals, wherein the controller has an incentive to push the state past the scheduler threshold and modify the estimation error across the network. This result implies that the optimal scheduler, observer and controller designs are coupled. However, we identified a dual predictor architecture, which resulted in the desired separation in design of the scheduler, observer and controller. The scheduling function in this architecture is constrained to be a symmetric function of its arguments, such that the resulting schedule is not a function of the past applied controls.

Analyzing the performance of a network of systems using the dual predictor architecture is a challenging direction of work for the future. Identifying control policies in the more general case of state-based schedulers with a dual effect has also been left for the future, as well as a complete extension to distributed measurements of the state.

Appendix
Derivation of the 2-Step Horizon Example

Here are the expressions for the estimation error covariances in Example VI-B. For a detailed derivation, refer to [38].

The estimation error covariance at time $k = 0$ is given by

$$P_{0|0} = \begin{cases} 0, & \delta_0 = 1, \\ R_{\tilde{x}_0}, & \delta_0 = 0, \end{cases} \tag{39}$$

where $R_{\tilde{x}_0} = \mathbb{E}[(x_0 - \bar{x}_{\delta 0})^2 | x_0 < 0.5] = \int_{-\infty}^{0.5-\bar{x}_{\delta 0}} x^2 \phi_{x_{\delta 0}}(x + \bar{x}_{\delta 0}) dx$. Also, $\bar{x}_{\delta 0} := \mathbb{E}[x_0 | x_0 < 0.5] = \int_{-\infty}^{0.5} x \phi_{x_{\delta 0}}(x) dx$, $\phi_{x_{\delta 0}}$ is the conditional probability distribution function (pdf) of x_0, conditioned on $x_0 < 0.5$. Thus, $\phi_{x_{\delta 0}}(x) = \phi_{x_0}(x) / Pr(x_0 < 0.5)$, where ϕ_{x_0} is the pdf of x_0. The probability of a non-transmission is given by $Pr(x_0 < 0.5) = \int_{-\infty}^{0.5} \phi_{x_0}(x) dx$.

Let us denote e_1 as the unknown part of x_1 before y_1 is received:

$$e_1 = \begin{cases} w_0, & \delta_0 = 1, \\ a x_0 + w_0, & \delta_0 = 0, \end{cases} \qquad \phi_e(\epsilon) = \begin{cases} \phi_{w_0}(\epsilon), & \delta_0 = 1, \\ \phi_{e_{\delta 0}}(\epsilon), & \delta_0 = 0, \end{cases}$$

where, ϕ_e is the pdf of e_1, ϕ_{w_0} is the pdf of w_0 and $\phi_{e_{\delta 0}}(\epsilon) = \int_{-\infty}^{0.5} \phi_{x_{\delta 0}}(x) \phi_{w_0}(\epsilon - a x) dx$. We denote $\tilde{e}_1$ as the error in estimating e_1 after y_1 arrives, and $\bar{e}_{\delta 0} = \mathbb{E}[a x_0 + w_0 | x_0 < 0.5, a x_0 + w_0 < 0.5 - b u_0]$. Now, the estimation error variance $P_{1|1}$ is given by

$$P_{1|1} = \begin{cases} 0, & \delta_1 = 1, \\ R_{e_1}, & \delta_1 = 0, \end{cases} \tag{40}$$

where $R_{e_1} = \mathbb{E}[\tilde{e}_1^2 | \delta_1 = 0]$ is given by

$$R_{e_1} = \begin{cases} \int_{-\infty}^{0.5 - a x_0 - b u_0 - \bar{w}_0} w^2 \frac{\phi_{w_0}(w + \bar{w}_0)}{Pr(w_0 < 0.5 - a x_0 - b u_0)} dw, & \delta_0 = 1, \\ \int_{-\infty}^{0.5 - b u_0 - \bar{e}_{\delta 0}} \epsilon^2 \frac{\phi_{\delta 0}(\epsilon + \bar{e}_{\delta 0})}{Pr(e_1 < 0.5 - b u_0)} d\epsilon, & \delta_0 = 0. \end{cases}$$

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