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Attitude coordinated control of multiple underactuated axisymmetric spacecraft

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***Abstract*—Attitude coordinated control of multiple underactuated spacecraft is studied in this paper. We adopt the parametrization proposed by Tsiotras et al. (1995) to describe attitude kinematics, which has been shown to be very convenient for control of underactuated axisymmetric spacecraft with two control torques. We first propose a partial attitude coordinated controller with angular velocity commands. The controller is based on the exchange of each spacecraft's information with local neighbors and a self damping term. Under a necessary and general connectivity assumption and by use of a novel Lyapunov function, we show that the symmetry axes of all spacecraft are eventually aligned. Full attitude control of multiple underactuated spacecraft is also considered and a discontinuous distributed control algorithm is proposed. It is shown that the proposed algorithm succeeds to achieve stabilization given that control parameters are chosen properly. Discussions on the cases without self damping are also provided for both partial and full attitude controls. Simulations are given to validate the theoretical results and different steady-state behaviors are observed.**

I. Introduction

Synchronization of multi-agent systems has received much attention recently due to its broad applications in power networks [2], biological networks [3], social networks [4], mechanical networks [5, 6] and so on. Distributed protocols were proposed for various agent networks, including general linear dynamical networks [7], nonlinear system networks [8], Lagrangian dynamical networks [9], and mobile robotic networks [10].

In this paper, the agents are specified as the attitudes of spacecraft and the relevant works on the attitude control of multiple spacecraft include [11–22]. In particular, by considering a ring communication topology structure, attitude synchronization problem of a group of rotating and translating rigid bodies was studied in [12]. Attitude direction cosine matrix was used in [16] to construct a leaderless attitude synchronization algorithm for undirected fixed communication topologies, while the observed-based controller was also proposed to solve the situation of directed switching communication topologies. Similar problems were considered in [18, 20], where the attitude was represented by the Euler angle in [18] and almost global convergence was shown in [20]. The authors of [11] proposed a cooperative attitude tracking protocol such that the follower spacecraft track a time-varying leader spacecraft using relative attitude and relative angular velocity information. Under a standing assumption that the states of the leader spacecraft are only available to a subset of follower spacecraft and the follower spacecraft only have local information exchange, a distributed cooperative attitude tracking algorithm was designed in [15]. A passivity-based group orientation approach was introduced in [13] to solve distributed attitude alignment problem, where the inertial frame information is not assumed to be available to the spacecraft. In addition, the attitude containment problem was considered in [17] and the influence of communication delay between spacecraft was studied in [14].

We focus on the attitude coordinated control problem of multiple *underactuated* spacecraft in this paper. As far as we know, this is the first attempt in the literature to study collective behaviors for coupled spacecraft systems in an underactuated setting and with joint connectivity. In particular, we consider axisymmetric spacecraft with two control torques and assume that angular velocity commands are possible. We adopt the special parametrization of attitude given in [23] to describe attitude kinematics and we propose attitude coordinated control algorithms to solve the problem. We show that both partial and full attitude control are achieved under a necessary and general connectivity assumption.

The organization of this paper is as follows. We first present some background and preliminaries on graph theory, switching communication topology, Dini derivatives, and the attitude parametrization in Section II. Then, partial and full attitude control of multiple underactuated spacecraft are studied in Sections III and IV, respectively. Finally, we give concluding remarks in Section V.

II. Background and preliminaries

A. Graph theory

Using graph theory, we can model the communication topology among spacecraft in the formation. A graph $\mathcal{G}$ consists of a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is a finite nonempty set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of ordered pairs of nodes. An edge (i, j) denotes that node j can obtain information from i. All the neighbors of node i are denoted as $\mathcal{N}_i := \{j | (j, i) \in \mathcal{E}\}$, where we assume that $i \notin \mathcal{N}_i$.

A directed path in a directed graph is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots$. If there exists a path from node i to j, then node j is said to be reachable from node i. $\mathcal{G}$ is said to be strongly connected if each node is reachable from any other node.

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B. Switching communication topology and joint connectivity

In order to implement the distributed algorithm for the multi-agent systems, each agent is often equipped with a communication unit. This raises a natural issue of the possible communication link failure and therefore makes the studying on the case of switching communication topology important. In this paper, we associate the switching communication topology with a time-varying graph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$, where $\sigma : [0, +\infty) \to \mathcal{P}$ is a piecewise constant function and $\mathcal{P}$ is finite set of all possible graphs. $\mathcal{G}_{\sigma(t)}$ remains constant for $t \in [t_\varsigma, t_{\varsigma+1})$, $\varsigma = 0, 1, \ldots$ and switches at $t = t_\varsigma$, $\varsigma = 1, \ldots$. In addition, we assume that $\inf_\varsigma(t_{\varsigma+1} - t_\varsigma) \geq \tau_d^* > 0$, $\varsigma = 1, \ldots$, where τ_d^* is a constant known as dwell time [24]. The joint graph of $\mathcal{G}_{\sigma(t)}$ during time interval $[t_1, t_2)$ is defined by $\mathcal{G}_{\sigma(t)}([t_1, t_2)) = \bigcup_{t \in [t_1, t_2)} \mathcal{G}(t) = (\mathcal{V}, \bigcup_{t \in [t_1, t_2)} \mathcal{E}(t))$. Moreover, j is a neighbor of i at time t when $(j, i) \in \mathcal{E}_{\sigma(t)}$, and $\mathcal{N}_i(\sigma(t))$ represents the set of agent i's neighbors at time t. In the remainder of this paper, we use $*$ to denote that a parameter is constant.

Definition 2.1. *$\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected if there exists a constant $T^* > 0$ such that $\mathcal{G}([t, t + T^*))$ is strongly connected for any $t \geq 0$.*

C. Dini derivatives

Let $D^+V(t, x(t))$ be the upper Dini derivative of $V(t, x(t))$ with respect to t, i.e., $D^+V(t, x) = \limsup_{\tau \to 0^+} \frac{V(t+\tau, x(t+\tau)) - V(t, x(t))}{\tau}$. The following Lemma holds [25].

Lemma 2.1. *Suppose for each $i \in \mathcal{V}$, $V_i : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable. Let $V(t, x) = \max_{i \in \mathcal{V}} V_i(t, x)$, and let $\overline{\mathcal{V}}(t) = \{i \in \mathcal{V} : V_i(t, x(t)) = V(t, x(t))\}$ be the set of indices where the maximum is reached at time t. Then $D^+V(t, x(t)) = \max_{i \in \overline{\mathcal{V}}(t)} \dot{V}_i(t, x(t))$.*

D. The (w, z) attitude parametrization

Since we are interested in the control of underactuated axisymmetric spacecraft, a special but efficient parametrization is used to represent the attitude of a spacecraft. This parametrization is based on a pair (w, z) of a complex variable w and a real variable z that was first introduced in [23].

It is known that the attitude direction cosine matrix $R = [R_{pq}] \in \mathbb{R}^{3\times3}$ is used to determine the orientation between the body frame and inertial frame and a basic parametrization of an attitude. The (w, z) parametrization can be derived from R by using the following relationship (see Lemma 1 of [26]): $\mathrm{w} = \frac{R_{23}}{1+R_{33}} - \mathbf{j}\frac{R_{13}}{1+R_{33}}$, $\cos(z) = \frac{1}{2}((1 + |\mathrm{w}|^2)\mathrm{trace}(R) + |\mathrm{w}|^2 - 1)$, $\sin(z) = \frac{(1+\mathrm{Re}(\mathrm{w}^2))R_{12} + \mathrm{Im}(\mathrm{w}^2)R_{22} + 2\mathrm{Im}(\mathrm{w})R_{32}}{1+|\mathrm{w}|^2}$, where $\mathrm{Re}(\mathrm{w})$ and $\mathrm{Im}(\mathrm{w})$ denote, respectively, the real part and imaginary part of a complex number $\mathrm{w} \in \mathbb{C}$, $\mathbf{j} = \sqrt{-1}$, trace$R$ denotes the sum of the elements on the main diagonal of a square matrix R, $\overline{\mathrm{w}}$ denotes the complex conjugate, and $|\mathrm{w}| = \sqrt{\mathrm{w}\overline{\mathrm{w}}}$ denotes the absolute value of w.

Based on the above relationship, we use (w, z) to describe attitude coordinates from now on. Consider n spacecraft with attitude (w_i, z_i), $i = 1, 2, \ldots, n$. In addition, ω_{i1}, ω_{i2}, and ω_{i3} are used to describe the components of the angular velocity of the body frame with respect to the inertia frame expressed in the body frame. The kinematic equation of each spacecraft is described by (see (28) and (38b) of [27]):

$$\dot{\mathrm{w}}_i = -\mathbf{j}\omega_{i3}^*\mathrm{w}_i + \frac{\omega_i}{2} + \frac{\overline{\omega_i}}{2}\mathrm{w}_i^2, \tag{1a}$$

$$\dot{z}_i = \omega_{i3}^* + \mathrm{Im}(\omega_i\overline{\mathrm{w}_i}), \tag{1b}$$

where $\mathrm{w}_i = \mathrm{w}_{i1} + \mathbf{j}\mathrm{w}_{i2}$, $\omega_i = \omega_{i1} + \mathbf{j}\omega_{i2}$. In this paper, we focus on angular velocity commands and assume that only angular velocity ω_i can be manipulated. The third axis of the spacecraft is considered as the underactuated axis and we know that for an axisymmetric spacecraft, $\omega_{i3} \equiv \omega_{i3}^*$, $i = 1, 2, \ldots, n$, remain constant since the torque input about the symmetry axis is zero and two of the principal moments of inertia are equal (see equations (1) and (2) of [23]). Since the input dimension is less than the state dimension, for the kinematics (1), only two-axis stabilization of pointing is possible for the general case when at least one ω_{i3}^*, $i \in \mathcal{V}$, is nonzero [23]. On the other hand, for the special case when $\omega_{i3}^* \equiv 0$, for all $i \in \mathcal{V}$, three-axis stabilization of pointing is possible, and in this case the kinematic equations become

$$\dot{\mathrm{w}}_i = \frac{\omega_i}{2} + \frac{\overline{\omega_i}}{2}\mathrm{w}_i^2, \tag{2a}$$

$$\dot{z}_i = \mathrm{Im}(\omega_i\overline{\mathrm{w}_i}). \tag{2b}$$

We will next focus on (1) and (2) to study partial and full attitude control problems, respectively. In this paper, we restrict the discussion to the corresponding kinematic parameters without specific mentioning.

III. Partial attitude control of multiple underactuated spacecraft

In this section, we focus on the kinematic (1a) and study the partial attitude control problem, where the manifold $\mathcal{W}$ is defined as $\mathcal{W} = \{(\mathrm{w}_1, z_1, \ldots, \mathrm{w}_n, z_n) : \mathrm{w}_1 = \cdots = \mathrm{w}_n = 0\}$. Note that (1b) is uncontrollable for the general case when at least one ω_{i3}^*, $i \in \mathcal{V}$ is nonzero.

The following attitude control algorithm is proposed for all the spacecraft,

$$\omega_i = -b_i(t)\mathrm{w}_i - \sum_{j \in \mathcal{N}_i(\sigma(t))} a_{ij}(t)(\mathrm{w}_i - \mathrm{w}_j),\ \forall i \in \mathcal{V}, \tag{3}$$

where $a_{ij}(t) \geq 0$ is the weight of arc (j, i) for $i, j \in \mathcal{V}$ at t and $b_i(t) \geq 0$ is a continuous function denoting the self damping weight. We also assume that $a_{ij}(t)$ and $b_i(t)$ satisfy the following condition:

Assumption 3.1. *There exist constants $a^* > 0$ and $a_* > 0$ such that for all $i, j \in \mathcal{V}$, and $t \geq 0$, $a_* \leq a_{ij}(t) \leq a^*$ when $a_{ij}(t) > 0$.*

Assumption 3.2. *There exist an agent $k \in \mathcal{V}$, a constant $\overline{T}^* > 0$ and a constant $b_* > 0$ such that for all $t \geq 0$, $\int_t^{t+\overline{T}^*} b_k(\tau)\mathrm{d}\tau \geq b_*$.*

Theorem 3.1. *Suppose that Assumption 3.1 and 3.2 hold and $\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected. For multiple*

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underactuated spacecraft kinematics (1a), *algorithm* (3) *guarantees that partial attitude control is achieved with respect to* $\mathcal{W}$, *i.e.*, $\lim_{t\to\infty} \mathrm{w}_i(t) = 0$, *for all bounded* $\mathrm{w}_i(0)$ *and all* $i \in \mathcal{V}$.

Proof. Consider the following Lyapunov function candidate $V(\mathrm{W}) = \max_{i\in\mathcal{V}} V_i(\mathrm{w}_i)$, where $V_i(\mathrm{w}_i) = \mathrm{w}_i\overline{\mathrm{w}_i} = |\mathrm{w}_i|^2$ for all $i \in \mathcal{V}$ and $\mathrm{W} = [\mathrm{w}_1, \mathrm{w}_2, \dots, \mathrm{w}_n]$. Note that for all $i \in \mathcal{V}$, $\dot{\overline{\mathrm{w}_i}} = \overline{\dot{\mathrm{w}}_i} = \mathbf{j}\omega_{i3}^*\overline{\mathrm{w}_i} + \frac{\overline{\omega_i}}{2} + \frac{\omega_i}{2}\overline{\mathrm{w}_i}^2$. Therefore, it follows that for all $i \in \mathcal{V}$, $\dot{V}_i = \frac{1+|\mathrm{w}_i|^2}{2}\left(\omega_i\overline{\mathrm{w}_i} + \overline{\omega_i}\mathrm{w}_i\right)$.

Let $\overline{\mathcal{V}}$ be the set containing all the agents that reach the maximum of $V_i, i \in \mathcal{V}$, at time t, i.e., $\overline{\mathcal{V}} = \{i \in \mathcal{V} | V_i(t) = V(t)\}$. It follows from Lemma 2.1 that the derivative of V can be calculated as

$$\begin{aligned} D^+V =& \max_{i\in\overline{\mathcal{V}}} \dot{V}_i = \max_{i\in\overline{\mathcal{V}}}\Big\{\frac{1+|\mathrm{w}_i|^2}{2}\left(-2b_i(t)|\mathrm{w}_i|^2\right. \\ &+ \left.\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)\left(\mathrm{w}_i\overline{\mathrm{w}_j} + \overline{\mathrm{w}_i}\mathrm{w}_j - 2|\mathrm{w}_i|^2\right)\right\} \\ \le& \max_{i\in\overline{\mathcal{V}}}\Big\{\frac{1+|\mathrm{w}_i|^2}{2}(-2b_i(t)|\mathrm{w}_i|^2 \\ &+ \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(|\mathrm{w}_j|^2 - |\mathrm{w}_i|^2))\Big\} \le 0, \end{aligned}$$

where we have used the fact $\mathrm{w}_i\overline{\mathrm{w}_j} + \overline{\mathrm{w}_i}\mathrm{w}_j \le |\mathrm{w}_i|^2 + |\mathrm{w}_j|^2$ for all $i, j \in \mathcal{V}$. This implies that $|\mathrm{w}_i(t)|^2 \le V^* < \infty$, for all $i \in \mathcal{V}$ and all $t \ge 0$, where $V^* := V(\mathrm{W}(0))$. The above deductions show that the proposed Lyapunov function V is nonincreasing and therefore the states of $V_i(t)$, $\forall i \in \mathcal{V}$ are bounded by V^* for all $t \ge 0$. We next show that $V(\mathrm{W}(t))$ actually converges to zero as $t \to \infty$.

Based on Assumption 3.2, we know that there exists an agent $k_1 \in \mathcal{V}$, a constant $\overline{T}^* > 0$, and a constant $b_* > 0$ such that $\int_0^{\overline{T}^*} b_{k_1}(\tau)\mathrm{d}\tau \ge b_*$. We first consider agent k_1 and the time interval $[0, \overline{T}^*]$. It follows that

$$\begin{aligned} \dot{V}_{k_1} \le& \frac{1+|\mathrm{w}_{k_1}|^2}{2}(-2b_{k_1}(t)|\mathrm{w}_{k_1}|^2 + \sum_{j\in\mathcal{N}_{k_1}(\sigma(t))} a_{k_1j}(t) \\ &\times(|\mathrm{w}_j|^2 - |\mathrm{w}_{k_1}|^2)) \\ \le& -b_{k_1}(t)V_{k_1} + \frac{1+|\mathrm{w}_{k_1}|^2}{2}\sum_{j\in\mathcal{N}_{k_1}(\sigma(t))} a_{k_1j}(t)(V^* - V_{k_1}) \\ \le& -b_{k_1}(t)V_{k_1} + \alpha(V^* - V_{k_1}), \end{aligned}$$

where $\alpha = \frac{(1+V^*)(n-1)a^*}{2}$. It then follows that

$$\begin{aligned} V_{k_1}(\overline{T}^*) \le& e^{-\int_0^{\overline{T}^*}(b_{k_1}(\tau)+\alpha)\mathrm{d}\tau}V_{k_1}(0) \\ &+ \alpha V^*\int_0^{\overline{T}^*} e^{-\int_s^{\overline{T}^*}(b_{k_1}(\tau)+\alpha)\mathrm{d}\tau}\mathrm{d}s \\ \le& e^{-(b_*+\alpha\overline{T}^*)}V^* + \alpha V^*\int_0^{\overline{T}^*} e^{-\alpha(\overline{T}^*-s)}\mathrm{d}s \\ \le& \hat{\alpha}_1 V^*, \end{aligned}$$

where $\hat{\alpha}_1 = 1 - e^{-\alpha\overline{T}^*}(1 - e^{-b_*}) < 1$, Gronwall's inequality has been used for the first inequality, Assumption 3.2 has been used for the second inequality, and relation $\int_0^{\overline{T}^*} e^{-\alpha(\overline{T}^*-s)}\mathrm{d}s = \frac{1}{\alpha}(1 - e^{-\alpha\overline{T}^*})$ has been used for the third inequality. Define $T_1 = T^* + 2\tau_d^*$, where T^* is defined in Definition 2.1. Therefore, we know that after a finite time period, the state evaluation on agent k_1 (i.e., V_{k_1}) is strictly less than the explicit upper bound V^*.

It then follows that for all $t \in [\overline{T}^*, \widetilde{T}]$ with $\widetilde{T} > \overline{T}^* + (n-1)T_1$ being an arbitrary constant, $\dot{V}_{k_1} \le \frac{1+|\mathrm{w}_{k_1}|^2}{2}\sum_{j\in\mathcal{N}_{k_1}(\sigma(t))} a_{k_1j}(t)(V^* - V_{k_1}) \le \alpha(V^* - V_{k_1})$. This shows that for all $t \in [\overline{T}^*, \widetilde{T}]$,

$$V_{k_1}(t) \le V^* + e^{-\alpha(\widetilde{T}-\overline{T}^*)}(V_{k_1}(\overline{T}^*) - V^*) \le \alpha_1^* V^*, \quad (4)$$

where $\alpha_1^* = 1 - e^{-\alpha(\widetilde{T}-\overline{T}^*)}e^{-\alpha\overline{T}^*}(1 - e^{-b_*}) < 1$. To this end, we have shown that V_{k_1} is strictly less than V^* for any $t \ge \overline{T}^*$.

We next consider the time interval $[\overline{T}^*, \overline{T}^* + T_1]$. Since the union graph $\mathcal{G}([\overline{T}^* + \tau_d^*, \overline{T}^* + \tau_d^* + T^*))$ is strongly connected, it follows that there exist a time instant t_2 and an agent $k_2 \in \mathcal{V}\backslash\{k_1\}$ such that there exists an arc (k_1, k_2) for all $t \in [t_2, t_2 + \tau_d^*) \subset [\overline{T}^*, \overline{T}^* + T_1]$. Due to this connection, we next show that agent k_2 will be "attracted" by k_1 for a time interval larger than τ_d^* and the state evaluation on agent k_2 (i.e., V_{k_2}) will be also strictly less than the explicit upper bound V^*. The analysis can be divided by two cases.

Case I: $V_{k_2}(t) > V_{k_1}(t)$ for all $t \in [t_2, t_2 + \tau_d^*)$. It then follows that

$$\begin{aligned} \dot{V}_{k_2} \le& \frac{1+|\mathrm{w}_{k_2}|^2}{2}\left(-2b_{k_2}(t)V_{k_2} + \sum_{j\in\mathcal{N}_{k_2}(\sigma(t))} a_{k_2j}(t)(V_j - V_{k_2})\right) \\ \le& \frac{1+|\mathrm{w}_{k_2}|^2}{2}\sum_{j\in\mathcal{N}_{k_2}(\sigma(t))} a_{k_2j}(t)(V_j - V_{k_2}) \\ \le& \frac{1+|\mathrm{w}_{k_2}|^2}{2}\sum_{j\in\mathcal{N}_{k_2}(\sigma(t))\backslash\{k_1\}} a_{k_2j}(t)(V_j - V_{k_2}) \\ &+ \frac{1+|\mathrm{w}_{k_2}|^2}{2}a_{k_2k_1}(V_{k_1} - V_{k_2}) \\ \le& \frac{1+V^*}{2}\sum_{j\in\mathcal{N}_{k_2}(\sigma(t))\backslash\{k_1\}} a_{k_2j}(t)(V^* - V_{k_2}) \\ &+ \frac{1+|\mathrm{w}_{k_2}|^2}{2}a_{k_2k_1}(V_{k_1} - V_{k_2}) \\ \le& \hat{\alpha}(V^* - V_{k_2}) + \frac{1+|\mathrm{w}_{k_2}|^2}{2}a_{k_2k_1}(V_{k_1} - V_{k_2}). \\ \le& \hat{\alpha}(V^* - V_{k_2}) + \frac{a_*}{2}(\alpha_1 V^* - V_{k_2}), \end{aligned}$$

where we have used Assumption 3.1 and the facts $V_j(t) \le V^*$, for all $j \in \mathcal{V}$ and all $t \ge 0$, and $\frac{1+|\mathrm{w}_{k_2}|^2}{2}a_{k_2k_1}(t) \ge \frac{a_*}{2}$ for $t \in [t_2, t_2 + \tau_d^*)$, $\hat{\alpha} = \frac{(1+V^*)(n-2)a^*}{2}$, $\alpha_1 = 1 - e^{-\alpha T_1}e^{-\alpha\overline{T}^*}(1 - e^{-b_*})$. It then follows that $V_{k_2}(t_2 + \tau_d^*) \le \left(e^{-(\hat{\alpha}+a_*/2)\tau_d^*} + (1 - e^{-(\hat{\alpha}+a_*/2)\tau_d^*}) \times \frac{\hat{\alpha}+a_*\alpha_1/2}{\hat{\alpha}+a_*/2}\right)V^* = \hat{\alpha}_2^* V^*$, where $\hat{\alpha}_2^* = \frac{\hat{\alpha}+a_*/2-\frac{a_*}{2}(1-e^{-(\hat{\alpha}+a_*/2)\tau_d^*})(1-\alpha_1)}{\hat{\alpha}+a_*/2} < 1$.

Case II: there exists a time $\bar{t}_2 \in [t_2, t_2 + \tau_d^*)$ such that $V_{k_2}(\bar{t}_2) \le V_{k_1}(\bar{t}_2)$. This implies from (4) that $V_{k_1}(t) \le \alpha_1 V^*$, for all $t \in [\overline{T}^*, \overline{T}^* + T_1]$. Therefore, since $\bar{t}_2 \in [\overline{T}^*, \overline{T}^* + T_1]$, we know that $V_{k_2}(\bar{t}_2) \le \alpha_1 V^*$. Following the similar analysis for equation (4), we know that $V_{k_2}(t_2 + \tau_d^*) \le (1 - e^{-\alpha\tau_d^*}(1-$

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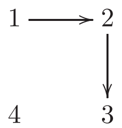

Fig. 1. The communication graph $\mathcal{G}^1$

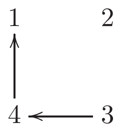

Fig. 2. The communication graph $\mathcal{G}^2$

$\alpha_1))V^*$. Define $\hat{\alpha}_2 = 1-\frac{a_*/2}{\hat{\alpha}+a_*/2}e^{-\alpha\tau_d^*}(1-e^{-(\hat{\alpha}+a_*/2)\tau_d^*})(1-\alpha_1)$. It is not hard to show that $\max\{\hat{\alpha}_2^*, \alpha_1, (1-e^{-\alpha\tau_d^*}(1-\alpha_1))\} \leq \hat{\alpha}_2 < 1$. We thus know that for all $k \in \{k_1, k_2\}$, $V_k(t_2+\tau_d^*) \leq \hat{\alpha}_2 V^*$. This in turn shows that for all $k \in \{k_1, k_2\}$, and all $t \in [\overline{T}^* + T_1, \widetilde{T}]$, $V_k(t) \leq (1-e^{-\alpha T_1}(1-\hat{\alpha}_2))V^* = \alpha_2 V^*$, where $\alpha_2 = 1-e^{-\alpha T_1}\frac{a_*/2}{\hat{\alpha}+a_*/2}e^{-\alpha\tau_d^*}(1-e^{-(\hat{\alpha}+a_*/2)\tau_d^*})(1-\alpha_1)$. To this end, we have shown that both V_{k_1} and V_{k_2} are strictly less than V^* for any $t \geq \overline{T}^* + T_1$.

We next consider the time interval $[\overline{T}^* + T_1, \overline{T}^* + 2T_1]$. Since $\mathcal{G}([\overline{T}^* + T_1 + \tau_d^*, \overline{T}^* + T_1 + T^* + \tau_d^*))$ is strongly connected, it follows that there exist a time instant t_3 and an agent $k_3 \in \mathcal{V}\backslash\{k_1, k_2\}$ such that there exists an arc (k, k_3) for all $t \in [t_3, t_3+\tau_d^*) \subset [\overline{T}^* + T^* + \tau_d^*, \overline{T}^* + 2T^* + 2\tau_d^*]$, where $k \in \{k_1, k_2\}$. Then, following the similar analysis on agent k_2, we know that for all $k \in \{k_1, k_2, k_3\}$, and all $t \in [\overline{T}^* + 2T_1, \widetilde{T}]$, $V_k(t) \leq \alpha_3 V^*$, where $\alpha_3 = 1-e^{-2\alpha T_1}\left(\frac{a_*/2}{\hat{\alpha}+a_*/2}\right)^2 e^{-2\alpha\tau_d^*}(1-e^{-(\hat{\alpha}+a_*/2)\tau_d^*})^2(1-\alpha_1)$. Therefore, we know that V_{k_1}, V_{k_2} and V_{k_3} are strictly less than V^* for any $t \geq \overline{T}^* + 2T_1$.

Finally, at the worst case, it follows that for all $i \in \mathcal{V}$ and all $t \in [\overline{T}^* + (n-1)T_1, \widetilde{T}]$, $V_i(t) \leq \alpha_n V^*$, where $\alpha_n = 1-e^{-(n-1)\alpha T_1}\left(\frac{a_*/2}{\hat{\alpha}+a_*/2}\right)^{n-1} e^{-(n-1)\alpha\tau_d^*}(1-e^{-(\hat{\alpha}+a_*/2)\tau_d^*})^{n-1}(1-\alpha_1)$. Thus, for all $t \in [\overline{T}^* + (n-1)T_1, \widetilde{T}]$, $V(t) \leq \alpha_n V^*$. The above inequality shows no agents will stay on the boundary V^* at a certain time instant and therefore, by the following arguments, we know that the Lyapunov function V is strictly shrinking.

Let ψ be the smallest positive integer satisfying $t \leq \psi N T_2$, where $T_2 = \overline{T}^* + (n-1)T_1$. It then follows that $V(t) \leq (1-\alpha_n)^{\psi-1}V^* \leq \frac{1}{1-\alpha_n}(1-\alpha_n)^{\frac{t}{NT_2}}V^* = \rho e^{-\varrho t}V^*$, where $\varrho = \frac{1}{NT_2}\ln\frac{1}{1-\alpha_n}$ and $\rho = \frac{1}{1-\alpha_n}$. This implies that $\lim_{t\to\infty} V(t) = 0$ and further shows that $\lim_{t\to\infty} \mathrm{w}_i(t) = 0$ for all $i \in \mathcal{V}$. ■

We next verify Theorem 3.1 using simulations and show numerically that the introduce of synchronization term helps to obtain better transient process in terms of achieving synchronization objective. In particular, we consider that there are four spacecraft ($n = 4$) in the group. The weights a_{ij} and b_i are chosen to be 1 when $(j, i) \in \mathcal{E}$. The communication graph $\mathcal{G}$ switches between $\mathcal{G}^1$ (Fig. 1) and $\mathcal{G}^2$ (Fig. 2) at time instants $t_\varrho = \varrho$, $\varrho = 0, 1, \ldots$. In addition, b_i, $\forall i \in \mathcal{V}$ switches between 0 and 1 at time instants $t_\varrho = \varrho$, $\varrho = 0, 1, \ldots$.

Fig. 3 shows the trajectories of w_{i1} and w_{i2} for all $i = 1, 2, 3, 4$ using algorithm (3) for (1a). We see that the partial attitudes of all the spacecraft converge to zero. We also

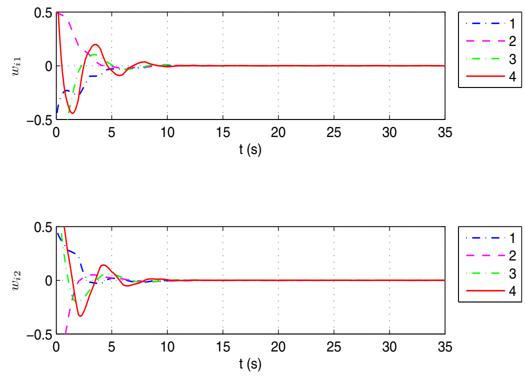

Fig. 3. Trajectories of partial attitudes

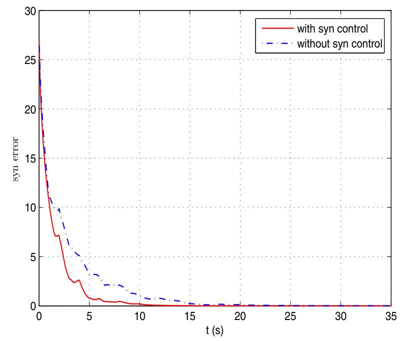

Fig. 4. Trajectories of synchronization errors

compare algorithm (3) with individual stabilization algorithm $\omega_i = -b_i(t)\mathrm{w}_i$ in terms of achieving synchronization objective, where we use quantity $\sum_{i=1}^n\sum_{j=1}^n |\mathrm{w}_{i1} - \mathrm{w}_{j1}| + \sum_{i=1}^n\sum_{j=1}^n |\mathrm{w}_{i2} - \mathrm{w}_{j2}|$ to evaluate the synchronization error during transient process. It is clear from Fig. 4 that algorithm (3) presents a better transient process in terms of achieving synchronization objective.

A. *Discussions on the case without self damping term*

We note that the absolute damping term is used in (3) and stabilization result is obtained. On the other hand, the study on case of leaderless algorithms attracts much attention recently [2, 4, 7] because abundant emergent behaviors can be observed for the multi-agent systems. This motivates us to observe and study the complex behaviors of the coupled underactuated spacecraft being controlled by a leaderless consensus-like algorithm. We will show later that the convergence results indicate the noticeable differences between the stabilization case and the synchronization case, as well as the classical coupled linear systems and the coupled underactuated attitude dynamical systems.

In particular, the algorithm without self damping term takes the following form for all the spacecraft

$$\omega_i = -\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(\mathrm{w}_i - \mathrm{w}_j), \quad \forall i \in \mathcal{V}, \tag{5}$$

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where $a_{ij}(t) \geq 0$ is the weight of arc (j, i) for $i, j \in \mathcal{V}$ at t.

Proposition 3.1. *Suppose that Assumption 3.1 holds and $\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected. For multiple underactuated spacecraft kinematic* (1a)*, algorithm* (5) *guarantees that partial attitude norm synchronization is achieved with respect to $\mathcal{W}_1$. In particular, it follows that $\lim_{t\to\infty} |\mathrm{w}_i(t)| = w^*$, for all bounded $\mathrm{w}_i(0)$ and all $i \in \mathcal{V}$, where w^* is a positive constant.*

Proof. Similar to the proof of Theorem 3.1, we use the Lyapunov function candidate

$$V(\mathrm{W}) = \max_{i\in\mathcal{V}} V_i(\mathrm{w}_i),$$

where $V_i(\mathrm{w}_i) = \mathrm{w}_i\overline{\mathrm{w}_i} = |\mathrm{w}_i|^2$ for all $i \in \mathcal{V}$ and $\mathrm{W} = [\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n]$.

Let $\overline{\mathcal{V}}$ be the set containing all the agents that reach the maximum of $V_i, i \in \mathcal{V}$, at time t, i.e., $\overline{\mathcal{V}} = \{i \in \mathcal{V} | V_i(t) = V(t)\}$. It follows from Lemma 2.1 that the derivative of V can be calculated as

$$\begin{aligned}
&D^+V \\
&= \max_{i\in\overline{\mathcal{V}}} \dot{V}_i \\
&= \max_{i\in\overline{\mathcal{V}}} \left\{ \frac{1+|\mathrm{w}_i|^2}{2} \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)\left(\mathrm{w}_i\overline{\mathrm{w}_j} + \overline{\mathrm{w}_i}\mathrm{w}_j - 2|\mathrm{w}_i|^2\right) \right\} \\
&\leq \max_{i\in\overline{\mathcal{V}}} \left\{ \frac{1+|\mathrm{w}_i|^2}{2} \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(|\mathrm{w}_j|^2 - |\mathrm{w}_i|^2) \right\} \\
&\leq 0,
\end{aligned}$$

where we have used the fact $\mathrm{w}_i\overline{\mathrm{w}_j} + \overline{\mathrm{w}_i}\mathrm{w}_j \leq |\mathrm{w}_i|^2 + |\mathrm{w}_j|^2$ for all $i, j \in \mathcal{V}$. This implies that $|\mathrm{w}_i(t)|^2 \leq V(\mathrm{W}(0))$, for all $i \in \mathcal{V}$ and all $t \geq 0$. It thus follows that $\lim_{t\to\infty} V(t) = \tilde{V}$, where $\tilde{V}$ is a positive constant. Therefore, we know that for any $\varepsilon > 0$, there exists a $t_1^*(\varepsilon) \geq 0$ such that

$$\tilde{V} - \varepsilon \leq V(t) \leq \tilde{V} + \varepsilon, \quad \forall t \geq t_1^*.$$

Suppose that $\lim_{t\to\infty} V_i(t) = \tilde{V}$ does not hold for certain $i \in \mathcal{V}$. Then, based on Assumption 3.1 and following the similar analysis of Lemma 4.3 given in [28], we can show the contradiction with the fact that $V(t^*) \geq \tilde{V} - \varepsilon$ by choosing ε sufficient small for some $t^* \geq t_1^*$. Such a selection of ε exists due to the fact that $\mathrm{w}_i(0)$ is bounded for all $i \in \mathcal{V}$. This indicates a contradiction and shows that $\lim_{t\to\infty} V_i(t) = \tilde{V}$ for all $i \in \mathcal{V}$. The desired result is proven. ■

We notice that Proposition 3.1 only claims that the norms of all spacecraft's partial attitudes reach synchronization. There is no affirmative assertion on convergence of all spacecraft's partial attitudes. We next focus on the special case when the angular velocity of the uncontrollable axis ω_{i3}^* remains zero for all $i \in \mathcal{V}$ and show that in addition to partial attitude norm synchronization, partial attitude synchronization is also achieved.

Proposition 3.2. *Suppose that Assumption 3.1 holds and $\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected. For multiple underactuated spacecraft kinematic* (1a) *with $\omega_{i3}^* \equiv 0$, for all $i \in \mathcal{V}$, algorithm* (5) *guarantees that $\lim_{t\to\infty}(\mathrm{w}_i(t) - \mathrm{w}_j(t)) = 0$, for all bounded $\mathrm{w}_i(0)$ and $i, j \in \mathcal{V}$.*

Proof. We prove that $\lim_{t\to\infty}(\mathrm{w}_i(t) - \mathrm{w}_j(t)) = 0$ for all $i, j \in \mathcal{V}$ using contradiction. We still use $V_i = |\mathrm{w}_i|^2$ for all $i \in \mathcal{V}$. Based on the result of Proposition 3.1, we know that for any $\varepsilon > 0$, there exists a $t_1^*(\varepsilon) \geq 0$ such that $\tilde{V} - \varepsilon \leq V_i(t) \leq \tilde{V} + \varepsilon, \ \ \forall i \in \mathcal{V}, \ \forall t \geq t_1^*$, where $\tilde{V}$ is a positive constant.

Suppose that there exist $l, k \in \mathcal{V}$ and $t_1 \geq t_1^*$ such that $\mathrm{w}_l(t_1) \neq \mathrm{w}_k(t_1)$. We next show that there exists $h \in \mathcal{V}$ and $t^* \geq t_1$ such that $V_h(t^*) < \tilde{V} - \varepsilon$, which will indicate a contradiction and prove the desired result.

Since $\mathcal{G}([t_1, t_1 + T^*))$ is strongly connected, we can define a time $t_2 = \inf_{t\in[t_1,t_1+T^*)}\{\exists i_1 \in \mathcal{V}\backslash\{l\} | (i_1, l) \in \mathcal{E}_{\sigma(t)}\}$, and a set $\mathcal{V}_1 = \{i_1 \in \mathcal{V}\backslash\{l\} | (i_1, l) \in \mathcal{E}_{\sigma(t_2)}\} \neq \emptyset$.

Note that for all $i \in \mathcal{V}$, it follows that for all $t \geq t_1$,

$$\begin{aligned}
\dot{V}_i &= \frac{1+|\mathrm{w}_i|^2}{2} \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)\left(\mathrm{w}_i\overline{\mathrm{w}_j} + \overline{\mathrm{w}_i}\mathrm{w}_j - 2|\mathrm{w}_i|^2\right) \\
&= \sum_{j\in\mathcal{N}_i(\sigma(t))} b_{ij}(\mathrm{w}_i, t)\left(-|\mathrm{w}_i - \mathrm{w}_j|^2 - (|\mathrm{w}_i|^2 - |\mathrm{w}_j|^2)\right) \\
&\leq -\sum_{j\in\mathcal{N}_i(\sigma(t))} b_{ij}(\mathrm{w}_i, t)|\mathrm{w}_i - \mathrm{w}_j|^2 + 2\bar{b}^*(n-1)\varepsilon.
\end{aligned}$$

where we define $b_{ij}(\mathrm{w}_i, t) = \frac{1+|\mathrm{w}_i|^2}{2} a_{ij}(t)$ and easily derive that $\bar{b}_* \triangleq \frac{1}{2}a_* \leq b_{ij} \leq \frac{1+V(\mathrm{W}(0))}{2}a^* \triangleq \bar{b}^*$ for all $i, j \in \mathcal{V}$. We have also used the fact that $\left||\mathrm{w}_i(t)|^2 - |\mathrm{w}_j(t)|^2\right| = |V_i(t) - V_j(t)| \leq 2\varepsilon$ for all $i, j \in \mathcal{V}$ and $t \geq t_1$. We next consider two cases.

Case I: there exists a $i_1 \in \mathcal{V}_1$ such that $\mathrm{w}_l(t_2) \neq \mathrm{w}_{i_1}(t_2)$. It then follows from the definition of t_2 that for all $t \in [t_2, t_2 + \tau_d^*]$, $\dot{V}_l \leq -\bar{b}_*|\mathrm{w}_l - \mathrm{w}_{i_1}|^2 + 2\bar{b}^*(n-1)\varepsilon$. Define a positive constant $\phi^* = |\mathrm{w}_l(t_2) - \mathrm{w}_{i_1}(t_2)|^2 > 0$ and a function $\phi_{\mu\nu}(t) = |\mathrm{w}_\mu(t) - \mathrm{w}_\nu(t)|^2$ for any pair $(\mu, \nu) \in \mathcal{V} \times \mathcal{V}$. It follows that $\phi_{\mu\nu}(t) = V_\mu + V_\nu - \mathrm{w}_\mu\overline{\mathrm{w}_\nu} - \overline{\mathrm{w}_\mu}\mathrm{w}_\nu$. By noting the fact that $|\omega_i| \leq 2(n-1)a^*\sqrt{\tilde{V} + \varepsilon}$, for all $i \in \mathcal{V}$, we know that $|\dot{\mathrm{w}}_i| \leq (1 + \tilde{V} + \varepsilon)(n-1)a^*\sqrt{\tilde{V} + \varepsilon}$ for all $i \in \mathcal{V}$. In addition, by noting that $|\mathrm{w}_\mu(t) - \mathrm{w}_\nu(t)|^2 \leq 2|\mathrm{w}_\mu(t)|^2 + 2|\mathrm{w}_\nu(t)|^2 = 2(V_\mu(t) + V_\nu(t))$ for all $t \geq 0$, $b_{ij} \leq \bar{b}^*$, for all $i, j \in \mathcal{V}$ and $V_i(t) \leq \tilde{V} + \varepsilon, \ \ \forall i \in \mathcal{V}$, we know that

$$-\bar{b}^*(n-1)(4\tilde{V} + 2\varepsilon) \leq \dot{V}_i \leq \bar{b}^*(n-1)(4\tilde{V} + 6\varepsilon), \quad \forall i \in \mathcal{V}, \ \forall t \geq t_1.$$

Therefore, it follows that

$$\dot{\phi}_{li_1} \geq -4(1+\tilde{V}+\varepsilon)(n-1)a^*(\tilde{V}+\varepsilon) - 2\bar{b}^*(n-1)(4\tilde{V}+2\varepsilon).$$

It thus follows that $\phi_{li_1}(t) \geq \frac{\phi^*}{2}$ for all $t \in [t_2, t_2 + \frac{\phi^*}{8(1+\tilde{V}+\varepsilon)(n-1)a^*(\tilde{V}+\varepsilon)+4\bar{b}^*(n-1)(4\tilde{V}+2\varepsilon)}]$. Define $\delta_1 = \min\{\tau_d^*, \frac{\phi^*}{8(1+\tilde{V}+\varepsilon)(n-1)a^*(\tilde{V}+\varepsilon)+4\bar{b}^*(n-1)(4\tilde{V}+2\varepsilon)}\}$. It then follows that

$$\begin{aligned}
V_l(t_2 + \delta_1) &\leq V_l(t_2) - \frac{\bar{b}_*\phi^*\delta_1}{2} + 2\bar{b}^*(n-1)\delta_1\varepsilon \\
&\leq \tilde{V} - \frac{\bar{b}_*\phi^*\delta_1}{2} + (2\bar{b}^*(n-1)\delta_1 + 1)\varepsilon.
\end{aligned}$$

We then derive a contradiction with the fact that $V_l(t_2+\delta_1) \geq \tilde{V}-\varepsilon$ by choosing ε sufficiently small. This proves the desired result.

Case II: for all $i_1 \in \mathcal{V}_1$, $\mathrm{w}_{i_1}(t_2) = \mathrm{w}_l(t_2)$. Then, based on the definitions of t_2 and $\mathcal{V}_1$, we know that the dynamics $\dot{\mathrm{w}}_l = \frac{\omega_l}{2} + \frac{\overline{\omega_l}}{2}\mathrm{w}_l^2$ is reduced to $\dot{\mathrm{w}}_l = 0$, $\forall t \in [t_1, t_2]$. Therefore, we know that $\mathrm{w}_l(t_2) = \mathrm{w}_l(t_1)$ and thus $\mathrm{w}_{i_1}(t_2) = \mathrm{w}_l(t_1)$, for all $i_1 \in \mathcal{V}_1$.

Next, we define the set $\overline{\mathcal{V}}_1 = \{l\} \bigcup \mathcal{V}_1$. Since $\mathcal{G}([t_1 + T^*, t_1 + 2T^*))$ is strongly connected, we can define a time $t_3 = \inf_{t\in[t_2,t_1+2T^*)}\{\exists i_2 \in \mathcal{V}\backslash\overline{\mathcal{V}}_1, i_1 \in \overline{\mathcal{V}}_1 | (i_2, i_1) \in \mathcal{E}_{\sigma(t)}\}$, and a set $\mathcal{V}_2 = \{i_2 \in \mathcal{V}\backslash\overline{\mathcal{V}}_1 | (i_2, i_1) \in \mathcal{E}_{\sigma(t_3)}\}$. Following the previous analysis on node l, we can show that $V_{i_1}(t_3+\delta_2) < \tilde{V} - \varepsilon$ for some $i_1 \in \mathcal{V}_1$ when $\mathrm{w}_{i_2}(t_3) \neq \mathrm{w}_{i_1}(t_3)$, for some $i_2 \in \mathcal{V}_2$. Otherwise, for the case that $\mathrm{w}_{i_2}(t_3) = \mathrm{w}_{i_1}(t_3)$ for all $i_2 \in \mathcal{V}_2$, we know that $\mathrm{w}_{i_1}(t)$ remain unchanged for all $i_1 \in \mathcal{V}_1$ during $t \in [t_2, t_3]$ based on the definitions of t_3 and $\mathcal{V}_2$. It then follows that $\mathrm{w}_{i_1}(t_3) = \mathrm{w}_{i_1}(t_2) = \mathrm{w}_l(t_1)$ for all $i_1 \in \mathcal{V}_1$ and thus $\mathrm{w}_{i_2}(t_3) = \mathrm{w}_l(t_1)$ for all $i_2 \in \mathcal{V}_2$.

Then, we define $\overline{\mathcal{V}}_2 = \overline{\mathcal{V}}_1 \bigcup \mathcal{V}_2$ and repeat the above analysis. At worst case, we finally have $\overline{\mathcal{V}}_{n-2} = \mathcal{V}\backslash\{k\}$ and the time $t_n = \inf_{t\in[t_{n-1},t_1+(n-1)T^*)}\{\exists i_{n-2} \in \overline{\mathcal{V}}_{n-2} | (k, i_{n-2}) \in \mathcal{E}_{\sigma(t)}\}$. By noting that $\mathrm{w}_{i_{n-2}}(t_n) = \mathrm{w}_l(t_1)$, for all $i_{n-2} \in \overline{\mathcal{V}}_{n-2}$ and the earliest condition of $\mathrm{w}_l(t_1) \neq \mathrm{w}_k(t_1)$, we know that it must have $\mathrm{w}_k(t_n) \neq \mathrm{w}_{i_{n-2}}(t_n)$, for some $i_{n-2} \in \overline{\mathcal{V}}_{n-2}$. Therefore, we derive a contradiction with the fact that $V_{i_{n-2}}(t_n + \tau_d^*) \geq \tilde{V} - \varepsilon$ by choosing ε sufficient small. This proves the desired result. ■

Remark 3.1. *Due to the existence of complex internal nonlinear dynamics and the leaderless couplings of spacecraft, we introduce in the proof of Proposition 3.2 a contradiction argument, instead of a direct Lyapunov approach as was used in the proof of Theorem 3.1. Also, note that the synchronization of* w_i, $i \in \mathcal{V}$ *in Proposition 3.2, does not necessarily guarantee that the symmetry axes of all spacecraft are eventually aligned. In fact, all symmetry axes of the spacecraft may converge to the surface of a cone centered around the third axis of the inertial frame. The emergence of such a collective behavior is caused by the underactuated attitude dynamics, and cannot be observed in the classical coupled linear systems.*

B. Constant and oscillatory steady-state behaviors

We notice that there is no affirmative assertion on the pattern of the steady-state behavior when multiple spacecraft reach partial attitude synchronization according to Proposition 3.1. It turns out that the uncontrollable angular velocity ω_{i3}^* plays an important role in forming the pattern of the steady-state behavior. We next show that different selections on ω_{i3}^* produce different steady-state behaviors using simulations.

1) Constant steady-state behavior: We first consider the case that angular velocities ω_{i3}^* are nonidentical for all $i \in \mathcal{V}$ and show that final attitudes of all the spacecraft present a constant steady-state behavior. In particular, we still consider that there are four spacecraft ($n = 4$) in the group. The weight a_{ij} is chosen to be 1 when $(j, i) \in \mathcal{E}$. The communication graph $\mathcal{G}$ switches between $\mathcal{G}^1$ (Fig. 1) and $\mathcal{G}^2$ (Fig. 2) at time instants $t_\varrho = \varrho$, $\varrho = 0, 1, \ldots$.

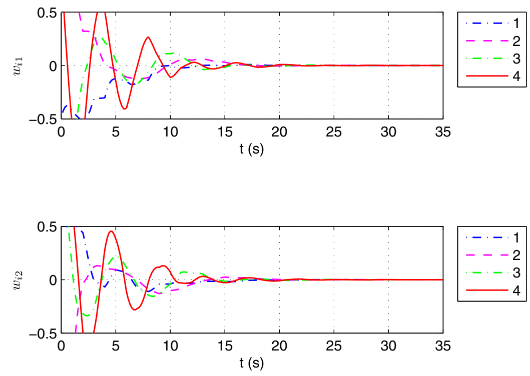

Fig. 5. Nonidentical ω_{i3}^*

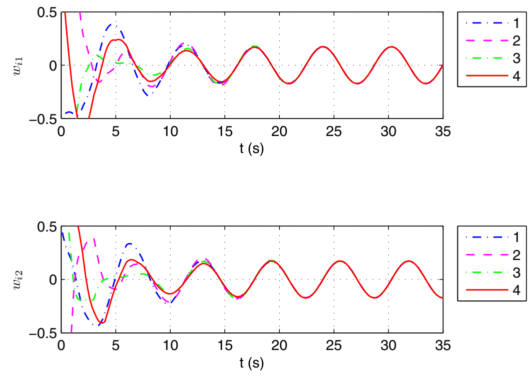

Fig. 6. Identically nonzero ω_{i3}^*

Fig. 5 shows the trajectories of w_{i1} and w_{i2} for all $i = 1, 2, 3, 4$. We see that attitude synchronization is achieved while the final attitudes of all the spacecraft converge to zero. This is an interesting observation since intuitively, a weakly coupled and strongly heterogeneous network may not display coherent behavior. However, the simulation result shows that final trajectories of all the agents converges to the constant origin as if each agent is commanded with an absolute damping. This also presents the coherent behavior. Note that the above behavior can be always observed as long as ω_{i3}^* are nonidentical for all $i \in \mathcal{V}$.

2) Oscillatory steady-state behavior: We next consider the case that angular velocities ω_{i3}^* are identically nonzero for all $i \in \mathcal{V}$ and show that final attitudes of all the spacecraft present an oscillatory steady-state behavior. Fig. 6 shows the trajectories of w_{i1} and w_{i2} for all $i = 1, 2, 3, 4$. We see that attitude synchronization is achieved while final attitudes of all the spacecraft converge to time-varying bounded curves. Note that the above behavior can be always observed as long as ω_{i3}^* are identically nonzero for all $i \in \mathcal{V}$. Also note that for the case of all zero ω_{i3}^*, time-varying bounded curves reduce to a consensus to a nonzero constant, which is similar to the coherent behavior presented for the consensus of multiple single integrators.

IV. Full attitude control of multiple underactuated spacecraft

In this section, we study full attitude control problem. We focus on the kinematic (2), where ω_{i3}^* is zero, for all $i \in \mathcal{V}$ and the manifold $\mathcal{P}$ is defined as $\mathcal{P} = \{(\mathrm{w}_1, z_1, \ldots, \mathrm{w}_n, z_n) : z_1 = \cdots = z_n = 0, \mathrm{w}_1 = \cdots = \mathrm{w}_n = 0\}$.

The following attitude control algorithm is proposed for all the spacecraft

$$\omega_i = -\gamma_1 \mathrm{w}_i - \mathbf{j}\frac{b_i(t)z_i + \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(z_i - z_j)}{\overline{\mathrm{w}_i}}, \quad \forall i \in \mathcal{V}, \tag{6}$$

where $a_{ij}(t) > 0$ is the weight of arc (j, i) for $i, j \in \mathcal{V}$ at t, $b_i(t) \geq 0$ is a continuous function denoting the self damping weight, and γ_1 is a positive constant to be determined. In addition to Assumption 3.2, we also assume that $b_i(t)$ satisfies the following condition:

Assumption 4.1. *There exists a constant $b^* > 0$ such that for all $i \in \mathcal{V}$, and $t \geq 0$, $b_i(t) \leq b^*$.*

Theorem 4.1. *Suppose that Assumptions 3.1, 3.2 and 4.1 hold and $\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected. Also assume that $z_i(0) \in (-\pi, \pi]$, $\mathrm{w}_i(0) \neq 0$ and bounded, for all $i \in \mathcal{V}$. For multiple underactuated spacecraft (2), algorithm (6) guarantees that full attitude control is achieved with respect to $\mathcal{P}$ if γ_1 is chosen sufficiently small. In particular, it follows that*

- *$\mathrm{w}_i(t) \neq 0$, for all $i \in \mathcal{V}$ and for all $t \geq 0$.*
- *$\lim_{t\to\infty} \mathrm{w}_i(t) = 0$, for all $i \in \mathcal{V}$.*
- *$\lim_{t\to\infty} z_i(t) = 0$, for all $i \in \mathcal{V}$.*
- *The control input ω_i remains bounded for all $i \in \mathcal{V}$ and all $t \geq 0$.*

Proof. It is clear that the closed-loop system (2) with distributed attitude algorithm (6) can be written as

$$\dot{\mathrm{w}}_i = -\frac{\gamma_1}{2}(1 + |\mathrm{w}_i|^2)\mathrm{w}_i - \mathbf{j}\frac{1}{2}\left(b_i(t)z_i + \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(z_i - z_j)\right)\left(\frac{1}{\overline{\mathrm{w}_i}} - \mathrm{w}_i\right), \tag{7a}$$

$$\dot{z}_i = -b_i(t)z_i - \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(z_i - z_j), \quad i \in \mathcal{V}. \tag{7b}$$

Note that the above equations hold for $(\mathbb{C}\backslash\{0\}) \times \mathbb{R} \times \cdots \times (\mathbb{C}\backslash\{0\}) \times \mathbb{R}$. We first show that $\mathrm{w}_i(t) \neq 0$, for all $i \in \mathcal{V}$ and for all $t \geq 0$ given $\mathrm{w}_i(0) \neq 0$, for all $i \in \mathcal{V}$.

Consider the Lyapunov function candidate

$$V_i = |\mathrm{w}_i|^2$$

for all $i \in \mathcal{V}$. The derivative of V_i along (7a) can be calculated as

$$\dot{V}_i = 2\mathrm{Re}(\dot{\mathrm{w}}_i\overline{\mathrm{w}_i}) = -\gamma_1(1 + |\mathrm{w}_i|^2)|\mathrm{w}_i|^2 = -\gamma_1(1 + V_i)V_i.$$

It is not hard to show that $V_i(t) = \frac{1}{c_i e^{\gamma_1 t} - 1}$, where $c_i = \frac{1+|\mathrm{w}_i(0)|^2}{|\mathrm{w}_i(0)|^2}$ is a bounded constant for all $i \in \mathcal{V}$. Therefore, it follows that $|\mathrm{w}_i(t)| = \sqrt{\frac{1}{c_i e^{\gamma_1 t} - 1}}$ based on the definition of V_i. Therefore, $\mathrm{w}_i(t) \neq 0$ for all $t \geq 0$ and we know that $\lim_{t\to\infty} \mathrm{w}_i(t) = 0$, for all $i \in \mathcal{V}$.

For system (7b), it is not hard to show that $\max_{i\in\mathcal{V}} |z_i(t)| \leq \rho_1 e^{-\varrho_1 t} \max_{i\in\mathcal{V}} |z_i(0)|$ by choosing a function $V_1 = \max_{i\in\mathcal{V}} z_i^2$ and following the similar analysis on the proof of Theorem 3.1, where ρ_1 and ϱ_1 are positive constants related to parameters n, T^*, τ_d^*, a^*, a_*, $\overline{T}^*$, and b_*. Therefore, $\lim_{t\to\infty} z_i(t) = 0$, for all $i \in \mathcal{V}$. Also, based on the fact that $D^+V_1 \leq 0$, we know that $z_i(t) \in (-\pi, \pi]$, for all $t \geq 0$ and $i \in \mathcal{V}$.

We finally show that ω_i is bounded, for all $i \in \mathcal{V}$. It follows from the fact $\max_{i\in\mathcal{V}} |z_i(t)| \leq \rho_1 e^{-\varrho_1 t} \max_{i\in\mathcal{V}} |z_i(0)|$ that for all $i \in \mathcal{V}$,

$$\begin{aligned}
&|\omega_i(t)| \\
\leq&\gamma_1|\mathrm{w}_i(t)| + \frac{b^*|z_i(t)| + 2(n-1)a^* \max_{i\in\mathcal{V}} |z_i(t)|}{|\mathrm{w}_i(t)|} \\
\leq&\gamma_1|\mathrm{w}_i(0)| + \rho_1(b^* + 2(n-1)a^*) \max_{i\in\mathcal{V}} |z_i(0)| e^{-\varrho_1 t}\sqrt{c_i e^{\gamma_1 t} - 1} \\
<&\gamma_1|\mathrm{w}_i(0)| + \rho_1(b^* + 2(n-1)a^*) \max_{i\in\mathcal{V}} |z_i(0)|\sqrt{c_i} e^{-(\varrho_1 - 0.5\gamma_1)t}.
\end{aligned}$$

Therefore, $|\omega_i| \leq \gamma_1|\mathrm{w}_i(0)| + \rho_1(b^* + 2(n-1)a^*) \max_{i\in\mathcal{V}} |z_i(0)|\sqrt{c_i}$, if we choose $\gamma_1 < 2\varrho_1$. ■

A. Discussions on the case without self damping term

In this section, we consider the case without self damping term $b_i(t)z_i$ and show that all $z_i(t)$, $i \in \mathcal{V}$, synchronize to a possible non-zero final state. The algorithm takes the following form for all the spacecraft

$$\omega_i = -\gamma_2 \mathrm{w}_i - \mathbf{j}\frac{\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(z_i - z_j)}{\overline{\mathrm{w}_i}}, \quad \forall i \in \mathcal{V}, \tag{8}$$

where $a_{ij}(t) > 0$ is the weight of arc (j, i) for $i, j \in \mathcal{V}$ at t and γ_2 is a positive constant to be determined.

Proposition 4.1. *Suppose that Assumption 3.1 holds and $\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected. Also assume that $z_i(0) \in (-\pi, \pi]$, $\mathrm{w}_i(0) \neq 0$ and bounded, for all $i \in \mathcal{V}$. For multiple underactuated spacecraft (2), algorithm (8) guarantees that full attitude synchronization is achieved if γ_2 is chosen sufficiently small. In particular, it follows that*

- *$\mathrm{w}_i(t) \neq 0$, for all $i \in \mathcal{V}$ and for all $t \geq 0$.*
- *$\lim_{t\to\infty} \mathrm{w}_i(t) = 0$, for all $i \in \mathcal{V}$.*
- *$\lim_{t\to\infty}(z_i(t) - z_j(t)) = 0$, for all $i, j \in \mathcal{V}$.*
- *The control input ω_i remains bounded for all $i \in \mathcal{V}$ and all $t \geq 0$.*

Proof. It is clear that the closed-loop system (2) with distributed attitude algorithm (8) can be written as

$$\dot{\mathrm{w}}_i = -\frac{\gamma_2}{2}(1+|\mathrm{w}_i|^2)\mathrm{w}_i - \mathbf{j}\frac{1}{2}\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(z_i - z_j)\left(\frac{1}{\overline{\mathrm{w}_i}} - \mathrm{w}_i\right), \tag{9a}$$

$$\dot{z}_i = -\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(z_i - z_j), \quad i \in \mathcal{V}. \tag{9b}$$

Note that the above equations hold for $(\mathbb{C}\backslash\{0\}) \times \mathbb{R} \times \cdots \times (\mathbb{C}\backslash\{0\}) \times \mathbb{R}$.

Following the similar analysis of Theorem 4.1, we can show that $\mathrm{w}_i(t) \neq 0$ for all $t \geq 0$ and $\lim_{t\to\infty} \mathrm{w}_i(t) = 0$, for all $i \in \mathcal{V}$.

We next show that $\lim_{t\to\infty}(z_i(t) - z_j(t)) = 0$, for all $i, j \in \mathcal{V}$. Consider the following function $U = \max_{\{i,j\}\in\mathcal{V}\times\mathcal{V}} V_{ij}$, where $V_{ij}(z) = (z_i(t) - z_j(t))^2$ for all $\{i, j\} \in \mathcal{V}\times\mathcal{V}$ and $z = [z_1, z_2, \ldots, z_n]^{\mathrm{T}}$. We first establish the following claim (proof can be found in Appendix).

Claim I: $U(z(t)) \leq \rho_2 e^{-\varrho_2 t} U(z(0))$, where $U(z(0)) = \max_{\{i,j\}\in\mathcal{V}\times\mathcal{V}} V_{ij}(z(0))$, $\rho_2 = \frac{1}{1-\beta_*^N}$ and $\varrho_2 = \frac{1}{NT_1}\ln\frac{1}{1-\beta_*^N}$ with $N = \frac{n(n-1)}{2}$, $T_1 = T^* + 2\tau_d^*$ and $\beta_* = (1 - e^{-((2n-3)a^*+a_*)\tau_d^*})\frac{a_*}{(2n-3)a^*+a_*}e^{-2(n-1)a^* N T_1}$.

It follows from Claim I that $\lim_{t\to\infty} U(t) = 0$ and thus $\lim_{t\to\infty}(z_i(t) - z_j(t)) = 0$, for all $i, j \in \mathcal{V}$.

We finally show that ω_i is bounded, for all $i \in \mathcal{V}$ using Claim I. It follows from the fact $\max_{\{i,j\}\in\mathcal{V}\times\mathcal{V}} |z_i(t) - z_j(t)| \leq \sqrt{\rho_2}e^{-\frac{\varrho_2}{2}t}\sqrt{U(z(0))}$ that for all $i \in \mathcal{V}$

$$\begin{aligned}|\omega_i(t)| \leq\ & \gamma_2|\mathrm{w}_i(t)| + \frac{(n-1)a^*\max_{\{i,j\}\in\mathcal{V}\times\mathcal{V}}|z_i(t)-z_j(t)|}{|\mathrm{w}_i(t)|}\\ \leq\ & \gamma_2|\mathrm{w}_i(0)| + \sqrt{\rho_2}(n-1)a^*\sqrt{U(z(0))}e^{-0.5\varrho_2 t}\sqrt{c_i e^{\gamma_2 t} - 1}\\ <\ & \gamma_2|\mathrm{w}_i(0)| + \sqrt{\rho_2}(n-1)a^*\sqrt{c_i}\sqrt{U(z(0))}e^{-0.5(\varrho_2-\gamma_2)t}.\end{aligned}$$

Therefore, $|\omega_i| \leq \gamma_2|\mathrm{w}_i(0)| + \sqrt{\rho_2}(n-1)a^*\sqrt{c_i}\sqrt{U(z(0))}$, if we choose that $\gamma_2 < \varrho_2$. ■

Proposition 4.1 is verified by the simulation in Figs. 7 and 8. In particular, the communication topologies are chosen the same as in Section III-B. The control gain γ_2 is chosen as $\gamma_2 = 0.1$. Figs. 7 and 8 show, respectively, the trajectories of attitudes w_{i1}, w_{i2}, z_i and the control inputs ω_{i1} and ω_{i2}, for $i = 1, 2, 3, 4$. We see that attitude synchronization is achieved while the control inputs remain bounded.

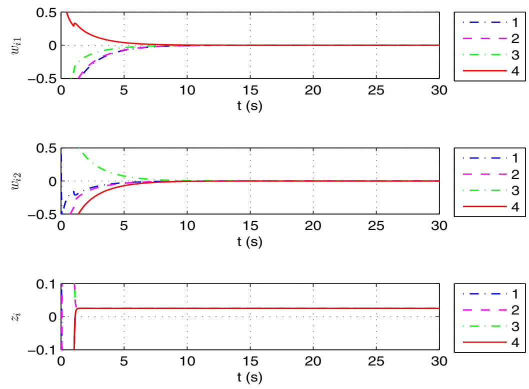

Fig. 7. Attitude trajectories

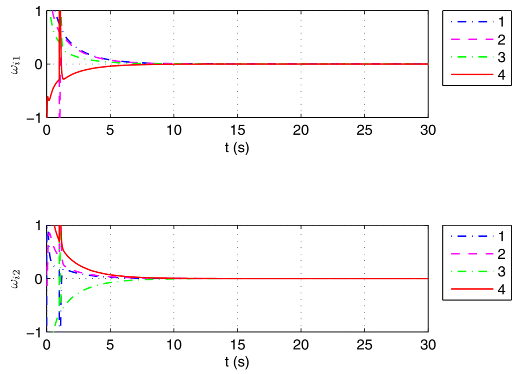

Fig. 8. Control input trajectories

V. Concluding Remarks

We studied attitude coordinated control problem of multiple underactuated spacecraft in this paper. A special parametrization was used to describe the attitude kinematics. We proposed partial attitude coordination protocols and full attitude coordination protocols. The symmetry axes of all the spacecraft were shown to be aligned under a general connectivity assumption. The discussions on the cases without self damping term provided some insight into the collective behaviors of multiple underactuated systems. Future work includes the extension of the kinematic study in this paper to the dynamic case and using rotation matrix as attitude parameterization to avoid singularity.

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Appendix: The Proof of Claim I

Proof. Let $\mathcal{V}^1 \times \mathcal{V}^2$ be the set containing all the agent pairs that reach the maximum of V_{ij}, $\{i,j\} \in \mathcal{V} \times \mathcal{V}$ at time t, i.e., $\mathcal{V}^1(t) \times \mathcal{V}^2(t) = \{\{i,j\} \in \mathcal{V} \times \mathcal{V} | V_{ij}(t) = U(t)\}$. The derivative of U can be calculated as $D^+U \leq 0$. Using comparison lemma (using 0 and $U(z(0))$ as the roles of $f(t,u)$ and u_0 for Lemma 3.4 of [29]) , we know that $U(z(t)) \leq U(z(0))$, for all $t \geq 0$.

Define $T_1 = T^* + 2\tau_d^*$. We analyze the trajectory of $U(z(t))$ during the time interval $[0, NT_1]$, where $N = \frac{n(n-1)}{2}$. Based on the definition of U, we know that for all $t \in [0, NT_1]$, $V_{ij}(z(t)) \leq U^*$, $\forall \{i,j\} \in \mathcal{V} \times \mathcal{V}$, where $U^* := U(z(0))$. Let us first consider agent $i_1 \in \mathcal{V}$ and focus on the time interval $[0, T_1]$. Since agent i_1 is the root, it follows from uniformly jointly strongly connected assumption that there exists a time t_1 and an agent $i_2 \in \mathcal{V} \setminus \{i_1\}$ such that $(i_1, i_2) \in \mathcal{E}$ during $t \in [t_1, t_1 + \tau_d^*) \subset [0, T_1]$. Then for all $t \in [t_1, t_1 + \tau_d^*)$, we know that $\dot{V}_{i_1 i_2} \leq -\alpha(V_{i_1 i_2} - \frac{(2n-3)a^*}{\alpha} U^*)$, where $\alpha = (2n-3)a^* + a_*$. It thus follows that $V_{i_1 i_2} \leq \hat{\alpha}_1 U^*$, where $\hat{\alpha}_1 = (1 - e^{-\alpha \tau_d^*}) \frac{(2n-3)a^*}{\alpha} + e^{-\alpha \tau_d^*} < 1$. Then, we know that for all $t \in [t_1 + \tau_d^*, NT_1]$,

$$\begin{aligned} \dot{V}_{i_1 i_2} \leq & - \sum_{k \in \mathcal{N}_{i_1}(\sigma(t))} a_{i_1 k}(t)(V_{i_1 i_2} - V_{i_2 k}) \\ & - \sum_{k \in \mathcal{N}_{i_2}(\sigma(t))} a_{i_2 k}(t)(V_{i_1 i_2} - V_{i_1 k}) \\ \leq & - 2(n-1)a^*(V_{i_1 i_2} - U^*). \end{aligned}$$

This implies that for all $t \in [t_1 + \tau_d^*, NT_1]$, $V_{i_1 i_2}(t) \leq \left((1 - e^{-\overline{\alpha} NT_1}) + e^{-\overline{\alpha} NT_1} \hat{\alpha}_1\right) U^*$, where $\overline{\alpha} = 2(n-1)a^*$. Note that $1 - (1 - \hat{\alpha}_1) e^{-\overline{\alpha} NT_1} < 1$. Therefore, we know that for all $t \in [t_1 + \tau_d^*, NT_1]$,

$$V_{i_1 i_2}(z(t)) \leq \alpha_1^* U^*,$$

where $\alpha_1^* = (1 - (1 - e^{-\alpha \tau_d^*}) \frac{a_*}{\alpha} e^{-\overline{\alpha} NT_1})$. We next focus on the time interval $[T_1, 2T_1]$ and consider $\mathcal{V}_1 = \{i_1, i_2\}$ together. We know from uniformly jointly strongly connected assumption that there exists a time instant t_2 and an arc from $h \in \mathcal{V}_1$ to $i_3 \in \mathcal{V} \setminus \{i_1, i_2\}$ during $[t_2, t_2 + \tau_d^*) \subset [T_1, 2T_1]$. We next bound V_{hi_3}. This involves two different cases.

Case I: $h = i_1$. Following the similar analysis on $V_{i_1 i_2}$, we know that for all $t \in [t_2 + \tau_d^*, NT_1]$,

$$V_{i_1 i_3}(z(t)) \leq \alpha_1^* U^*.$$

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Case II: $h \in \mathcal{V}_1 \backslash \{i_1\}$. It then follows that for all $t \in [t_2, t_2 + \tau_d^*)$, $\dot{V}_{i_1 i_3} \leq -a_{i_3 h}(V_{i_1 i_3} - V_{i_1 h})$. To this end, we have two subcases.

Case II.a: $V_{i_1 i_3}(z(t)) > V_{i_1 h}(z(t))$ for all $t \in [t_2, t_2 + \tau_d^*)$. It then follows that

$$\begin{aligned} \dot{V}_{i_1 i_3} \leq & - (2n-3)a^*(V_{i_1 i_3} - U^*) - a_*(V_{i_1 i_3} - \alpha_1^* U^*) \\ \leq & - \alpha(V_{i_1 i_3} - \frac{(2n-3)a^* + a_* \alpha_1^*}{\alpha} U^*) \end{aligned}$$

This shows that $V_{i_1 i_3}(z(t_2 + \tau_d^*)) \leq \hat{\alpha}_2 U^*$, where $\hat{\alpha}_2 = 1 - (1 - e^{-\alpha \tau_d^*}) \frac{a_*(1-\alpha_1^*)}{\alpha}$. It follows that for all $t \in [t_2 + \tau_d^*, NT_1]$,

$$V_{i_1 i_3}(z(t)) \leq \left(1 - (1 - \hat{\alpha}_2) e^{-\overline{\alpha} N T_1}\right) U^* = (1 - \beta_*^2) U^*,$$

where $\beta_* = (1 - e^{-\alpha \tau_d^*}) \frac{a_*}{\alpha} e^{-\overline{\alpha} N T_1}$.

Case II.b: there exists a time $t^* \in [t_2, t_2 + \tau_d^*]$ such that $V_{i_1 i_3}(z(t^*)) \leq V_{i_1 h}(z(t^*)) \leq \alpha_1^* U^*$. It then follows that for all $t \in [t^*, NT_1]$, $V_{i_1 i_3}(t) \leq \left(1 - e^{-\overline{\alpha} N T_1}(1 - \alpha_1^*)\right) U^*$.

Combining the above analysis, it is not hard to show that for all $t \in [t_2 + \tau_d^*, NT_1]$, $V_{i_1 i_3}(z(t)) \leq (1 - \beta_*^2) U^*$. Then, we consider $\mathcal{V}_2 = \{i_1, i_2, i_3\}$ together. We have actually shown that for all $t \in [2T_1, NT_1]$, $V_{i_1 k}(z(t)) \leq (1 - \beta_*^2) U^*$, for all $k \in \mathcal{V}_2 \backslash \{i_1\}$, since $(1 - \beta_*^2) \geq \alpha_1^* = 1 - \beta_*$.

Continuing the above analysis, we can show that for all $t \in [(n-1)T_1, NT_1]$ and for all $k \in \mathcal{V} \backslash \{i_1\}$,

$$V_{i_1 k}(z(t)) \leq (1 - \beta_*^{n-1}) U^*. \tag{10}$$

Next, we consider agent i_2 as the root. Note that we have shown that $V_{i_2 i_1}$ satisfies (10). Therefore, we can similarly show that that for all $t \in [(2n-3)T, NT_1]$, $V_{i_2 k}(z(t)) \leq (1 - \beta_*^{2n-3}) U^*$, for all $k \in \mathcal{V} \backslash \{i_2\}$.

Finally, it follows that for all $i, j \in \mathcal{V}$,

$$V_{ij}(z(NT_1)) \leq (1 - \beta_*^N) U^*.$$

Then, let ψ be the smallest positive integer satisfying $t \leq \psi N T_1$. It then follows that

$$\begin{aligned} U(z(t)) & \leq (1 - \beta_*^N)^{\psi - 1} U^* \\ & \leq \frac{1}{1 - \beta_*^N} (1 - \beta_*^N)^{\frac{t}{NT_1}} U^* \\ & = \rho_2 e^{-\varrho_2 t} U^*, \end{aligned}$$

where $\varrho_2 = \frac{1}{NT_1} \ln \frac{1}{1 - \beta_*^N}$ and $\rho_2 = \frac{1}{1 - \beta_*^N}$. This proves the desired claim. ■

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