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Hierarchical Model Decomposition for Distributed Design of Glocal Controllers

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***Abstract*— Since modern network systems are managed by multiple operators, practical distributed controller design is required to be independently performed in a distributed manner. The independent design of distributed controllers, referred to as distributed design, enables the synthesis process to be scalable. Nevertheless, distributed design methods have not yet been fully developed because of its difficulty. As a novel scheme for control of network systems, this paper presents a distributed design method of glocal (global/local) controllers. In the glocal structure, a global controller is introduced into the controller to be designed in addition to local decentralized controllers. The key idea to realize distributed design is to represent the original network system as a hierarchical cascaded system composed of reduced-order models each of which stands for the dynamics of global and local behaviors, here referred to as hierarchical model decomposition. Distributed design is achieved by designing controllers for the reduced-order models owing to the cascade structure. A numerical example demonstrates the effectiveness of the proposed glocal control.**

I. Introduction

Modern highly complex network systems are independently developed and managed by multiple operators to be scalable. For example, power grids are typically governed by various system operators each of which manages the corresponding subsystem in an independent manner [1]. In the context of control theory, it is referred to as *distributed design* to independently design each controller with access to its corresponding partial model information [2], while it is referred to as centralized design to integratedly design all controllers with access to the entire model information simultaneously. Most existing controller design methods have been built on the premise of centralized design [3], [4] and distributed design has not yet been fully explored in the literature because of its difficulty. The goal of this study is to characterize a suitable information structure of controllers under distributed design and to develop a specific distributed design method of the structured controller.

Retrofit control [5], [6] has recently been proposed as a distributed design method for decentralized controllers, in which controllers determine their own control input depending only on its local measurement signal without communication with one another. In the retrofit control framework, each operator regards the entire network system as an interconnected system comprising the subsystem of interest and its unknown environment. Each local controller is designed only with the information on the subsystem model of interest to guarantee stability of the entire closed-loop system for any possible environment. Since the decentralized structure is the most fundamental information structure, the approach can handle a broad range of network systems.

It has been, however, reported that there exists a case where adequate control performance cannot be achieved when the model information of the environment is completely unavailable in [7]. It is well-known that the behavior of a consensus network system can be captured as coordinated states that depend on local clustered subsystems and the global network structure. [8]. From the viewpoint of the local and global behaviors, as shown in this paper, a retrofit controller designed only with the local subsystem model information works to suppress local oscillating behavior while global behavior arisen from inter-area modes still remains. The untouched global behavior is the primary cause of the performance degradation.

To deal with the above undesirable behavior, we propose a novel approach based on the concept of *glocal (global/local) control* [9]. In our proposed glocal control framework, we provide a controller having the glocal structure in which a global controller is introduced in addition to local decentralized controllers. The global and local controllers are designed based on reduced-order global and local dynamics and hence the design procedure for each controller is relatively simple even if the original network system is complex. The aim of this paper is to develop a distributed design method for glocal control.

The key idea for this aim is to represent the network system to be controlled as a hierarchical cascaded system consisting of local reduced-order models and a global reduced-order model. Distributed design is achieved by designing corresponding stabilizing controllers for the derived reduced-order models. We refer to the hierarchical cascaded system as *hierarchical model decomposition,* which can be regarded as a generalization of hierarchical state-space expansion in retrofit control [5]. Owing to the cascade structure, the stability of the entire closed-loop system can be assured as long as all reduced-order models are stabilized. Thus independent design of the glocal controllers is carried out just by designing stabilizing controllers for each reduced-order model. In this paper, considering the situation where all subsystems are divided into given clusters of subsystems beforehand, we derive a geometric characterization

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for existence of hierarchical model decomposition from the perspective of controllable subspaces from each cluster. Subsequently, we give a specific representation of hierarchical model decomposition under the above condition. We also discuss implementation of designed control policies and show that controllers that preserve the hierarchical structure can be implemented by using functional observers [10], [11].

We compare the proposed glocal control with the existing methods on distributed design [12], [13], [14], composite control [15], and control strategies with combination of global and local controllers [9], [16], [17]. A few studies on distributed design can be found in [12], [13], [14]. These approaches are different from one another and the achievable goals are also different. For instance, in localized system level synthesis [14], each local controller confines the closed-loop map to its local region with communication among them. It is, however, unclear whether this approach is effective in dealing with network systems having inter-area modes. Composite control [15], the basic philosophy of which is to divide the entire system dynamics into global and local models behaviors, appears to be similar to the proposed glocal control. Actually, however, composite control is based on singular perturbation under the assumption on time-scale separation, that is, attenuation of the non-coherent modes is assumed to be sufficiently fast compared with that of the inter-area mode [18]. On the contrary, the proposed approach does not require any assumption on time-scales of global and local behaviors. Finally, the notion of glocal control has been originally proposed in [9] but the specific approach for realizing glocal control is not fully developed. Although a similar idea, combining local controllers and a global controller, can be found in the literature [16], [17], distributed design of the controllers is not focused on there. A distributed design method of glocal controllers has been proposed in [19] by the authors, but this study provides generalized results. Specifically, while a sufficient condition for exiting of hierarchical model decomposition, introduced in this study, is derived for a particular class of network systems in [19], a necessary and sufficient condition is derived for arbitrary network systems in this paper.

This paper is organized as follows. In Sec. II, we give a description of the network system under the information structure treated in this study and formulate the distributed design problem of glocal controllers. Sec. III provides a motivating example that deals with a second-order network system. The example shows that retrofit control without environment modeling is ineffective as global behavior becomes dominant. Further, we outline the proposed method through the specific example. In Sec. IV, the precise definition of hierarchical model decomposition is introduced and several mathematical conditions for the representation are derived. In Sec. VI, the effectiveness of the proposed method is demonstrated through the example in Sec. III. Finally, Sec. VII draws conclusion. The proofs of the theorems derived in this paper are omitted due to the page limit.

Notation

We denote the n-dimensional identity matrix by I_n, the $n \times m$ zero matrix by $0_{n\times m}$, the ith canonical vector by e_i, the n-dimensional all-ones vector by $\mathbf{1}_n$, the vector where x_i for $i \in \mathcal{I}$ are concatenated vertically by $\mathrm{col}(x_i)_{i\in\mathcal{I}}$. The subscript for the variables is omitted when the dimension is clear from the context. We denote the Kronecker product by $\otimes$, the transpose of a matrix M by M^{T}, the direct sum and the sum space of linear subspaces $\mathcal{X}$ and $\mathcal{Y}$ by $\mathcal{X} \oplus \mathcal{Y}$, $\mathcal{X} + \mathcal{Y}$, respectively, the controllable subspace with respect to the pair (A, B) by $\mathcal{R}(A, B)$, and the $\mathcal{H}_2$ norm of a system Σ by $\|\Sigma\|_{\mathcal{H}_2}$.

II. Problem Formulation

A. System Description

We consider a linear time-invariant interconnected system composed of N_0 subsystems

$$\Sigma_{[k]} : \begin{cases} \dot{x}_{[k]} = A_{[k]}x_{[k]} + L_{[k]} \sum_{l\in\mathcal{N}_{[k]}} v_{[l]} + B_{[k]}u_{[k]} \\ w_{[k]} = \Gamma_{[k]}x_{[k]} \\ y_{[k]} = C_{[k]}x_{[k]} \end{cases}$$

for $k = 1, \ldots, N_0$, where $x_{[k]}, v_{[k]}, w_{[k]}, u_{[k]}, y_{[k]}$ denote the state, the interconnection signals, the control input, and the measurement signal and $\mathcal{N}_{[k]}$ denotes the index set associated with the neighborhood of $\Sigma_{[k]}$, interconnected through

$$\begin{bmatrix} v_{[1]} \\ \vdots \\ v_{[N_0]} \end{bmatrix} = \begin{bmatrix} M_{[11]} & \cdots & M_{[1N_0]} \\ \vdots & & \vdots \\ M_{[N_01]} & \cdots & M_{[N_0N_0]} \end{bmatrix} \begin{bmatrix} w_{[1]} \\ \vdots \\ w_{[N_0]} \end{bmatrix}$$

with transfer matrices $M_{[ij]}$ for $i, j = 1, \ldots, N_0$. We assume that the dimensions of the signals are identical with respect to k. and also assume that dimensions of $u_{[k]}$ and $y_{[k]}$ are one for notational simplicity.

Let us suppose that the network system is managed by N independent operators and N clusters $\mathcal{I}_i \subset \{1, \ldots, N_0\}$ for $i = 1, \ldots, N$ are given to represent the indices of each operator's subsystems. It is assumed that there is no overlap and $\bigcup_{i=1}^{N} \mathcal{I}_i = \{1, \ldots, N_0\}$ holds. We describe the dynamics of the subsystems in $\mathcal{I}_i$ as

$$\Sigma_i : \begin{cases} \dot{x}_i = A_i x_i + L_i \sum_{j\in\mathcal{N}_i} v_j + B_i u_i \\ w_i = \Gamma_i x_i \\ y_i = C_i x_i \end{cases}$$

and the network with respect to $\{\Sigma_i\}_{i=1}^{N}$ as

$$\begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} M_{11} & \cdots & M_{1N} \\ \vdots & & \vdots \\ M_{N1} & \cdots & M_{NN} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

where the state is defined by $x_i := \mathrm{col}(x_{[k]})_{k\in\mathcal{I}_i}$ and other signals are defined in a similar manner. Let n_i and r_i be the dimension of the state and the cardinality of $\mathcal{I}_i$, respectively. Note that the dimensions of the control input and the measurement output both are r_i from the assumption. Note also that although the clusters are assumed to be given in the present setting the clustering process could be included in the system design.

Consider introducing a supervising global controller in addition to local controllers designed by each operator. Then the control input is supposed to be composed of global and local control inputs. We now assume that the dimensions of the signals in each cluster are identical. Moreover, we also assume that the input matrices in each cluster are identical as well, that is, $B_{[k]} = B_{[l]}$ for $k, l \in \mathcal{I}_i$, and that the same global input is injected into the subsystems belonging to $\mathcal{I}_i$ in a broadcast manner for any $i \in \{1, \ldots, N\}$ in accordance with [20]. Then the ith control input can be represented by $u_i = \mathbf{1}_{r_i}\hat{u}_{0,i} + \hat{u}_i$, where $\hat{u}_{0,i} \in \mathbb{R}$ and $\hat{u}_i \in \mathbb{R}^{r_i}$ are global and local control inputs, respectively. Similarly, the global controller is supposed to utilize the aggregated measurement signal y_0 defined by $y_0 := \mathrm{col}\left(\sum_{k\in\mathcal{I}_i} y_{[k]}\right)_{i=1}^{N}$. Under the setting, the dynamics of the entire system can be represented by

$$\Sigma : \begin{cases} \dot{x} = Ax + P_0 B_0 \hat{u}_0 + \sum_{i=1}^{N} P_i B_i \hat{u}_i \\ y_0 = C_0 P_0^{\mathsf{T}} x \\ y_i = C_i P_i^{\mathsf{T}} x \end{cases} \tag{1}$$

where $x := \mathrm{col}(x_i)_{i=1}^{N}, \hat{u}_0 := \mathrm{col}(\hat{u}_{0,i})_{i=1}^{N}$, P_0 and P_i, which correspond to broadcasting and embedding matrices, are defined by

$$P_0 := \begin{bmatrix} \mathbf{1}_{r_1} \otimes I_{n_{0,1}} & \cdots & 0_{n_1 \times n_{0,N}} \\ \vdots & \ddots & \vdots \\ 0_{n_N \times n_{0,1}} & \cdots & \mathbf{1}_{r_N} \otimes I_{n_{0,N}} \end{bmatrix}, \; P_i := \begin{bmatrix} 0_{n_1 \times n_1} \\ \vdots \\ I_{n_i} \\ \vdots \\ 0_{n_N \times n_N} \end{bmatrix}, \tag{2}$$

where $n_{0,i}$ is the dimension of the subsystem's state in $\mathcal{I}_i$, and A, B_0, and C_0 are defined to be compatible with the above signals and matrices. For the given system Σ, we consider developing a distributed design method of the global and local controllers $\hat{u}_i = K_i y_i$ for $i = 0, 1, \ldots, N$ with dynamical linear controllers K_i. Note that K_0 and $K_1, \ldots, K_N$ correspond to the global and the local controllers, respectively.

B. Problem Formulation: Distributed Design of Glocal Controllers

To formulate the problem of distributed design for glocal control, we present the definition of distributed design treated in this paper. In order to allow independent design of controllers, we design controller sets $\mathcal{K}_i$ instead of controllers K_i themselves. We say that distributed design of glocal controllers is achieved if the closed-loop system composed of the original network system Σ and any glocal controllers K_i belonging to $\mathcal{K}_i$ has a desired property, e.g., internal stability. In other words, when

$$\begin{aligned} &K_i \in \mathcal{K}_i, \quad i = 0, \ldots, N \\ &\Rightarrow \text{the system } \Sigma \text{ with } \{K_i\}_{i=0}^{N} \text{ is internally stable} \end{aligned}$$

holds then we say that distributed design is achieved. In this study we employ stability as the desired property but other quantitative performance measure can be adopted.

Specifically, we design the controller sets $\mathcal{K}_i$ through the following procedure. We build $\hat{\Sigma}_i$ for $i = 0, \ldots, N$ as virtual global and local models for design of glocal controllers. Based on the derived models $\hat{\Sigma}_i$, $\mathcal{K}_i$ is decided to be the set of controllers by which the corresponding model $\hat{\Sigma}_i$ is stabilized. The problem here is how to design the virtual models $\hat{\Sigma}_i$ and also the controller sets $\mathcal{K}_i$.

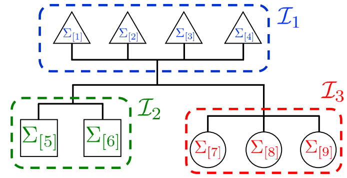

Fig. 1. motivating example: second-order network system.

We address the following problem of distributed design for glocal control.

Problem: For a given network system Σ and clusters $\{\mathcal{I}_i\}_{i=1}^{N}$, establish $\hat{\Sigma}_i$ and $\mathcal{K}_i$ that achieve distributed design.

Although clustering should be included in the design process for practical systems, the clusters $\{\mathcal{I}_i\}_{i=1}^{N}$ are assumed to be given in the problem as a preliminary step.

III. Brief Overview of Proposed Method through Motivating Example

In this section, a brief overview of the proposed glocal control is given through a specific example of a second-order network system.

A. Motivating Example

Consider the network system shown in Fig. 1. Each subsystem is given by the following second-order system $m_{[k]}\ddot{\theta}_{[k]} + d_{[k]}\dot{\theta}_{[k]} + v_{[k]} + u_{[k]} = 0$ with the interconnection signal $v_{[k]} := \sum_{l\in\mathcal{N}_{[k]}} \alpha_{[kl]}(\theta_{[k]} - \theta_{[l]})$. It is assumed that the parameters of the subsystems represented by the same shape in Fig. 1 are identical for simplicity. Following this point, we provide the clusters as $\mathcal{I}_1 := \{1, 2, 3, 4\}, \quad \mathcal{I}_2 := \{5, 6\}, \quad \mathcal{I}_3 := \{7, 8, 9\}$. Let the parameters of the subsystems in each cluster be given by

$$m_{[k]} := \begin{cases} 1, & k \in \mathcal{I}_1 \\ 2, & k \in \mathcal{I}_2 \\ 3, & k \in \mathcal{I}_3 \end{cases}, \quad d_{[k]} := \begin{cases} d, & k \in \mathcal{I}_1 \\ 2d, & k \in \mathcal{I}_2 \\ 3d, & k \in \mathcal{I}_3 \end{cases}$$

where $d > 0$ is a positive scalar that determines damping rates. Set the strength of the interconnection among the subsystems as $\alpha_{[kl]} = 1, \quad \forall k, l \in \{1, \ldots, 9\}$. The purpose of control for the example is to regulate the initial condition response. Regarding the subsystems belonging to a cluster as a single subsystem, we introduce retrofit control only with local model information [5]. The detail of retrofit control is omitted, but it should be noted that retrofit control provides a distributed design method of the local controllers $K_1, \ldots, K_N$ without the global controller K_0 by setting $\hat{\Sigma}_i$ to be Σ_i itself. The design parameter of retrofit control, namely, the locally stabilizing controller, is determined by a linear quadratic regulator and a state observer.

For examining how retrofit controllers work, we see the initial condition response under the initial value $\theta_{[k]}(0) = 0, \ \forall k \in \{1, \ldots, 9\}$ and $\dot{\theta}_{[1]}(0) = 1, \dot{\theta}_{[2]}(0) = 1.25, \dot{\theta}_{[3]}(0) = 1.5, \dot{\theta}_{[4]}(0) = 2, \dot{\theta}_{[k]}(0) = 0$ for $k \notin \mathcal{I}_1$. First, the response of $\dot{\theta}_{[k]}$ in the case $d = 1$ is shown in Fig. 2. The top

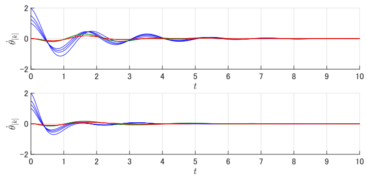

Fig. 2. top: $\dot{\theta}_{[k]}$ with no control, bottom: $\dot{\theta}_{[k]}$ with retrofit control when $d = 1$.

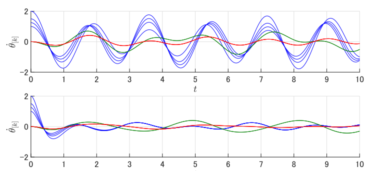

Fig. 3. top: $\dot{\theta}_{[k]}$ with no control, bottom: $\dot{\theta}_{[k]}$ with retrofit control when $d = 0.01$.

and bottom figures correspond to the responses with no control and retrofit control, respectively. The blue, green, and red lines are associated with the clusters $\mathcal{I}_1, \mathcal{I}_2$, and $\mathcal{I}_3$, respectively. In this case, we can observe that retrofit control successfully suppresses the response. Next, the response of $\dot{\theta}_{[k]}$ in the case $d = 0.01$ is shown in Fig. 3. Unlike the previous case, though amplitude is made small, stationary inter-area oscillation remains even with retrofit control. The same problem has been pointed out in the existing study [**?**] and it has been confirmed that achievable improvement by introducing retrofit controllers becomes smaller as the influence by global interconnection becomes larger.

B. Brief Overview of Proposed Method

It can be expected that the behavior of the network system can be represented as a superposition of global inter-area behavior and local oscillating behavior since the curves in Fig. 3 show that the subsystems in each cluster exhibit synchronized behavior by introducing retrofit controllers. Let the state of each subsystem be $x_{[k]} := [\theta_{[k]}\ \dot{\theta}_{[k]}]^{\mathsf{T}}$ and denote the variables and the system matrices according to Sec. II-A. Then the whole state can be represented by

$$x(t) = \textstyle\sum_{i=1}^{3} P_i \xi_i(t) + P_0 \xi_0(t), \quad \forall t \geq 0 \tag{3}$$

where ξ_i for $i = 0, \ldots, 3$ are the states of the following hierarchical cascaded system

$$\Xi : \left\{ \begin{array}{l} \dot{\xi}_i = P_i^{\mathsf{T}} A P_i \xi_i + B_i \hat{u}_i, \quad i = 1,2,3 \\ \dot{\xi}_0 = P_0 A P_0 \xi_0 + \sum_{i=1}^{3} R_i \xi_i + B_0 \hat{u}_0 \end{array} \right.$$

and

$$R_i := \sum_{j \neq i} e_j \otimes \left(\mathbf{1}_{r_i}^{\mathsf{T}} \otimes \begin{bmatrix} 0 & 0 \\ 1/m_{[k]} & 0 \end{bmatrix} \right), \quad k \in \mathcal{I}_j$$

provided that $x(0) = \sum_{i=1}^{3} P_i \xi_i(0) + P_0 \xi_0(0)$. The matrices P_i for $i = 0, \ldots, 3$ are specifically given by

$$\begin{array}{l} P_0 = \begin{bmatrix} \mathbf{1}_4 \otimes [I_2\ 0_{2\times 2}\ 0_{2\times 2}] \\ \mathbf{1}_2 \otimes [0_{2\times 2}\ I_2\ 0_{2\times 2}] \\ \mathbf{1}_3 \otimes [0_{2\times 2}\ 0_{2\times 2}\ I_2] \end{bmatrix}, \\ P_1 = \begin{bmatrix} I_8 \\ 0_{4\times 8} \\ 0_{6\times 8} \end{bmatrix}, \ P_2 = \begin{bmatrix} 0_{8\times 4} \\ I_4 \\ 0_{6\times 4} \end{bmatrix}, \ P_3 = \begin{bmatrix} 0_{8\times 6} \\ 0_{4\times 6} \\ I_6 \end{bmatrix}. \end{array} \tag{4}$$

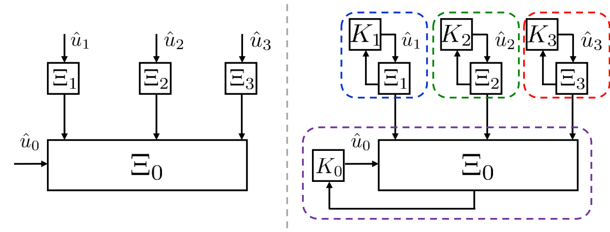

Fig. 4. left: block diagram of Ξ, right: block diagram of Ξ with controllers.

From (3) and (4), ξ_0 and ξ_1, ξ_2, ξ_3 are interpreted as representing global and local behaviors of the original state x, respectively. Note that the dimensions of ξ_i are small compared with that of the original state x.

A block diagram of Ξ is depicted in the left part of Fig. 4 where Ξ_i represents the dynamics with respect to ξ_i. What should be emphasized here is that the system is composed of reduced-order subsystems only with cascade and parallel interconnections, i.e., no feedback connections. We refer to the system representation Ξ as a *hierarchical model decomposition*. Owing to the interconnection structure, stability of Ξ is guaranteed by introducing glocal controllers according to the right part of Fig. 4 provided that each controller stabilizes the corresponding small closed-loop system and then stability of the original system Σ is also guaranteed from (3). Following the procedure, we can achieve distributed design of glocal controllers because the controllers can be designed independently. A problem here is that K_i cannot be directly implemented because the measurement signals in Fig. 4 are different from $\{y_i\}$ themselves and we will discuss an implementation method to settle the problem in Sec. V-A.

The above process is a brief overview of the proposed method for the specific example. The resulting responses under the proposed glocal controllers will be shown in Sec. VI. We will develop a framework for applying the approach to general systems from the next section.

IV. Hierarchical Model Decomposition

As seen in the previous section, the basic idea of the proposed distributed design for glocal control totally relies on hierarchical model decomposition composed of reduced-order subsystems. Let us give the definition of hierarchical model decomposition for general systems.

Definition: For the system Σ in (1), consider the hierarchical cascaded system

$$\Xi : \left\{ \begin{array}{l} \dot{\xi}_i = \hat{A}_i \xi_i + B_i \hat{u}_i, \quad i = 1, \ldots, N \\ \dot{\xi}_0 = \hat{A}_0 \xi_0 + \sum_{i=1}^{N} \hat{R}_i \xi_i + B_0 \hat{u}_0. \end{array} \right. \tag{5}$$

The system Ξ is said to be a hierarchical model decomposition of Σ for $\{P_i\}_{i=0}^{N}$ in (2) when

$$x(t) = \textstyle\sum_{i=1}^{N} P_i \xi_i(t) + P_0 \xi_0(t), \quad \forall t \geq 0$$

holds for arbitrary $\{\hat{u}_i\}_{i=0}^{N}$ under the condition $x(0) = \sum_{i=1}^{N} P_i \xi_i(0) + P_0 \xi_0(0)$.

We provide mathematical conditions for existence and specific representations of the decomposition in the subsequent subsections.

A. Existence Condition

We first give a necessary and sufficient condition for existence of hierarchical model decomposition from the viewpoint of controllable subspaces

Theorem 1: A hierarchical model decomposition of Σ in (1) for $\{P_i\}_{i=0}^N$ in (2) exists if and only if the condition

$$\begin{cases} \mathcal{R}(A, P_i) \subset \operatorname{im} P_i + \operatorname{im} P_0, \quad i = 1, \ldots, N & \text{(6a)} \\ \mathcal{R}(A, P_0) \subset \operatorname{im} P_0 & \text{(6b)} \end{cases}$$

holds.

The conditions of Theorem 1 are derived based on the following procedure. First, we represent the initial condition response as a response for an impulsive input through the system $\dot{x} = Ax + \delta x(0)$ with the delta function δ. Decompose the input matrix according to $\operatorname{im} I = \mathbb{R}^n = \bigoplus_{i=1}^N \operatorname{im} P_i$, each of which corresponds to $\hat{u}_i$, and consider the controllable subspaces with respect to the pairs $\{(A, P_i)\}$. The first condition of Theorem 1 implies that $\operatorname{im} P_0$ covers the exterior of $\operatorname{im} P_i$ in $\mathcal{R}(A, P_i)$ for every i under the decomposition of inputs. From the perspective of a network system, the condition means that the behavior excited by local external input through P_i can be represented by local and global behaviors in $\operatorname{im} P_i$ and $\operatorname{im} P_0$, respectively. The second condition simply implies that $\operatorname{im} P_0$ is an invariant subspace of A and trajectories inside $\operatorname{im} P_0$ remain in the same space (see Fig. 4). Theorem 1 provides a condition to discriminate existence of hierarchical model decomposition for a given system Σ and clusters $\{P_i\}_{i=0}^N$.

B. Representation

Based on Theorem 1, we derive a necessary and sufficient condition for the system Ξ to be a hierarchical model decomposition.

Theorem 2: The system Ξ in (5) is a hierarchical model decomposition of Σ for $\{P_i\}$ if and only if the condition

$$\begin{cases} AP_i - P_0 \hat{R}_i - P_i \hat{A}_i = 0, \quad i = 1, \ldots, N \\ \hat{A}_0 = P_0^{\dagger} A P_0 \end{cases} \tag{7}$$

holds.

Theorem 2 gives a construction method of a hierarchical model decomposition through the equation (7), which can be readily solved since (7) is a linear equation with respect to the matrix variables.

Remark: Set $N = N_0$ and $\mathcal{I}_i = \{k\}$. Then $P_0 = I$ that satisfies the conditions of Theorem 1 and the matrices are determined $\hat{R}_i = AP_i - P_i \hat{A}_i$, $\hat{A}_0 = A$ with free parameters $\hat{A}_i$. This hierarchical model decomposition is the hierarchical state-space expansion for retrofit control introduced in [5] itself. In this sense, hierarchical model decomposition is a generalized version of hierarchical state-space expansion.

V. Proposed Glocal Control

A. Implementation through Functional Observer

Consider designing controllers $K_0, \ldots, K_N$ that stabilize the subsystems of ξ_i based on the hierarchical model decomposition obtained through Theorem 2. Assuming that the controllers are represented by static gain K_i for simplicity, we design the controllers such that the closed-loop systems

$$\begin{cases} \dot{\xi}_i = \hat{A}_i \xi_i + B_i \hat{u}_i \\ \hat{u}_i = K_i C_i \xi_i \end{cases}, \quad i = 0, \ldots, N \tag{8}$$

are internally stable. However, although the control input in (8) depends on ξ_i, ξ_i is a virtual variable and unavailable for feedback signals. Consequently, it is impossible to directly implement K_i in (8).

To tackle with this problem, we consider estimating $C_i \xi_i$ utilizing functional observers [10], [11]. In fact, the cascade structure of Ξ is preserved by using functional observers and stability of the entire network system can be assured with the combination of the above designed controllers and functional observers as follows.

Theorem 3: Assume that Ξ is a hierarchical model decomposition of Σ for $\{P_i\}$ and $K_0, \ldots, K_N$ stabilize the closed-loop systems (8). If the dynamical systems

$$\Phi_i : \begin{cases} \dot{\phi}_i = \mathbf{A}_i \phi_i + \mathbf{B}_{yi} y_i + \mathbf{B}_{ui} \hat{u}_i + \mathbf{B}_{u0} \hat{u}_0 \\ \psi_i = \mathbf{C}_i \phi_i + \mathbf{D}_i y_i \end{cases}$$

for $i = 1, \ldots, N$ are functional observers of $C_i \xi_i$ for Ξ, i.e., $\lim_{t \to +\infty} (C_i \xi_i(t) - \psi_i(t)) = 0$ holds for any initial condition and inputs, then the closed-loop system composed of Σ and the controllers $\hat{u}_0 = K_0 y_0$ and $\hat{u}_i = K_i \psi_i$ for $i = 1, \ldots, N$ is internally stable.

What should be emphasized in Theorem 3 is that the properties that Φ_i must hold are independent of K_j for $j \neq i$ since the dynamics of ξ_i is also independent of K_j. Hence, functional observers with respect to i can be independently designed and distributed design is achieved including the design of Φ_i.

We next provide specific functional observers. For simplicity, we suppose the situation where the interconnection signal v_i is available in addition to the measurement signal y_i and the control inputs $\hat{u}_i$ and $\hat{u}_0$.

Theorem 4: Assume that A_i and $\hat{A}_i$ are stable. Let

$$\Phi_i : \begin{cases} \dot{\phi}_i = \hat{A}_i \phi_i + (A_i - \hat{A}_i) \hat{x}_i + L_i v_i + P_i^{\mathsf{T}} P_0 B_0 \hat{u}_0 \\ \dot{\hat{x}}_i = A_i \hat{x}_i + B_i \hat{u}_i + L_i v_i + P_i^{\mathsf{T}} P_0 B_0 \hat{u}_0 \\ \psi_i = -C_i \phi_i + y_i \end{cases} \tag{9}$$

and then Φ_i is a functional observer for $C_i \xi_i$.

The states ϕ_i and $\hat{x}_i$ in (9) are estimated values of the variables $P_i^{\mathsf{T}} P_0 \xi_0$ and x_i, respectively. Although we put the assumption that A_i and $\hat{A}_i$ are stable for simplicity, we can stabilize and improve the convergence rate by making a certain error feedback even without the assumption. Moreover, it is expected that the assumption on interconnection signal v_i can be removed based on the idea in [5], [21].

In Theorem 4, the parameters of Φ_i depend only on Σ_i. Thus, it suffices to utilize only the parameters of Φ_i and Σ_i for making the virtual system $\hat{\Sigma}_i$ for distributed design.

B. Proposed Glocal Control

Based on the above results, we summarize the proposed glocal control as the solution of the formulated problem.

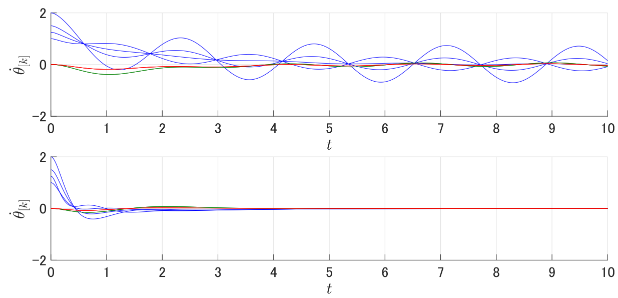

Fig. 5. top: $\dot{\theta}_{[k]}$ only with global control, bottom: $\dot{\theta}_{[k]}$ with glocal control when $d = 0.01$.

1) Discriminate existence of a hierarchical model decomposition for the given Σ and $\{P_i\}_{i=0}^N$ based on Theorem 1.
2) Construct a hierarchical model decomposition based on Theorem 2 provided that the condition in Theorem 1 holds.
3) Establish $\hat{\Sigma}_i$ from the derived Ξ_i and Σ_i and determine $\mathcal{K}_i$ to be the set whose elements are the controllers composed of the functional observers in (9) and internal controllers that stabilize (8). Then distributed design is achieved using the set $\mathcal{K}_i$ obtained through the above process from Theorem 3 and 4.

When the condition in Theorem 1 is not satisfied, we cannot find a hierarchical decomposition and consequently the procedure fails. This situation means that the given clusters are not suitable for the proposed glocal control and we have to rearrange the clusters.

VI. Numerical Example

We examine how the proposed method works through a numerical example. In particular, we continue to deal with the second-order network system used in Sec. III. The design parameters of K_i, which are internal stabilizing controllers, are determined as a linear quadratic regulator with a state-observer under a certain weight.

To verify effectiveness of the proposed method, we show Fig. 5 illustrating that the initial responses of $\dot{\theta}_{[k]}$ when the damping coefficient $d = 0.01$, which corresponds to Fig. 3. The top and bottom in Fig. 5 show the responses only with the global controller and with the glocal controllers, respectively. It is observed that locally oscillating behavior in the cluster $\mathcal{I}_1$ is not suppressed when introducing only the global controller. In contrast, both of global and local behaviors are suppressed by introducing the glocal controllers. This result indicates effectiveness of the proposed method.

VII. Conclusion

We have proposed a distributed design method for glocal control based on hierarchical model decomposition. The next issue is to investigate detailed properties of the proposed clustering method and to develop a computationally efficient algorithm. Moreover, the condition for hierarchical model decomposition derived in this paper is strict in the sense that the original state is perfectly reproduced as a superposition of the states of the decomposition. It is necessary to develop a robust theory for systems perturbed away from the ideal situation to apply the proposed method to practical systems. Also, developing a clustering method that provides an appropriate clusters for a given network system is included in the future work.

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