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On Some Communication Schemes for Distributed Pursuit–Evasion Games

Alberto Speranzon, Karl Henrik Johansson
Department of Signals, Sensors and Systems
Royal Institute of Technology
Osquldas väg 10
100 44 Stockholm, Sweden
{albspe,kallej}@s3.kth.se

***Abstract*— A probabilistic pursuit–evasion game from the literature is used as an example to study constrained communication in multi-robot systems. Communication protocols based on time-triggered and event-triggered synchronization schemes are considered. It is shown that by limiting the communication to events when the probabilistic map updated by the individual pursuer contains new information, as measured through a map entropy, the utilization of the communication link can be considerably improved compared to conventional time-triggered communication.**

I. INTRODUCTION

Multi-robot systems have many advantages compared to single-robot systems, including improved flexibility, sensing, and reliability. For most mobile robot systems, one needs to address challenges related to sensor noise, self-localization, and partial knowledge of the environment. For a multi-robot system, the inter-robot communication adds to this list. In practice, every communication channel has a limited bandwidth, which is due to the physical laws on the achievable data rate and to that channels might be shared with other users. The performance of the multi-robot system is often highly dependent on the utilization of the communication network. Still, integrated design of communication protocols has so far gained little attention in the literature of multi-robot systems, cf., [1], [2].

The main contribution of this paper is to illustrate how information theory [3], [4] can be used in the design of a multi-robot system, in order to optimize the communication utilization with respect to a control performance. We let a pursuit–evasion game [5], [6], [7], [8] with several pursuers serve as a prototype system, since it is a good representative for many multi-robot tasks. In particular we consider a probabilistic approach for pursuit–evasion where each pursuer build a probabilistic map of the environment [7], [9]. Map entropy is used as an information measure of the probabilistic map. It establishes an event-triggered communication scheme for the pursuers, which is compared with a time-triggered scheme with periodic communications. The considered multi-robot problem can be viewed as a realistic benchmark problem for the design of integrated control and communication systems. Recent work in that area has focused mainly on stabilizability under limited communication, e.g., [10], [11]. For a stochastic control system, the advantage of event-triggered control compared to time-triggered was discussed by Åström and Bernhardsson [12]. For distributed real-time systems, design specifications sometimes lead to that a time-triggered scheme is instead preferable, as advocated by Kopetz [13].

The paper is organized as follow. In Section II we extend the pursuit–evasion model of Hespanha et al. [7] by introducing an explicit broadcasting communication protocol for the pursuer, in which they synchronize their probabilistic maps by broadcasting the current map to each other. Two particular communication schemes are discussed in Section III: time-triggered and event-triggered. The synchronization events are in the latter based on the probabilistic map entropy. In Section IV quantization is utilized to cope with bandwidth limitations. It is shown that the map entropy can be used to quantize the probabilistic map in an efficient way. Simulation results are presented in Section V.

II. PURSUIT–EVASION WITH COMMUNICATION

We consider a pursuit–evasion game with $n_p > 1$ pursuers, $P_1, \dots, P_{n_p}$, and one randomly moving evader. Following Hespanha et al. [7], we suppose that the game is played in a finite-dimensional square space, uniformly partitioned in $n_c^2 < \infty$ cells denoted

$$\mathcal{X} = \{1, 2, \dots, n_c\} \times \{1, 2, \dots, n_c\}$$

Each cell can be occupied by the evader, a pursuer or an obstacle. Neither the evader nor the pursuers can occupy a cell with an obstacle, although the evader and a pursuer can share a cell. The latter corresponds to a capture of the evader. We assume discrete time $t \in \mathcal{T} = \{0, 1, 2, \dots\}$. The motions of the pursuers and the evader are modelled as a controlled Markov chain, see [8] for details. Pursuer P_i, $i = 1, \dots, n_p$, senses at each time instance $t \in \mathcal{T}$ the triple

$$\mathbf{z}_i(t) = \{\mathbf{s}_i(t), \mathbf{o}_i(t), \mathbf{e}_i(t)\}$$

where $\mathbf{s}_i(t) \in \mathcal{X}$ is the position of the pursuer, $\mathbf{o}_i(t) \subset \mathcal{X}$ is a measurement of the obstacle locations sensed by the pursuer, and $\mathbf{e}_i(t) \in \mathcal{X}$ is the corresponding measurement of the evader[1]. We assume that all sensors (detecting position, obstacles, and evader) are ideal, and thus are not affected by measurement noise etc. The measurement space is denoted $\mathcal{Z} = \mathcal{X} \times 2^{\mathcal{X}} \times \mathcal{X}$, where $2^{\mathcal{X}}$ is the power set of $\mathcal{X}$.

[1]Boldface indicates a random variable and the normal typeface its realization.

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A. Synchronization

We extend the pursuit–evasion model of Hespanha et al. by introducing limited pursuer communication. Pursuers gather individual sensor information, make local decisions and communicate at synchronization time instances $\tau \in \mathcal{T}$.

Definition 2.1: A synchronization is a complete broadcasting communication in which all pursuers exchange information with each other. Denote the data received by pursuer P_i

$$\mathbf{Y}^i(\tau) = \left\{\mathbf{y}^j(\tau)\right\}_{j \neq i}$$

where $\mathbf{y}^j(\tau)$ is the data transmitted by P_i.

Transmitted data Y^i is in this paper a probabilistic map, as introduced in next section. A synchronization is depicted in Figure 1: the network is simultaneously accessed by all pursuers when a synchronization is performed. In the paper we consider two types of synchronization: time-triggered and event-triggered.

Definition 2.2: A time-triggered synchronization is a synchronization that occurs at a time $\tau_{\mathrm{t}} \in \{\Delta, 2\Delta, \dots\}$, where $\Delta \in \mathcal{T}$ is the synchronization period. An event-triggered synchronization occurs at a time $\tau_{\mathrm{e}} \in \mathcal{T}$, at which a pre-specified event takes place.

B. Probabilistic Map

The probabilistic map for a pursuer is the probability mass function for the position of the evader conditioned on the available data up to time t.

Definition 2.3: An element of the probabilistic map of pursuer P_i is given by

$$\tilde{p}^i_{t+1}(x_e, Z_t) = P(\mathbf{x}_e(t+1) = x_e | \mathbf{Z}^i_t = Z_t) \qquad (1)$$

where $\mathbf{Z}^i_t \in \{\mathbf{z}_i(0), \dots, \mathbf{z}_i(t)\}$, is the sequence of measurements taken by pursuer P_i up to time t and $\mathbf{x}_e(t) \in \mathcal{X}$ is the position of the evader at time t.

The probabilistic map is a square matrix, where each element is given by equation (1). The map is updated through a two-step algorithm: a measurement step in which $P(\mathbf{x}_e(t) = x_e | \mathbf{Z}^i_t = Z_t)$ is computed using the current measurements, and a prediction step in which $\tilde{p}^i_{t+1}(x_e, Z_t)$ is computed using an evader motion model, see [7] for details. The game starts with an a-priori probabilistic map $\tilde{p}^i_{0|-1}(x_e, \emptyset)$ that we assume to be the uniform distribution.

C. Control Policy

Let $\mathbf{u}_i(t)$ denote the control action of P_i, which gives the position of P_i at time $t+1$. We consider greedy control policies with constrained motion for the pursuer, i.e.,

$$\mathbf{u}_i(t) = \arg \max_{v \in \mathcal{N}(s_i)} p^i_{t+1}(v, Z_t, Y_t) \qquad (2)$$

where $\mathcal{N}(s_i)$ are all neighboring cells of the current position s_i of P_i. Thus, at t the control policy moves P_i to a neighboring cell v, which maximizes the conditional

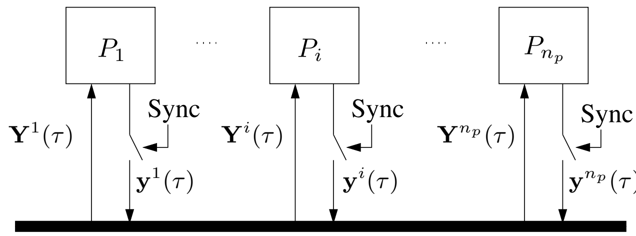


Fig. 1. At a synchronization time $\tau \in \mathcal{T}$, each pursuer P_i broadcasts $\mathbf{y}^i(\tau)$ to the network and receives $\mathbf{Y}^i(\tau) = \{\mathbf{y}^j(\tau)\}_{j \neq i}$.

probability of finding the evader at $t+1$. The greedy policy does not maximize the local probabilistic map $\tilde{p}^i_{t+1}$, but the probabilistic map p^i_{t+1} which we let depend also on the probabilistic maps received through the network; hence the greedy policy (2) depends on the local measurement Z_t and communicated data Y_t. The fusion of probabilistic maps at a synchronization time is computed as a normalized product of all maps, i.e., as an independent opinion pool [14]. This is based on an assumption that the pursuers are far from each other between consecutive synchronization instances. Measurements are thus considered to be approximately independent, cf., [15].

III. Entropy-triggered synchronization

In this section we introduce an event-triggered synchronization scheme based on the map entropy. We use the notation

$$m^i_x(t) = p^i_{t+1}(x, Z_t, Y_t)$$

for element $x = (k, \ell) \in \mathcal{X}$ of the probabilistic map of P_i and we let $M^i(t)$ denote the corresponding matrix. Inspired by information theory [4] we make the following definitions.

Definition 3.4: The map entropy H of a probabilistic map M is

$$H(M) = -\frac{1}{2 \log n_c} \sum_{x \in \mathcal{X}} m_x \, \log m_x$$

Definition 3.5: The relative map entropy D of two probabilistic maps M and N is

$$D(M||N) = \sum_{x \in \mathcal{X}} m_x \, \log \frac{m_x}{n_x}$$

In particular, the relative map entropy between the probabilistic map M and the uniform probabilistic map $N = n_c^{-2} \, \mathbb{1}_{n_c \times n_c}$ is given by

$$D(M||N) = \frac{1 - H(M)}{2 \log n_c}$$

In the following we consider two event-triggered synchronization schemes: one based on the map entropy and the other based on the relative map entropy.

A. Synchronization based on dynamic threshold

Let a synchronization event be triggered whenever $H(M^i(t)) < \lambda(t)$, where $\lambda(t)$ is the synchronization threshold. The threshold is updated according to

$$\lambda(t+1) = \begin{cases} \alpha\,\lambda(\tau_k), & \text{if } t+1 = \tau_k; \\ \lambda(t), & \text{otherwise.} \end{cases}$$
$$\text{with } \lambda(0) = \lambda_0 > 0$$

with τ_k being the time for synchronization k and $0 < \alpha < 1$. The decreasing dynamic threshold is natural since the map entropy is in most cases decreasing with time.

B. Synchronization based on relative map entropy

Let us consider a synchronization scheme based on the relative map entropy D that triggers a synchronization event whenever the local probabilistic map differs sufficiently much from the previously broadcasted map. Synchronization k is carried out when

$$D(M(t)||M(\tau_{k-1}) > \xi \qquad \tau_{k-1} < t$$

i.e., a synchronization is performed when the relative entropy between the current probabilistic map and the last synchronized probabilistic map is larger than a positive constant ξ.

C. Discussion

The idea behind event-triggered synchronization based on the map entropy is that the map entropy should reflect the amount of information in the probabilistic map. It is then natural to expect that the map entropy decreases as a pursuer moves around gathering more and more information about the environment. Simulations show that this is often the case, in particular in the earlier part of a game. However, it is easy to construct counter examples in which the map entropy increases at one or more times before the evader is captured; thus in general $H(M^i(t))$ is not a decreasing function. On the other hand, for a static evader it is straightforward to show that $H(M^i(t))$ is a decreasing function. This follows from that at each step the number of zero elements $z(t)$ of $M^i(t)$ is non-decreasing, while all other elements are equal to $1/(1-z(t))$.

IV. Bandwidth limitations

In order to cope with communication bandwidth limitations, it is natural to send only the part of the probabilistic map M that contains most of the information[2]. The idea is to transform the map M into a new map K, denoted reproduction probabilistic map. The map K should contain almost all information in M but should require less bits to be encoded. We consider a vector quantization

$$Q : \mathbb{R}^{n_c \times n_c} \to \mathbb{R}^{n_c \times n_c} : M \mapsto K = Q(M)$$

[2]In this section, the pursuer index i and the time dependence are suppressed.

where Q defines a complete partition of the matrix M into square sub-matrices $\mathcal{M}_1, \ldots, \mathcal{M}_N$ of order $n_1, \ldots, n_N$ such that $\sum_{i=1}^{N} n_i = n_c$. The reproduction probabilistic map K is block partitioned correspondingly into $\mathcal{K}_1, \ldots, \mathcal{K}_N$ with

$$\mathcal{K}_i = \frac{1}{n_i^2}\,\mathbb{1}_{n_i \times 1}\mathcal{M}_i\mathbb{1}_{1 \times n_i}\mathbb{1}_{n_i \times n_i} \tag{3}$$

Each element of $\mathcal{K}_i$ is thus given by the average of the elements of $\mathcal{M}_i$. Associated with the quantization Q, we define a distortion measure

$$d(M,K) = |H(M) - H(K)|$$

The choice of granularity in the block partition, i.e., the order of the sub-matrices, should be chosen such that $d(M,K)$ is small. This corresponds to a small loss of information in the quantization. Trade-off between quantization granularity and distortion is treated by the rate distortion theory [16]. An analytical characterization of this trade-off seems to be hard to obtain in our case. We therefore consider two heuristic approaches.

A. Uniform quantization

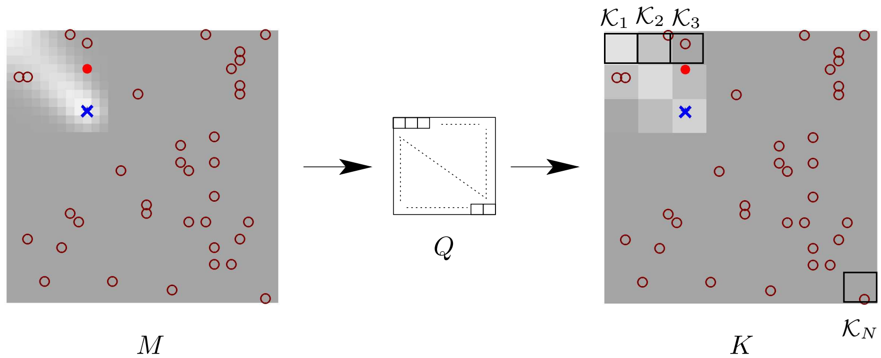
For uniform quantization, the block partition of Q is such that the order of all blocks are equal, $n_i = n$. An illustrative example is shown in Figure 2(a). In this case the number of elements of K is equal to n_c^2/n^2, while the number of elements of M is n_c^2.

B. Non-uniform quantization

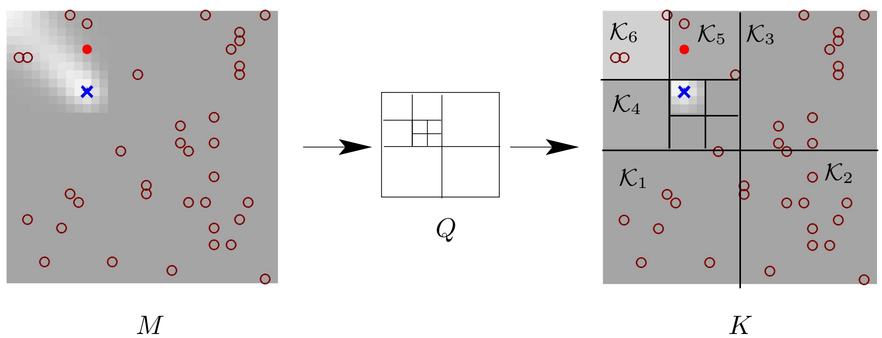
A possible non-uniform quantization is illustrated in Figure 2(b). This corresponds to a "divide-and-conquer" scheme, which is known as vector quantization with QuadTree map [17]. The partition $\mathcal{M}_1, \ldots, \mathcal{M}_N$ imposed by the quantization Q is in this case carried out recursively, such that

$$\dim \mathcal{M}_1 = \frac{1}{4}\dim M$$
$$\dim \mathcal{M}_{i+1} = \begin{cases} \dim \mathcal{M}_i, & \text{if } i \bmod 3 = 0; \\ \frac{1}{4}\dim \mathcal{M}_i, & \text{otherwise} \end{cases} \tag{4}$$

In each recursion step, the current block is divided into four sub-matrices. Three of them are quantized using (3), while the remaining sub-block is partitioned into four sub-blocks, and so on. The recursion stops when the smallest block has reached a preassigned dimension n. Compared with uniform quantization, one advantage of the non-uniform quantization is the possibility of an on-line termination of the quantization if the loss of information is too high, i.e., if the distortion measure $d(M,K)$ is too large. Solving the recursion (4), we find that the number of values to transmit is in the order of $\log n_c^2 + n^2$.



(a) Uniform quantization



(b) Non-uniform quantization

Fig. 2. Vector quantization $K = Q(M)$ of probabilistic map M.

V. Simulation Results

Sets of hundred Monte Carlo simulations have been performed in order to evaluate the proposed synchronization and quantization strategies. The capture time T^* and the number of synchronization instances S are used as performance indices. Figures 3 and 4 show the results for a game with two pursuers and one evader on a grid with $n_c^2 = 24^2$ cells. Three different synchronization schemes are compared: time-triggered, event–triggered based on dynamic threshold, and event-triggered based on relative map entropy. The time-triggered synchronization has a synchronization period of $\Delta = 20$. We see in Figure 3 that the capture time T^* is varying considerably over the set of experiments. The mean capture time $\overline{T}^*$ is similar for all synchronization schemes as indicated by the dashed lines (dashed-dotted lines indicate the standard deviations). The result is collected in the following table:

Synchronization schemes	$\overline{T}^*$	$\overline{S}$
Time-triggered	68	3.9
Event-triggered dynamic threshold	64	2.1
Event-triggered relative map entropy	66	2.6

Note that the mean number of synchronization times $\overline{S}$ is much smaller for the event-triggered schemes than for the time-triggered, while the average capture time $\overline{T}^*$ is about the same. Hence, event-triggered synchronization allows a more efficient utilization of the communication channel. This fact is also illustrated in Figure 4. The main difference between the two event-triggered schemes is mainly the distribution of synchronization times. We see that when using relative map entropy the pursuers tend to communicate more regularly. This is due to that new information is available quite regularly for the pursuers and this information triggers the synchronization events in this scheme. In Figure 5 the uniform and non-uniform quantization strategies are compared. The results are for a game with two pursuers and one evader on an environment that consists of $n_c^2 = 32^2$ cells. The synchronization is time-triggered with period $\Delta = 20$. The quantization map Q has been chosen so that the dimension of the sub-matrices $\mathcal{K}_i$ is $n^2 = 8^2$. Figure 5 shows T^* for the following cases: no quantization ($n = 1$), uniform quantization with $n = 8$ and non-uniform quantization with $n = 8$. The experimental results are collected in the following table:

Quantization	$\overline{T}^*$	$\overline{d}(M, K)$	$\overline{V}$
Uniform ($n = 1$)	91	0.	1024
Uniform ($n = 8$)	141	0.1	16
Non-uniform ($n = 8$)	120	0.04	70

Here $\overline{d}(M, K)$ denotes the average distortion over all

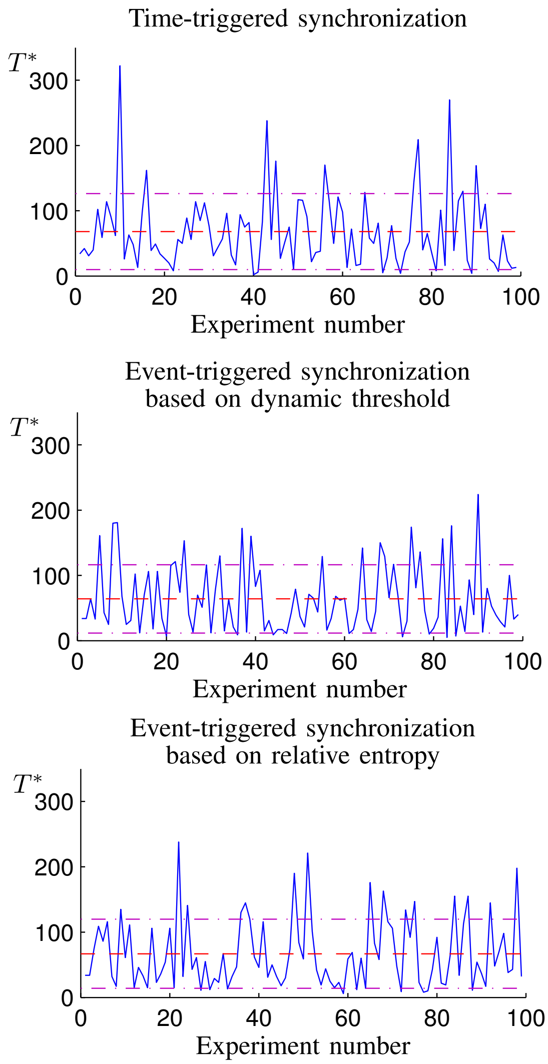


Fig. 3. Capture time T^* for hundred Monte Carlo experiments and the three proposed synchronization schemes. The dashed line is the mean capture times $\overline{T}^*$ and the dashed-dotted is line the standard deviations. The map size is $n_c^2 = 24^2$.

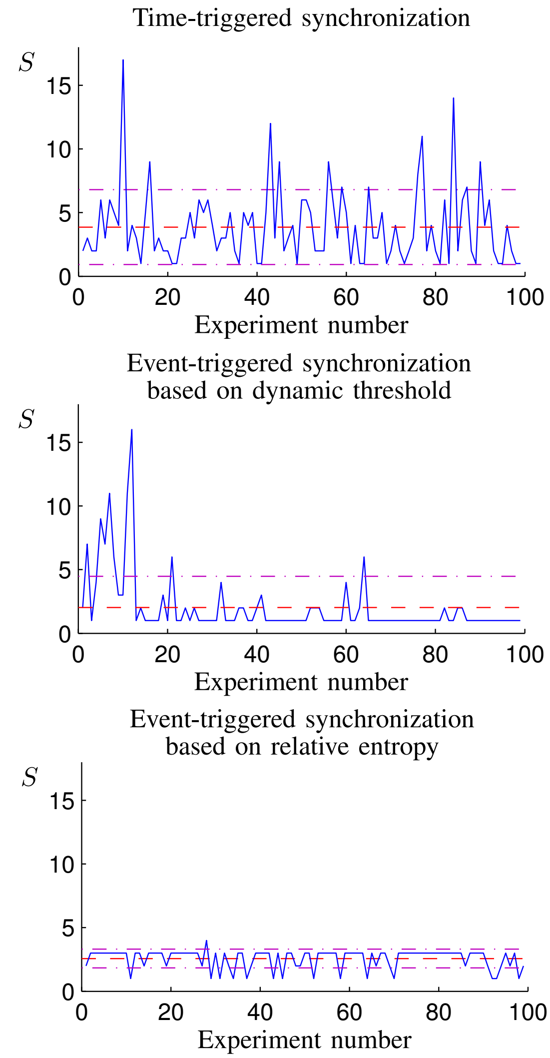


Fig. 4. Number of synchronization instances S for the same hundred Monte Carlo experiments as in Figure 3.

experiments and $\overline{V}$ the average number of broadcasted matrix elements at each synchronization. Note that $\overline{V}$ is one to two magnitudes smaller for the quantized cases compared to the non-quantized case. Still the mean capture time is only about 50% larger. The uniform quantization compared with the non-uniform quantization has quite high average distortion $\overline{d}(M, K)$. This implies a relevant loss of information that makes $\overline{T}^*$ larger in this case. On the other hand the average number of transmitted data $\overline{V}$ is considerably reduced.

VI. Conclusions and Future Work

In this paper, we have presented communication protocols based on time-triggered and event-triggered synchronization for a distributed pursuit–evasion game. The event-triggered schemes were based on the entropy of the probabilistic map. Simulations showed that by limiting the communication to certain events, the utilization of the communication link can be considerably improved compared to conventional time-triggered communication with uniformly distributed synchronization times. Two vector quantization maps were considered in order to cope with bandwidth limitations. A distortion measure based on the map entropy was introduced to evaluate the compression of the probabilistic map. The communication schemes developed in the paper can be applied in cases when a probabilistic map has to be sent through a network channel to a decision maker. This is common in mobile robotics: examples include localization of robots using occupancy grids [18], [2]. A related problem of our current interest is optimal localization of mobile sensors, which share a bandwidth limited communication channel.

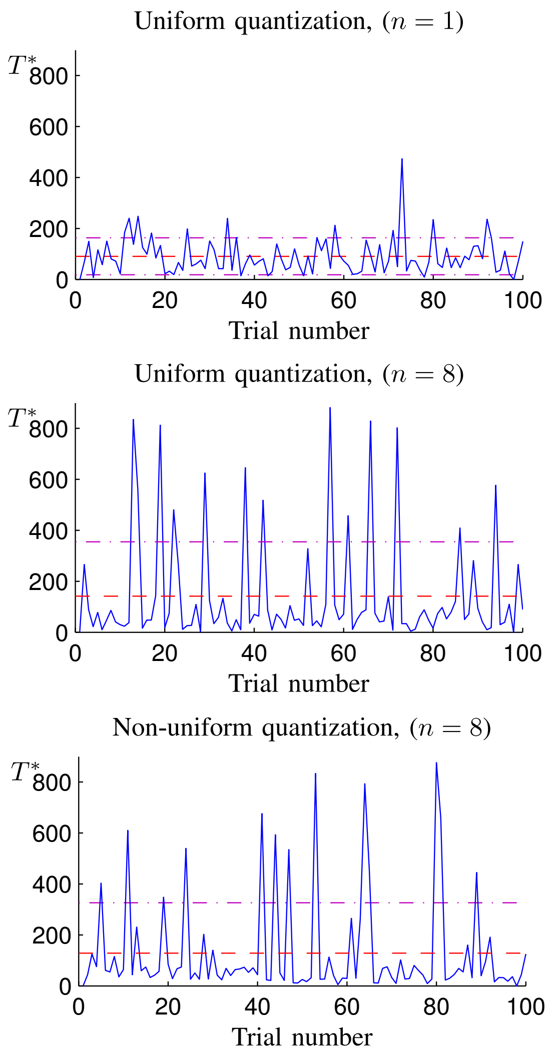


Fig. 5. Capture time T^* for hundred Monte Carlo experiments. In this case the size of the map is $n_c^2 = 32^2$.

VII. ACKNOWLEDGMENTS

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