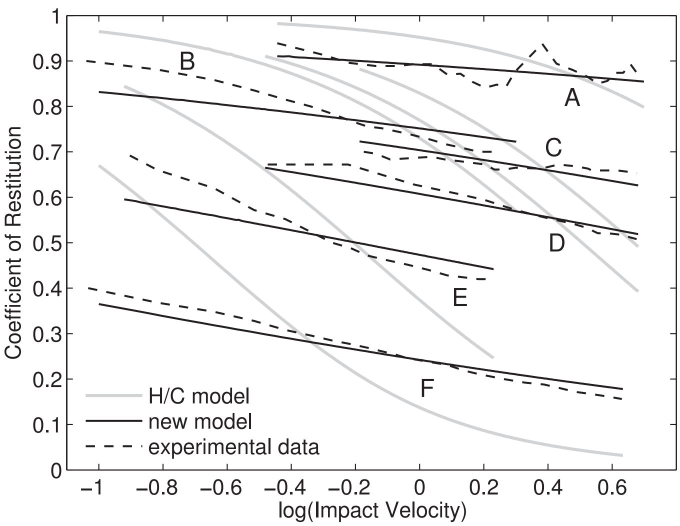

Fig. 5. Coefficient of restitution versus logarithm of impact velocity for the contact between (A) brass spheres ($R_1 = R_2 = 1.5$ cm), (B) a steel sphere ($R = 1.27$ cm) and a cast iron plate, (C) cork spheres ($R_1 = R_2 = 1.66$ cm), (D) a steel sphere ($R = 1.65$ cm) and a cork plate, (E) a steel sphere ($R = 1.27$ cm) and a brass plate, and (F) a steel sphere ($R = 1.27$ cm) and a cold-worked lead plate.

The empirical data were obtained by hand measurements from [4, Fig. 172] and [7, Fig. 3]. The model curves were obtained by calculating k from the appropriate material properties and sphere radius using (5), and tuning λ to obtain a least-squares best fit to the data points. Some of the curves appear to fit the data better at higher velocities than at lower ones, but this is simply a consequence of the distribution of data points along the data curves (more points at the high end because of the logarithmic scale used) and the fact that all data points were weighted equally.

From Fig. 5, we can immediately see that the data follow approximately straight lines, and that the new model fits the data much better than the Hunt/Crossley model. Furthermore, in cases (A), (C), (D), and (F), the model fits the data very well. Although the fit is not so good in cases (B) and (E), it is still better than any fit that can be achieved with either the linear or the Hunt/Crossley model. One possible reason for the less accurate fit in cases (B) and (E) is that these are the two cases in which the sphere and plate are made of different materials having similar stiffnesses; therefore, it is possible that the assumption in Section II does not hold.

It has been frequently assumed in the literature that the coefficient of restitution varies linearly with the impact velocity; for example, in [4], [5], [9], and [10]. However, this is not a good assumption because, as Fig. 5 clearly shows, the coefficient of restitution actually varies linearly with the logarithm of impact velocity.

IV. Conclusion

A new nonlinear model of contact normal force during compliant contact has been presented. It differs by only a single exponent from the well-known model of Hunt and Crossley [5]. However, detailed comparisons between published experimental measurements of the coefficients of restitution between spheres and plates of various materials, and the coefficients of restitution predicted by the new model and the Hunt/Crossley model, show that the new model provides a substantially more accurate fit to the experimental data. It is, therefore, likely to be a better choice when physically accurate simulations are required.

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Leader–Follower Coordinated Tracking of Multiple Heterogeneous Lagrange Systems Using Continuous Control

Ziyang Meng, Dimos V. Dimarogonas, and Karl H. Johansson

***Abstract*—In this paper, we study the coordinated tracking problem of multiple heterogeneous Lagrange systems with a dynamic leader. Only nominal parameters of Lagrange dynamics are assumed to be available. Under the local interaction constraints, i.e., the followers only have access to their neighbors’ information and the leader being a neighbor of only a subset of the followers, continuous coordinated tracking algorithms with adaptive coupling gains are proposed. Except for the benefit of the chattering-free control achieved, the proposed algorithm also has the attribute that it does not require the neighbors’ generalized coordinate derivatives. Global asymptotic coordinated tracking is guaranteed, and the tracking errors between the followers and the leader are shown to converge to zero. Examples are given to validate the effectiveness of the proposed algorithms.**

***Index Terms*—Continuous control algorithms, coordinated tracking, multiple heterogeneous Lagrange systems.**

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I. Introduction

Coordination of multiagent systems has been extensively studied for the past two decades due to its broad range of applications. One fundamental problem is coordinated tracking with a time-varying global objective [1], [3]. The goal is to control a group of followers to track a time-varying global objective function (often denoted as a leader) by using only local information interactions [23]. The coordinated tracking problem was introduced and studied in [12], where the followers were modeled as single integrators and the input delays were considered. With the emphasis on the delay effect analysis, Meng *et al.* [21] studied the stability conditions for the leader–follower tracking problem for both single integrator networks and double integrator networks. Recently, in [4], algorithms using variable structure approaches are proposed. Both the case of multiple single integrators and that of multiple double integrators were considered, and the tracking errors were shown to be zero using the proposed discontinuous control algorithms.

In this paper, instead of modeling the follower dynamics as single or double integrators, we study the coordinated tracking problem of multiple heterogeneous Lagrange systems with a dynamic leader. Here, a Lagrange system is used to represent a mechanical system, such as autonomous vehicles, robotic manipulators, and walking robots [27]. Therefore, the study on the coordination control of multiple Lagrange systems may provide some basic ideas for the applications on the formation control of multiple mobile robots and the coordinated object grabbing of multiple robot manipulators. Existing works on the coordination control of multiple Lagrange systems include [5]–[8], [11], [13], [14], [17], [20], [22], and [25] with different emphasis. For example, time-varying delays, limited communication rates, and nonvanishing bounded disturbances were considered in [11], coordinated tracking with finite-time convergence was studied in [13], and a class of nonlinear function was introduced in [8] to alleviate the chattering issues raised by the discontinuous coordinated tracking algorithm. The influence of communication delays was studied in [14] and [22], a flocking behavior was guaranteed in [7], and the containment control with group dispersion and group cohesion behaviors was reconstructed in [20]. In addition, the applications of coordination algorithms of multiple Lagrange systems on the shape and formation control were given in [10], and the application to task-space synchronization of multiple robotic manipulators was given in [15].

In this paper, by focusing on the leader–follower coordinated tracking problem of multiple Lagrange systems, we improve the existing works in three aspects. First, the proposed zero-error coordinated tracking algorithm is distributed, continuous, and guaranteeing zero-error tracking. Note that discontinuous control algorithms were proposed in [13], [17], and [20] to ensure zero-error coordinated tracking, the leader is assumed to be available to all the followers in [22], and the tracking errors were shown to be bounded instead of approaching zero in [5], although the proposed algorithms are continuous. Second, in contrast with [17] and [18], where the eigenvalues of the interaction Laplacian matrix and the upper bound of states of the bounded time-varying leader are assumed to be available to all the followers, the proposed algorithm in this paper is purely distributed in the sense that both the control input and coupling gain depend only on local information. Third, the neighbors' generalized coordinate derivative information is not required to be available in the proposed algorithm. Thus, such an approach may provide a solution to the case when the agents are not equipped with the sensors capable of obtaining relative generalized coordinate derivative information (e.g., relative velocity measurements). Moreover, since we do not need the neighbors' generalized coordinate derivative information, the communication capacities may be reduced. This is particularly important when the number of agents is large and when the communication structure is complex.

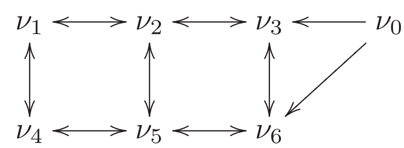

Fig. 1. Information flow associated with the leader and the six followers.

The outline of the paper is as follows. In Section II, we formulate the problem of coordinated tracking of multiple Lagrange systems and give some basic notations and definitions. The main results are presented in Section III. Numerical studies are carried out in Section IV to validate the theoretical results, and a brief concluding remark is given in Section V.

II. Problem Statement and Preliminaries

A. Problem Statement

Suppose that there are n follower agents in the group, labeled by $\nu_1, \nu_2, \ldots, \nu_n$. The system dynamics of the followers are described by the Lagrange equations

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, i = 1, 2, \ldots, n \quad (1)$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the $p \times p$ inertia (symmetric) matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i$ is the Coriolis and centrifugal terms, $g_i(q_i)$ is the vector of gravitational force, and $\tau_i \in \mathbb{R}^p$ is the control force. The dynamics of a Lagrange system satisfies the following properties [27]:

1) $0 < k_{\underline{M}} I_p \le M_i(q_i) \le k_{\overline{M}} I_p$, $\|C_i(x, y)\| \le k_C \|y\|$ for all vectors $x, y \in \mathbb{R}^p$, and $\|g_i(q_i)\| \le k_g$, where $k_{\underline{M}}, k_{\overline{M}}, k_C$, and k_g are positive constants.
2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.
3) The left-hand side of the dynamics can be parameterized, i.e., $M_i(q_i)y + C_i(q_i, \dot{q}_i)x + g_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\theta_i, \forall x, y \in \mathbb{R}^p$, where $Y_i \in \mathbb{R}^{p\times p_\theta}$ is a regression matrix with a constant parameter vector $\theta_i \in \mathbb{R}^{p_\theta}$.

From Property 3, we know that the nominal dynamics (available dynamics) satisfy

$$\widehat{M}_i(q_i)\ddot{q}_i + \widehat{C}_i(q_i, \dot{q}_i)\dot{q}_i + \widehat{g}_i(q_i) = Y_i(q_i, \dot{q}_i, \dot{q}_i, \ddot{q}_i)\widehat{\theta}_i$$

where $\widehat{M}_i(q_i), \widehat{C}_i(q_i, \dot{q}_i), \widehat{g}_i(q_i)$, and $\widehat{\theta}_i$ are nominal dynamics terms.

In addition to the n followers, we denote the global information as a leader agent in the group, labeled as agent ν_0 with the desired time-varying generalized coordinate $q_0 \in \mathbb{R}^p$ and the desired time-varying generalized coordinate derivative $\dot{q}_0 \in \mathbb{R}^p$. The objective of this paper is to design *continuous* coordinated tracking algorithms for follower dynamics (1) such that $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$ by using only local interactions, i.e., the leader's states q_0 and $\dot{q}_0$ are only available to a subset of the followers, and the followers only have access to their local neighbors' information.

Considering that there are six followers ($n = 6$) in the group, Fig. 1 gives an example of information flow among the leader and six followers. Note that the leader's states are only available to followers ν_3 and ν_6, and the followers only have access to their neighbors' information.

B. Basic Definitions in Graph Theory

We use graphs to represent the communication topology among agents. A directed graph $\mathcal{G}_n$ consists of a pair $(\mathcal{V}_n, \mathcal{E}_n)$, where $\mathcal{V}_n = \{\nu_1, \nu_2, \ldots, \nu_n\}$ is a finite, nonempty set of nodes and $\mathcal{E}_n \subseteq \mathcal{V}_n \times \mathcal{V}_n$ is a set of ordered pairs of nodes. An edge (ν_i, ν_j) denotes that node

ν_j has access to the information from node ν_i. An undirected graph is defined such that $(\nu_j, \nu_i) \in \mathcal{E}_n$ implies $(\nu_i, \nu_j) \in \mathcal{E}_n$. A directed path in a directed graph or an undirected path in an undirected graph is a sequence of edges of the form $(\nu_i, \nu_j), (\nu_j, \nu_k), \ldots$. The neighbors of node ν_i are defined as the set $N_i := \{\nu_j | (\nu_j, \nu_i) \in \mathcal{E}_n\}$.

For a follower graph $\mathcal{G}_n$, its adjacency matrix $\mathcal{A}_n = [a_{ij}] \in \mathbb{R}^{n\times n}$ is defined such that a_{ij} is positive if $(\nu_j, \nu_i) \in \mathcal{E}_n$ and $a_{ij} = 0$ otherwise. Here, we assume that $a_{ii} = 0, \forall i = 1, 2, \ldots, n$ and $a_{ij} = a_{ji}, \forall i, j = 1, 2, \ldots, n$. The Laplacian matrix $\mathcal{L}_n = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathcal{A}_n$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$. For the leader–follower graph $\mathcal{G}_{n+1} := (\mathcal{V}_{n+1}, \mathcal{E}_{n+1})$, the adjacency matrix $\mathcal{A}_{n+1} = [a_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$ is defined such that a_{i0} is positive if $(\nu_0, \nu_i) \in \mathcal{E}_{n+1}$ and $a_{i0} = 0$ otherwise, $\forall i = 1, 2, \ldots, n$.

Assumption 1: The global information q_0 and $\dot{q}_0$ are available to at least one follower, *i.e.*, $a_{i0} > 0$ for at least one $i, i = 1, 2, \ldots, n$. In addition, the follower graph $\mathcal{G}_n$ is undirected and connected.

Note that Fig. 1 is an example that satisfies Assumption 1. Letting $\mathcal{M} = \mathcal{L}_n + \mathrm{diag}(a_{10}, a_{20}, \ldots, a_{n0})$ ($\mathcal{L}_n$ is the Laplacian matrix associated with $\mathcal{G}_n$), we recall the following result.

Lemma 1 [12]: Under Assumption 1, $\mathcal{M}$ is positive definite (symmetric).

C. Filippov Solution and Nonsmooth Analysis

Consider the vector differential equation

$$\dot{x} = f(x, t) \tag{2}$$

where $f : \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}^p$ is measurable and essentially locally bounded. A vector function $x(t)$ is called a solution of (2) on $[t_0, t_1]$ if $x(t)$ is absolutely continuous on $[t_0, t_1]$ and for almost all $t \in [t_0, t_1], \dot{x} \in \mathbb{K}[f](x, t)$. Here, $\mathbb{K}[f](x, t) = \bigcap_{\delta>0} \bigcap_{\mu \overline{N}=0} \overline{co} f(B(x, \delta) \setminus \overline{N}, t)$, $\bigcap_{\mu \overline{N}=0}$ denotes the intersection over all sets $\overline{N}$ of Lebesgue measure zero, $\overline{co}(X)$ is the convex closure of X, and $B(x, \delta)$ denotes the open ball of radius δ centered at x.

For a locally Lipschitz function $V : \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}$, the generalized gradient of V at (x, t) is defined by $\partial V(x, t) = \overline{co}\{\lim \nabla V(x, t) | (x_i, t_i) \to (x, t), (x_i, t_i) \notin \Omega_V\}$, where Ω_V is the set of measure zero where the gradient of V is not defined. The generalized time derivative of V with respect to (2) is defined as $\dot{\tilde{V}} := \bigcap_{\zeta\in\partial V} \zeta^{\mathrm{T}} \begin{pmatrix} \mathbb{K}[f](x,t) \\ 1 \end{pmatrix}$. In addition, $f(x, t) : \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}$ is called regular if for all ψ, the usual one-sided directional derivative $f'(x; \psi)$ exists, and $f'(x; \psi) = f^o(x; \psi)$, where $f^o(x; \psi) = \lim_{y\to x, t\downarrow 0} \sup \frac{f(y+t\psi)-f(y)}{t}$ [24], [26].

Lemma 2 [9]: Let (2) be essentially locally bounded and $0 \in \mathbb{K}[f](x, t)$ in a region $\mathbb{R}^p \times [0, \infty)$. Furthermore, suppose that $f(0, t)$ is uniformly bounded for all $t \geq 0$. Let $V : \mathbb{R}^p \times [0, \infty) \to \mathbb{R}$ be locally Lipschitz in t, and regular such that $\forall t \geq 0, W_1(x) \leq V(t, x) \leq W_2(x)$, $\dot{\tilde{V}}(x, t) \leq -W(x)$, where $W_1(x)$ and $W_2(x)$ are continuous positive definite functions, and $W(x)$ is a continuous positive semidefinite function. Here, $\dot{\tilde{V}}(x, t) \leq -W(x)$ means that $\psi \leq -W, \forall \psi \in \dot{\tilde{V}}$. Then, all Filippov solutions of (2) are bounded and satisfy $W(x(t)) \to 0$, as $t \to \infty$.

D. Other Notation

Given a vector $x = [x_1, x_1, \ldots, x_n]^{\mathrm{T}}$, we define $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \mathrm{sgn}(x_2), \ldots, \mathrm{sgn}(x_n)]^{\mathrm{T}}$, and $|x| = [|x_1|, |x_2|, \ldots, |x_n|]^{\mathrm{T}}$. In addition, $\mathrm{diag}(x)$ denotes the diagonal matrix of a vector x, $\|x\|_1 = \sum_{i=1}^n |x_i|$ denotes 1-norm of a vector x, $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote respectively the minimum and maximum eigenvalues of the matrix P, and $P > 0$ and $P \geq 0$ mean that P is positive definite and positive semidefinite, respectively.

III. Main Result

The objective here is to drive the states of the followers to converge to those of the global objective. Note that the global objective is available to only a portion of the followers and we use nominal parameters of Lagrange dynamics. We also assume that the neighbors' generalized coordinate derivative information is not available. The following *continuous* control algorithm is proposed for each follower:

$$\tau_i = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)\widehat{\theta}_i - \alpha_i(t)s_i, \quad i = 1, 2, \ldots, n, \tag{3}$$

where Y_i is defined in Section II-A, and $\dot{\alpha}_i = \overline{\alpha}_i s_i^{\mathrm{T}} s_i$, with $\overline{\alpha}_i > 0, i = 1, 2, \ldots n$ being an arbitrary positive constant. The sliding surface and the adaptive control term are designed by

$$s_i = \dot{q}_i - \dot{q}_{ri} \tag{4}$$

$$\dot{\widehat{\theta}}_i = -\kappa_i Y_i^{\mathrm{T}}(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)s_i \tag{5}$$

where $\kappa_i > 0, i = 1, 2, \ldots n$ is an arbitrary positive constant, and, motivated by [17] and [18], the virtual reference trajectory $\dot{q}_{ri}$ and the leader's generalized coordinate derivative estimator $\widehat{v}_i$ are proposed, respectively, as

$$\dot{q}_{ri} = \widehat{v}_i - \left(\sum_{j=1}^n a_{ij}(q_i - q_j) + a_{i0}(q_i - q_0)\right) \tag{6}$$

$$\dot{\widehat{v}}_i(t) = -2\widehat{v}_i(t) - \int_0^t \left(k_{2i}(\tau)\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau)) + \beta_i(\tau)\mathrm{sgn}\left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau))\right)\right) \mathrm{d}\tau \tag{7}$$

where $\widehat{v}_0(t) = \dot{q}_0(t), a_{ij}, i, j = 1, 2, \ldots n$, is the (i, j)th entry of $\mathcal{A}_n$ associated with $\mathcal{G}_n$ defined in Section II-B, $a_{i0} > 0$ if the follower i has access to the global information ν_0 and $a_{i0} = 0$ otherwise

$$k_{2i}(t) = \frac{1}{2}\overline{k}_{2i}\left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(t) - \widehat{v}_j(t))\right)^{\mathrm{T}}\left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(t) - \widehat{v}_j(t))\right) + \overline{k}_{2i}\int_0^t \left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau))\right)^{\mathrm{T}} \times \left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau))\right)\mathrm{d}\tau \tag{8}$$

and

$$\beta_i(t) = \overline{\beta}_i\left\|\sum_{j=0}^n a_{ij}(\widehat{v}_i(t) - \widehat{v}_j(t))\right\|_1 + \overline{\beta}_i\int_0^t \left\|\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau))\right\|_1 \mathrm{d}\tau \tag{9}$$

with $\overline{k}_{2i} > 0$ and $\overline{\beta}_i > 0, i = 1, 2, \ldots n$ being arbitrary positive constants.

Before moving on, we need the following assumption and lemmas.

Assumption 2: $\dot{q}_0$ is bounded up to its third derivative.

Note that the assumption on that $\dot{q}_0, \ddot{q}_0$ are bounded is a necessary assumption to ensure zero-error tracking of generalized coordinates and generalized coordinate derivatives for the adaptive case. The assumption on $\dddot{q}_0, \ddddot{q}_0$ being bounded is necessary to ensure the convergence for the leader's generalized coordinate derivative estimator. In addition, note that in contrast with [13], [17], [18], and [20], the upper bound on any derivative of q_0 is not assumed to be available in the design of the controllers. Generally speaking, Assumption 2 is a mild assumption.

Lemma 3 [2]: Let S be a symmetric matrix partitioned as $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix}$, where S_{22} is square and nonsingular. Then, $S > 0$ if and only if $S_{22} > 0$ and $S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} > 0$.

Lemma 4 [28]: Define $\xi(t) \in \mathbb{R}$ as $\xi = (\mu + \dot{\mu})^{\mathrm{T}}(-\overline{\beta}\mathrm{sgn}(\mu) + N_d)$, where $\mu(t) \in \mathbb{R}^p, \overline{\beta}$ is a positive constant, and $N_d(t) \in \mathbb{R}^p$ is a bounded disturbance. Then, we have that $\int_0^t \xi(\tau)\mathrm{d}\tau \le \mathcal{B}$, if $\overline{\beta} > \sup_t\{\|N_d(t)\|_\infty + \|\dot{N}_d(t)\|_\infty\}$, where $\mathcal{B} = \overline{\beta}\|\mu(0)\|_1 - \mu^{\mathrm{T}}(0)N_d(0) > 0$.

Theorem 1: Let Assumptions 1 and 2 hold. Under the local continuous coordinated tracking algorithm (3)–(9), the states of the followers governed by the Lagrange dynamics (1) converge to those of the leader, i.e., $\lim_{t\to\infty}(q_i(t) - q_0(t)) = 0$ and $\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0$, $\forall i = 1, 2, \ldots, n$.

Proof: It follows from Property 3 of Lagrange dynamics in Section II-A that $M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)\theta_i - M_i(q_i)\sum_{j=0}^n a_{ij}(\dot{q}_i - \dot{q}_j)$. We then further have that $M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)\triangle\theta_i + M_i(q_i)\sum_{j=0}^n a_{ij}(\dot{q}_i - \dot{q}_j) - \alpha_i s_i$, where $\triangle\theta_i = \widehat{\theta}_i - \theta_i$.

It also follows from (7) that

$$\ddot{\overline{v}}_i = -2\dot{\overline{v}}_i - k_{2i}\left(\sum_{j=1}^n a_{ij}(\overline{v}_i - \overline{v}_j) + a_{i0}\overline{v}_i\right) - \beta_i \mathrm{sgn}\left(\sum_{j=1}^n a_{ij}(\overline{v}_i - \overline{v}_j) + a_{i0}\overline{v}_i\right) + N_{di}$$

where $\overline{v}_i = \widehat{v}_i - \dot{q}_0, N_{di} = -2\ddot{q}_0 - \dddot{q}_0$, for all $i = 1, 2, \ldots, n$. We then have that for $i = 1, 2, \ldots, n$

$$\ddot{\overline{v}}_i(t) = -2\dot{\overline{v}}_i(t) - k_{2i}(t)\left(\sum_{j=1}^n m_{ij}\overline{v}_j(t)\right) - \beta_i(t)\mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j(t)\right) + N_{di}(t) \tag{10}$$

where m_{ij} denotes the (i,j)th entry of $\mathcal{M}$ defined after Assumption 1. Note that the right-hand side of (10) is discontinuous. Because the signum function sgn is measurable and essentially locally bounded, we can rewrite (10) in terms of differential inclusions as

$$\ddot{\overline{v}}_i \in^{a.e.} \mathbb{K}\left[-2\dot{\overline{v}}_i - k_{2i}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right) - \beta_i \mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right) + N_{di}\right] \tag{11}$$

where *a.e.* stands for "almost everywhere," and $\mathbb{K}$ is defined in Section II-C. Define $\eta_i = \overline{v}_i + \dot{\overline{v}}_i$. It also follows from (8) and (9) that for $i = 1, 2, \ldots, n$

$$\dot{k}_{2i} = \overline{k}_{2i}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right)^{\mathrm{T}}\left(\sum_{j=1}^n m_{ij}\eta_j\right) \tag{12}$$

and from the fact that the signum function sgn is measurable and locally essentially bounded

$$\dot{\beta}_i \in^{a.e.} \mathbb{K}\left[\overline{\beta}_i\left(\sum_{j=1}^n m_{ij}\eta_j\right)^{\mathrm{T}} \mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right)\right]. \tag{13}$$

We then construct a Lyapunov function candidate as

$$V = V_0 + \frac{1}{2}\sum_{i=1}^n s_i^T M_i(q_i)s_i + \sum_{i=1}^n \frac{1}{2\kappa_i}(\triangle\theta_i)^{\mathrm{T}}\triangle\theta_i + \frac{1}{2}\overline{q}^{\mathrm{T}}\overline{q} + \frac{1}{2}\eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\eta + \frac{1}{2}\overline{k}\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p)\overline{v} + \sum_{i=1}^n \frac{1}{2\overline{k}_{2i}} \times (k_{2i} - \overline{k})^2 + \sum_{i=1}^n \frac{1}{2\overline{\beta}_i}(\beta_i - \overline{\beta})^2 + \sum_{i=1}^n \frac{1}{2\overline{\alpha}_i}(\alpha_i - \overline{\alpha})^2$$

where

$$V_0 = \sum_{i=1}^n \mathcal{B}_i - \sum_{i=1}^n \int_0^t \left(\sum_{j=1}^n m_{ij}\eta_j(\tau)\right)^{\mathrm{T}} \times \left(-\overline{\beta}\mathrm{sgn}\left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau))\right) + N_{di}(\tau)\right)\mathrm{d}\tau$$

$\eta = [\eta_1^{\mathrm{T}}, \eta_2^{\mathrm{T}}, \ldots, \eta_n^{\mathrm{T}}], \overline{v} = [\overline{v}_1^{\mathrm{T}}, \overline{v}_2^{\mathrm{T}}, \ldots, \overline{v}_n^{\mathrm{T}}]$, $\overline{q}_i = q_i - q_0, \overline{q} = [\overline{q}_1^{\mathrm{T}}, \overline{q}_2^{\mathrm{T}}, \ldots, \overline{q}_n^{\mathrm{T}}]^{\mathrm{T}}, \mathcal{B}_i = \overline{\beta}\|\sum_{j=1}^n m_{ij}\overline{v}_j(0)\|_1 - (\sum_{j=1}^n m_{ij}\overline{v}_j(0))^{\mathrm{T}}N_{di}(0)$. In addition, we select $\overline{\beta}$ and $\overline{k}$ as two positive constants satisfying that $\overline{\beta} > \sup_t\{2\|\dddot{q}_0(t)\|_\infty + 3\|\ddddot{q}_0(t)\|_\infty + \|\dddddot{q}_0(t)\|_\infty\}$ and $\overline{k} > \overline{b} + \frac{1}{4\lambda_{\min}(\mathcal{M})}$ and $\overline{b} > \frac{1}{4\lambda_{\min}^3(\mathcal{M})}$. In addition, $\overline{\alpha}$ is a constant to be determined later. It follows from Lemma 4 that $V_0 > 0$ when $\overline{\beta} > \sup_t\{2\|\dddot{q}_0(t)\|_\infty + 3\|\ddddot{q}_0(t)\|_\infty + \|\dddddot{q}_0(t)\|_\infty\}$. It follows that the generalized time derivative of V (see the definition of $\dot{\tilde{V}}$ in Section II-C) can be evaluated as

$$\dot{\tilde{V}} = \bigcap_{\xi\in\partial\|\mu\|_1} -((\mathcal{M}\otimes I_p)\eta)^{\mathrm{T}}\left(-\overline{\beta}\xi + N_d\right) + \mathbb{K}\Bigg[\sum_{i=1}^n s_i^{\mathrm{T}}\Big(Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)\triangle\theta_i - \alpha_i s_i + M_i(q_i) \times \sum_{j=0}^n a_{ij}(\dot{q}_i - \dot{q}_j)\Big) - \sum_{i=1}^n(\triangle\theta_i)^{\mathrm{T}}Y_i^{\mathrm{T}}(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)s_i + \sum_{i=1}^n\left(\sum_{j=1}^n m_{ij}\eta_j\right)^{\mathrm{T}}\Big(-k_{2i}\sum_{j=1}^n m_{ij}\overline{v}_j - \dot{\overline{v}}_i - \overline{v}_i + \overline{v}_i + N_{di} - \beta_i\mathrm{sgn}\Big(\sum_{j=1}^n m_{ij}\overline{v}_j\Big)\Big) + \overline{k}\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p)(\eta - \overline{v}) + \sum_{i=1}^n(k_{2i} - \overline{k})\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right)^{\mathrm{T}} \times \left(\sum_{j=1}^n m_{ij}\eta_j\right) + \sum_{i=1}^n(\beta_i - \overline{\beta})\left(\sum_{j=1}^n m_{ij}\eta_j\right)^{\mathrm{T}} \times\mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right) + \sum_{i=1}^n(\alpha_i - \overline{\alpha})s_i^{\mathrm{T}}s_i + \overline{q}^{\mathrm{T}}\dot{\overline{q}}\Bigg]$$

$$
\begin{aligned}
&= \bigcap_{\xi\in\partial\|\mu\|_1} -((\mathcal{M}\otimes I_p)\eta)^{\mathrm{T}}\left(-\overline{\beta}\xi + N_d\right)\\
&\quad + ((\mathcal{M}\otimes I_p)\eta)^{\mathrm{T}}\left(-\overline{\beta}\partial\|\mu\|_1 + N_d\right)\\
&\quad + \sum_{i=1}^{n} s_i^{\mathrm{T}}\left(-\alpha_i s_i + M_i(q_i)\sum_{j=0}^{n} a_{ij}(\dot{q}_i - \dot{q}_j)\right)\\
&\quad - \eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\eta + \eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\overline{v} - \overline{k}\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p)\\
&\quad \times \overline{v} + \sum_{i=1}^{n}(\alpha_i - \overline{\alpha})s_i^{\mathrm{T}} s_i + \overline{q}^{\mathrm{T}}(s - (\mathcal{M}\otimes I_p)\overline{q} + \overline{v})
\end{aligned}
$$

where $N_d = [N_{d1}^{\mathrm{T}}, N_{d2}^{\mathrm{T}}, \ldots, N_{dn}^{\mathrm{T}}]^{\mathrm{T}}$

$$
\mu = (\mathcal{M}\otimes I_p)\overline{v}, \partial|\mu_k| = \begin{cases} \{-1\}, \mu_k \in \mathbb{R}^- \\ \{1\}, \mu_k \in \mathbb{R}^+ \\ [-1, 1], \mu_k = 0 \end{cases}
$$

and μ_k is kth entry of μ. In addition, we have used (11)–(13), Property 2 of Lagrange dynamics in Section II-A, and the fact that $\mathbb{K}[f] = \{f\}$ if f is continuous [24].

If $\dot{\tilde{V}} \neq \emptyset$, suppose that $\phi \in \dot{\tilde{V}}$. By following a similar analysis as the one given in the example in [26, Sec. II] and noting that $\bigcap_{\xi_2\in[-1,1]}[\xi_2 - 1, \xi_2 + 1] = 0$, we know that

$$
\begin{aligned}
\phi &= \sum_{i=1}^{n} s_i^{\mathrm{T}}\left(-\alpha_i s_i + M_i(q_i)\sum_{j=0}^{n} a_{ij}(\dot{q}_i - \dot{q}_j)\right)\\
&\quad - \eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\eta + \eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\overline{v} - \overline{k}\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p)\\
&\quad \times \overline{v} + \sum_{i=1}^{n}(\alpha_i - \overline{\alpha})s_i^{\mathrm{T}} s_i + \overline{q}^{\mathrm{T}}(s - (\mathcal{M}\otimes I_p)\overline{q} + \overline{v}).
\end{aligned}
$$

It is clear to see that $\dot{\tilde{V}}$ is a singleton. We then have that

$$
\begin{aligned}
\dot{\tilde{V}} \leq\ & -\overline{\alpha}\sum_{i=1}^{n} s_i^{\mathrm{T}} s_i + s^{\mathrm{T}} M(q)(\mathcal{M}\otimes I_p)s\\
& - s^{\mathrm{T}} M(q)(\mathcal{M}^2\otimes I_p)\overline{q} + s^{\mathrm{T}} M(q)(\mathcal{M}\otimes I_p)\overline{v}\\
& - \overline{q}^{\mathrm{T}}(\mathcal{M}\otimes I_p)\overline{q} + \overline{q}^{\mathrm{T}} s + \overline{q}^{\mathrm{T}}\overline{v} - \overline{b}\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p)\overline{v}\\
& - (\overline{k} - \overline{b})\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p)\overline{v} - \eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\eta\\
& + \eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\overline{v}
\end{aligned}
$$

where $M(q) = \mathrm{diag}(M_1(q_1), M_2(q_2), \ldots, M_n(q_n)), \overline{b} > \frac{1}{4\lambda_{\min}^3(\mathcal{M})}$ is a constant and we have used the fact that $\sum_{j=0}^{n} a_{ij}(\dot{q}_i - \dot{q}_j) = \sum_{j=1}^{n} m_{ij}(\dot{q}_j - \dot{q}_0) = \sum_{j=1}^{n} m_{ij}(\dot{q}_j - \widehat{v}_j + \overline{v}_j) = \sum_{j=1}^{n} m_{ij}(s_j - \sum_{j=1}^{n} m_{ij}\overline{q}_j + \overline{v}_j)$. It then follows that

$$
\begin{aligned}
\dot{\tilde{V}} \leq\ & -[\eta^{\mathrm{T}} \quad \overline{v}^{\mathrm{T}}]\begin{bmatrix} \mathcal{M}\otimes I_p & -\dfrac{\mathcal{M}\otimes I_p}{2} \\ -\dfrac{\mathcal{M}\otimes I_p}{2} & (\overline{k} - \overline{b})\mathcal{M}^2\otimes I_p \end{bmatrix}\begin{bmatrix} \eta \\ \overline{v} \end{bmatrix}\\
& - [s^{\mathrm{T}} \quad \overline{q}^{\mathrm{T}} \quad \overline{v}^{\mathrm{T}}]\,\Omega\begin{bmatrix} s \\ \overline{q} \\ \overline{v} \end{bmatrix}\\
& \stackrel{\triangle}{=} -W(\eta, \overline{q}, \overline{v}, s)
\end{aligned}
$$

where

$$
\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{12}^{\mathrm{T}} & \mathcal{M}\otimes I_p & -\dfrac{1}{2}I_{pn} \\ \Omega_{13}^{\mathrm{T}} & -\dfrac{1}{2}I_{pn} & \overline{b}\mathcal{M}^2\otimes I_p \end{bmatrix}, \Omega_{11} = \overline{\alpha}I_{pn} - \frac{1}{2}\left(M(q)(\mathcal{M}\otimes I_p)\right.
$$

$$
\left. + (\mathcal{M}\otimes I_p)M(q)\right), \Omega_{12} = \frac{M(q)(\mathcal{M}^2\otimes I_p)}{2} - \frac{I_{pn}}{2}
$$

and $\Omega_{13} = -\frac{M(q)(\mathcal{M}\otimes I_p)}{2}$.

Note that $\overline{b} > \frac{1}{4\lambda_{\min}^3(\mathcal{M})}$ guarantees that $\begin{bmatrix} \mathcal{M}\otimes I_p & -\frac{1}{2}I_{pn} \\ -\frac{1}{2}I_{pn} & \overline{b}\mathcal{M}^2\otimes I_p \end{bmatrix}$ is positive definite. Then, it follows that Ω is positive definite from Lemma 3 when $\overline{\alpha}$ is chosen large enough satisfying $\overline{\alpha} > k_{\overline{M}}\overline{\lambda} + \frac{(1+k_{\overline{M}}\overline{\lambda}^2)^2\overline{b}\underline{\lambda}^2 + k_{\overline{M}}\overline{\lambda}(1+k_{\overline{M}}\overline{\lambda}^2) + (k_{\overline{M}}\overline{\lambda})^2\underline{\lambda}}{4\overline{b}\underline{\lambda}^3 - 1}$, where $\overline{\lambda}$ and $\underline{\lambda}$ denote, respectively, $\lambda_{\max}(\mathcal{M})$ and $\lambda_{\min}(\mathcal{M})$. Therefore, $W(\eta, \overline{q}, \overline{v}, s) \geq 0$ when $\overline{k} > \overline{b} + \frac{1}{4\lambda_{\min}(\mathcal{M})}$. It follows that $\int_0^t W(\eta(\tau), \overline{q}(\tau), \overline{v}(\tau), s(\tau))\mathrm{d}\tau$ is bounded. Thus, we know that V is bounded and therefore $s_i, \triangle\theta_i, \forall i = 1, 2, \ldots, n, \overline{v}, \dot{\overline{v}}, \eta$, and $\overline{q}$ are bounded. It then follows that $\dot{q}_{ri}, \forall i = 1, 2, \ldots, n$ are bounded from (6) and the facts that $\overline{v}_i, \forall i = 1, 2, \ldots, n, \dot{q}_0, \overline{q}$ are bounded and $\mathcal{M}$ is positive definite. This in turn shows that $\dot{q}_i, \forall i = 1, 2, \ldots, n$, are bounded from (4). This further implies that $\ddot{q}_{ri}, \forall i = 1, 2, \ldots, n$ are bounded since $\dot{\overline{v}}_i, \forall i = 1, 2, \ldots, n$, and $\ddot{q}_0$ are bounded. In addition, based on the first property of Lagrange dynamics given in Section II-A and the relationship of $M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)\theta_i - M_i(q_i)\sum_{j=0}^{n} a_{ij}(\dot{q}_i - \dot{q}_j), \forall i = 1, 2, \ldots, n$, we know that $Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)$ is bounded, $\forall i = 1, 2, \ldots, n$. It therefore shows that $\dot{s}_i, \forall i = 1, 2, \ldots, n$ are bounded. In addition, we know that $\dot{\eta}_i, \forall i = 1, 2, \ldots, n$ are bounded based on the fact that $\dddot{q}_0$ is bounded and (11). We then know that $s_i(t), \eta_i(t), \overline{q}_i(t)$, and $\overline{v}_i(t), \forall i = 1, 2, \ldots, n$ are uniformly continuous in t. This shows that $W(\eta(t), \overline{q}(t), \overline{v}(t), s(t))$ is uniformly continuous in t. Therefore, it follows from Lemma 2 that $W(\eta(t), \overline{q}(t), \overline{v}(t), s(t)) \to 0$, as $t \to \infty$. This shows that $\eta(t) \to 0, \overline{v}(t) \to 0, \overline{q}(t) \to 0$, and $s(t) \to 0$, as $t \to \infty$. It follows from (4) and (6) that $\dot{\overline{q}}_i = -\sum_{j=1}^{n} m_{ij}\overline{q}_j + s_i + \overline{v}_i$. We can then easily have that $\lim_{t\to\infty}(q_i(t) - q_0(t)) = 0$ and $\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0, \forall i = 1, 2, \ldots, n$. ■

Remark 1: The proposed algorithm possesses the following attributes. First, it is distributed, i.e., the leader's information is available to only a portion of the followers and the followers only have local interactions. This is a rather mild communication topology assumption compared with those in existing works, such as [6] and [22], where the leader's information is assumed to be available to all the followers. Second, the proposed algorithm is continuous and the tracking errors are shown to converge to zero, even when the leader's generalized coordinate derivative is time-varying. This improves the ultimate boundedness results reported in [5] and avoids the chattering phenomenon in the discontinuous designs of [13], [17], and [20]. Third, by introducing an adaptive gain scheduling technique, the coupling gain no longer relies on a certain bound relevant to the global information and the exact value of the upper bound of states of the time-varying leader is not required to be available. Therefore, in contrast with [17] and [18], the proposed algorithm is purely distributed in the sense that both the control input and the coupling gains depend only on the local information interactions and is feasible as long as that the leader's generalized coordinate derivative is bounded up to its third derivative. Fourth, the neighbors' generalized coordinate derivative information is not required to be available. This reduces the communication as the relative velocity measurements do not need to be exchanged between neighbors.

Remark 2: Coordination algorithms without using neighbors' generalized coordinate derivative information were proposed in [16] for static leader–follower regulation and leaderless synchronization of multiple Lagrange systems. One necessary assumption of [16] is that the target generalized coordinate derivative is constant. In contrast, the proposed algorithm (3)–(9) in this paper can be applied to the case when the leader's generalized coordinate derivative is time-varying.

IV. Simulation Results

In this section, numerical simulation results are given to validate the effectiveness of the theoretical results obtained in this paper. We assume that there exist six followers ($n = 6$) in the group. The system dynamics of the followers are given by the Lagrange dynamics of the two-link manipulators [17], [27]

$$\begin{bmatrix} M_{11,i} & M_{12,i} \\ M_{21,i} & M_{22,i} \end{bmatrix} \begin{bmatrix} \ddot{q}_{ix} \\ \ddot{q}_{iy} \end{bmatrix} + \begin{bmatrix} C_{11,i} & C_{12,i} \\ C_{21,i} & C_{22,i} \end{bmatrix} \begin{bmatrix} \dot{q}_{ix} \\ \dot{q}_{iy} \end{bmatrix} + \begin{bmatrix} g_{1,i} \\ g_{2,i} \end{bmatrix} = \begin{bmatrix} \tau_{ix} \\ \tau_{iy} \end{bmatrix}, i = 1, 2, \ldots, 6$$

where $M_{11,i} = \theta_{1i} + 2\theta_{2i} \cos q_{iy}, M_{12,i} = M_{21,i} = \theta_{3i} + \theta_{2i} \cos q_{iy}$, $M_{22,i} = \theta_{3i}, C_{11,i} = -\theta_{2i} \sin q_{iy} \dot{q}_{iy}, C_{12,i} = -\theta_{2i} \sin q_{iy} (\dot{q}_{ix} + \dot{q}_{iy})$, $C_{21,i} = \theta_{2i} \sin q_{iy} \dot{q}_{ix}, C_{22,i} = 0, g_{1,i} = \theta_{4i} g \cos q_{ix} + \theta_{5i} g \cos(q_{ix} + q_{iy}), g_{2,i} = \theta_{5i} g \cos(q_{ix} + q_{iy})$, and $g = 9.8$. In addition, $\theta_{1i} = m_{1i} l_{c1,i}^2 + m_{2i}(l_{1i}^2 + l_{c2,i}^2) + J_{1i} + J_{2i}, \theta_{2i} = m_{2i} l_{1i} l_{c2,i}, \theta_{3i} = m_{2i} l_{c2,i}^2 + J_{2i}, \theta_{4i} = m_{1i} l_{c1,i} + m_{2i} l_{1i}, \theta_{5i} = m_{2i} l_{2i}$. We choose $m_{1i} = 1 + 0.3i, m_{2i} = 1.5 + 0.3i, l_{li} = 0.2 + 0.06i, l_{2i} = 0.3 + 0.06i$, $l_{c1,i} = 0.1 + 0.03i, l_{c2,i} = 0.15 + 0.03i, J_{1i} = \frac{m_{1i} l_{li}^2}{12}, J_{2i} = \frac{m_{2i} l_{2i}^2}{12}$, $i = 1, 2, \ldots, 6$. According to property 3 of Lagrange dynamics given in Section II-A, the dynamics of the followers can be parameterized as $Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) = [y_{pq}]_i \in \mathbb{R}^{2\times5}$ [27].

The initial states of the followers are given by $q_{ix}(0) = 0.6i$, $q_{iy}(0) = 0.4i - 1, \dot{q}_{ix}(0) = 0.05i - 0.2, \dot{q}_{iy}(0) = -0.05i + 0.2, i = 1, 2, \ldots, 6$. The leader–follower communication topology is given in Fig. 1. The adjacency matrix $\mathcal{A}_n$ of the generalized coordinate derivatives associated with $\mathcal{G}_n$ is chosen to be

$$\mathcal{A}_n = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

and $a_{10} = 0, a_{20} = 0, a_{30} = 1, a_{40} = 0, a_{50} = 0, a_{60} = 1$. The initial estimations for $\theta_{1i}, \theta_{2i}, \theta_{3i}, \theta_{4i}$, and θ_{5i} for each follower $i = 1, 2, \ldots, 6$ are given by $\widehat{\theta}_{1i}(0) = 0, \widehat{\theta}_{2i}(0) = 0, \widehat{\theta}_{3i}(0) = 0, \widehat{\theta}_{4i}(0) = 0$, and $\widehat{\theta}_{5i}(0) = 0$.

For the case of coordinated tracking without using neighbors' generalized coordinate derivative information [algorithms (3)–(9)], the trajectories of the leader are given by $q_{0x}(t) = \cos(\frac{\pi}{15}t)$ and $q_{0y}(t) = \sin(\frac{\pi}{15}t)$. The constant control parameters are chosen by $\kappa_i = 2, \alpha_i = 1, \overline{k}_{2i} = 0.001$, and $\overline{\beta}_i = 0.1, \forall i = 1, 2, \ldots, 6$. The initial states of k_{2i} and β_i for each follower $i = 1, 2, \ldots, 6$ are given by $k_{2i}(0) = 0$ and $\beta_i(0) = 0$. The initial states of $\widehat{v}_i$ for each follower $i = 1, 2, \ldots, 6$ are given by $\widehat{v}_i(0) = \dot{\widehat{v}}_i(0) = [0, 0]^{\mathrm{T}}$ and the initial states of α_i for each follower $i = 1, 2, \ldots, 6$ are given by $\alpha_i(0) = 0$. The control parameters are chosen by $\overline{\alpha}_i = 1, \forall i = 1, 2, \ldots, 6$. Under the feedback algorithm (3)–(9), the generalized coordinates, the generalized coordinate derivatives, and the control torques of the followers and the leader are shown in Fig. 2(a) and (b). We see that the coordinated tracking is achieved for a group of heterogeneous Lagrange systems without using neighbors' generalized coordinate derivative information.

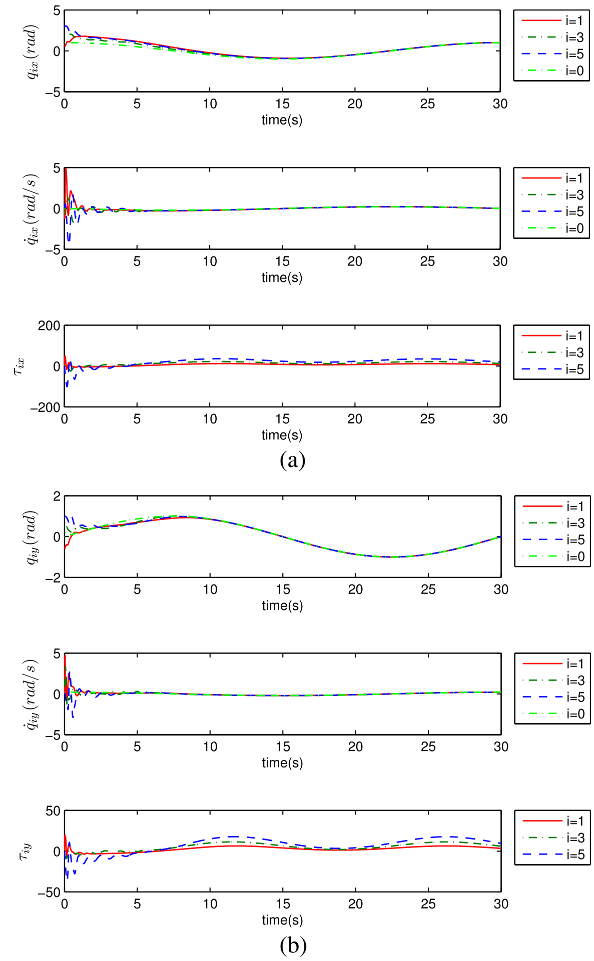

Fig. 2. States and the control torques of system (1) under algorithm (3)-(9). (a) Trajectories of the states and the control torques of the followers and the leader in x-coordinate. (b) Trajectories of states and the control torques of the followers and the leader in y-coordinate.

V. Concluding Remarks

In this paper, we study the leader–follower coordinated tracking problem for multiple heterogeneous Lagrange systems. The continuous coordinated tracking algorithms with uncertain parameter adaptive control and the leader's generalized coordinate derivative estimator are proposed. Except for benefit of the chattering-free control, the proposed algorithm also has the attribute that does not require the neighbors' generalized coordinate derivatives. Global asymptotic coordinated tracking is guaranteed and the tracking errors between the followers and the leader are shown to converge to zero. Simulations are given to validate the effectiveness of the proposed *continuous* coordinated tracking algorithms. Further directions include the study of directed communication topology and an arbitrary varying leader for the leader–follower coordinated tracking problems of multiple Lagrange systems.

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Stability Analysis of a Hierarchical Architecture for Discrete-Time Sensor-Based Control of Robotic Systems

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***Abstract*—The stability of discrete time kinematic sensor-based control of robots is investigated in this paper. A hierarchical inner-loop/outer-loop control architecture common for a generic robotic system is considered. The inner loop is composed of a servo-level joint controller and higher level kinematic feedback is performed in the outer loop. Stability results derived in this paper are of interest in several applications including visual servoing problems, redundancy control, and coordination/synchronization problems. The stability of the overall system is investigated taking into account input/output delays and the inner loop dynamics. A necessary and sufficient condition that the gain of the outer feedback loop has to satisfy to ensure local stability is derived. Experiments on a Kuka K-R16 manipulator have been performed in order to validate the theoretical findings on a real robotic system and show their practical relevance.**

***Index Terms*—Calibration and identification, discrete-time stability, kinematics, output feedback control, redundant robots, velocity control.**

I. INTRODUCTION

Industrial robot manipulators have mainly been applied in highly tailored situations, where preprogrammed motions are sufficient for task completion. As the industry is looking to extend the use of manipulators to unstructured environments, pure motion control is no longer viable and sensor-based control must be introduced.

Feedback for motion control of robot manipulators in the control literature is usually considered in the continuous time framework, assuming direct torque input [1, Ch. 6]. These assumptions may hold for some model research platforms, whereas control interfaces of most

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