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Brief paper

Coordinated output regulation of heterogeneous linear systems under switching topologies[☆]

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ABSTRACT

In this paper, we construct a framework to describe and study the coordinated output regulation problem for multiple heterogeneous linear systems. Each agent is modeled as a general linear multiple-input multiple-output system with an autonomous exosystem which represents the individual offset from the group reference for the agent. The multi-agent system as a whole has a group exogenous state which represents the tracking reference for the whole group. Under the constraints that the group exogenous output is only locally available to each agent and that the agents have only access to their neighbors' information, we propose observer-based feedback controllers to solve the coordinated output regulation problem using output feedback information. A high-gain approach is used and the information interactions are allowed to be switching over a finite set of networks containing both graphs that have a directed spanning tree and graphs that do not. Simulations are shown to validate the theoretical results.

1. Introduction

Coordinated control of multi-agent systems has recently drawn large attention due to its broad applications in physical, biological, social, and mechanical systems (Bai, Arcak, & Wen, 2011; Chopra & Spong, 2009; Cortes, Martinez, & Bullo, 2006; Meng, Dimarogonas, & Johansson, 2014; Meng et al., 2013; Tanner, Jadbabaie, & Pappas, 2007). The key idea of a coordination algorithm is to realize a global emergence using only local information interactions (Jadbabaie, Lin, & Morse, 2003; Olfati-Saber, Fax, & Murray, 2007). The coordination problem of a single-integrator network has been fully studied with an emphasis on the system robustness to the input time delays and switching communication topologies (Blondel, Hendrickx, Olshevsky, & Tsitsiklis, 2005; Jadbabaie et al., 2003; Olfati-Saber et al., 2007; Ren & Beard, 2005), discrete-time dynamical models (Moreau, 2005; You & Xie, 2011), nonlinear couplings (Lin, Francis, & Maggiore, 2007), convergence speed (Cao, Morse, & Anderson, 2008), and leader–follower tracking (Shi, Hong, & Johansson, 2012). The coordination of multiple general linear dynamic systems has recently been studied. For example, the authors of Wieland, Kim, and Allgöwer (2011) generalize the coordination of multiple single-integrator systems to the case of multiple linear time-invariant high-order systems. For a network of neutrally stable systems and polynomially unstable systems, the author of Tuna (2009) proposes a design scheme for achieving synchronization. The case of switching communication topologies is considered in Scardovi and Sepulchre (2009) and a so-called consensus-based observer is proposed to guarantee leaderless synchronization of multiple identical linear dynamic systems under a jointly connected communication topology. Similar problems are also considered in Ni and Cheng (2010) and Wang, Cheng, and Hu (2008), where a frequently connected communication topology is studied in Wang et al. (2008) and an assumption on the neutral stability is imposed in Ni and Cheng (2010). The authors of Li, Duan, Chen, and Huang (2010) propose a neighbor-based observer to solve the synchronization problem for general linear time-invariant systems. In addition, the classical Laplacian matrix is generalized in Yang, Roy, Wan, and Saberi (2011) to a so-called interaction matrix and a D-scaling approach is used to stabilize this interaction

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matrix. Synchronization of multiple heterogeneous linear systems has been investigated under both fixed and switching communication topologies (Alvergue, Pandey, Gu, & Chen, 2013; Grip, Yang, Saberi, & Stoorvogel, 2012; Lunze, 2012; Wieland, Sepulchre, & Allgöwer, 2011). In Grip et al. (2012), a high-gain approach is proposed to dominate the non-identical dynamics of the agents. The cases of frequently connected and jointly connected communication topologies are studied in Kim, Shim, Back, and Seo (2013) and Vengertsev, Kim, Shim, and Seo (2010), respectively, where a slow switching condition and a fast switching condition are presented. Recently, the generalizations of coordination of multiple linear dynamic systems to the cooperative output regulation problem are studied in Ding (2013), Kim, Shim, and Seo (2011), Su and Huang (2012), Wang, Hong, Huang, and Jiang (2012) and Xiang, Wei, and Li (2009). In addition, the study on the synchronization of homogeneous and heterogeneous networks with nonlinear couplings is considered in Cao, Chen, and Li (2008), Cao, Wang, and Sun (2007) and He, Du, Qian, and Cao (2013).

In this paper, we generalize the classical output regulation problem of a single linear system to the coordinated output regulation problem of multiple heterogeneous linear systems. We consider the case where each agent has an individual offset and simultaneously there is a group tracking reference. The individual offset and the group reference are generated by autonomous systems (*i.e.*, systems without inputs). Each individual offset is available to its corresponding agent while the group reference can be obtained only through constrained communication among the agents, *i.e.*, the group reference trajectory is available to only a subset of the agents. Our goal is to find an observer-based feedback controller for each agent such that the output of each agent converges to a given trajectory determined by the combination of the individual offset and the group reference. Motivated by the approach in Grip et al. (2012), we propose a unified observer to solve the coordinated output regulation problem of multiple heterogeneous general linear systems, where the open-loop poles of the agents can be exponentially unstable and the dynamics are allowed to be different both with respect to dimensions and parameters. This relaxes the common assumption of identical dynamics (Li et al., 2010; Ni & Cheng, 2010; Scardovi & Sepulchre, 2009; Su & Huang, 2012; Tuna, 2009; Vengertsev et al., 2010; Xiang et al., 2009), or open-loop poles at most polynomially unstable (Ni & Cheng, 2010; Scardovi & Sepulchre, 2009; Su & Huang, 2012; Wieland, Sepulchre et al., 2011), or relative degree and minimum phase requirement (Kim et al., 2011). In addition, in this work, the information interaction is allowed to be switching from a graph set containing both a directed spanning tree set and a disconnected graph set. This extends the existing works considering fixed communication topologies (Grip et al., 2012; Kim et al., 2011; Li et al., 2010; Tuna, 2009; Wang et al., 2012).

The remainder of the paper is organized as follows. In Section 2, we give some basic definitions on the network model. In Section 3, we formulate the problem of coordinated output regulation of multiple heterogeneous linear systems. We then propose the state feedback control law with a unified observer design in Section 4. Numerical studies are carried out in Section 5 to validate our design and a brief concluding remark is drawn in Section 6.

2. Network model

We use graph theory to model the communication topology among agents. A directed graph G consists of a pair $(\mathbf{V}, \mathbf{E})$, where $\mathbf{V} = \{\nu_1, \nu_2, \ldots, \nu_n\}$ is a finite, nonempty set of nodes and $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$ is a set of ordered pairs of nodes. An edge (ν_i, ν_j) denotes that node ν_j can obtain information from node ν_i. All neighbors of node ν_i are denoted as $N_i := \{\nu_j | (\nu_j, \nu_i) \in \mathbf{E}\}$. For an edge (ν_i, ν_j) in a directed graph, ν_i is the parent node and ν_j is the child node. A directed path in a directed graph is a sequence of edges of the form $(\nu_i, \nu_j), (\nu_j, \nu_k), \ldots$. A directed tree is a directed graph, where every node has exactly one parent except for one node, called the root, which has no parent, and the root has a directed path to every other node. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all other nodes.

For a leader–follower graph $\bar{G} := (\bar{\mathbf{V}}, \bar{\mathbf{E}})$, we have $\bar{\mathbf{V}} = \{\nu_0, \nu_1, \ldots, \nu_n\}$, $\bar{\mathbf{E}} \subseteq \bar{\mathbf{V}} \times \bar{\mathbf{V}}$, where ν_0 is the leader and $\nu_1, \nu_2, \ldots, \nu_n$ denote the followers. The leader–follower adjacency matrix $\bar{A} = [a_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$ is defined such that a_{ij} is positive if $(\nu_j, \nu_i) \in \bar{\mathbf{E}}$ while $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$, $i = 0, 1, \ldots, n$, and the leader has no parent, *i.e.*, $a_{0j} = 0$, $j = 0, 1, \ldots, n$. The leader–follower "grounded" Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\bar{A}$ is defined as $l_{ii} = \sum_{j=0}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

We assume that the leader–follower communication topology $\bar{G}_{\sigma(t)}$ is time-varying and switched from a finite set $\{\bar{G}_k\}_{k\in\Gamma}$, where $\Gamma = \{1, 2, \ldots, \delta\}$ is an index set and $\delta \in \mathbb{N}$ indicates its cardinality. We impose the technical condition that $\bar{G}_{\sigma(t)}$ is right continuous, where $\sigma : [t_0, \infty) \to \Gamma$ is a piecewise constant function of time, i.e., $\bar{G}_{\sigma(t)}$ remains constant for $t \in [t_\ell, t_{\ell+1})$, $\ell = 0, 1, \ldots$ and switches at $t = t_\ell$, $\ell = 1, 2, \ldots$. In addition, we assume that $\inf_\ell (t_{\ell+1} - t_\ell) \geq \tau_d > 0$, $\ell = 0, 1, \ldots$, with $\lim_{\ell\to\infty} t_\ell = \infty$, where τ_d is a constant known as the dwell time (Liberzon & Morse, 1999).

Let the sets $\{\bar{A}_k\}_{k\in\Gamma}$ and $\{L_k\}_{k\in\Gamma}$ be the leader–follower adjacency matrices and leader–follower grounded Laplacian matrices associated with $\{\bar{G}_k\}_{k\in\Gamma}$, respectively. Consequently, the time-varying leader–follower adjacency matrix and time-varying leader–follower grounded Laplacian matrix are defined as $\bar{A}_{\sigma(t)} = [a_{ij}(t)]$ and $L_{\sigma(t)} = [l_{ij}(t)]$.

Other notations in this paper: $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote, respectively, the minimum and maximum eigenvalues of a real symmetric matrix P, P^{T} denotes the transpose of P, I_n denotes the $n \times n$ identity matrix, and $\mathrm{diag}(A_1, A_2, \ldots, A_n)$ denotes a block diagonal matrix with the main diagonal blocks matrices. A square matrix A is called a Hurwitz matrix if every eigenvalue of A has strictly negative real part.

3. Problem formulation

3.1. Agent dynamics

Suppose that we have n agents modeled by the linear multiple-input multiple-output (MIMO) systems for each $\nu_i \in \mathbf{V}$:

$$\dot{x}_i = A_i x_i + B_i u_i, \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the agent state, $u_i \in \mathbb{R}^{m_i}$ is the control input, $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times m_i}$, and n_i and m_i are positive integers, for all $\nu_i \in \mathbf{V}$.

Also suppose that there is an individual autonomous exosystem for each $\nu_i \in \mathbf{V}$:

$$\dot{\omega}_i = S_i \omega_i, \tag{2}$$

where $\omega_i \in \mathbb{R}^{q_i}$, $S_i \in \mathbb{R}^{q_i\times q_i}$, and q_i is a positive integer, for all $\nu_i \in \mathbf{V}$.

In addition, there is a group autonomous exosystem for the multi-agent system as a whole:

$$\dot{x}_0 = A_0 x_0, \tag{3}$$

where $x_0 \in \mathbb{R}^{n_0}$, $A_0 \in \mathbb{R}^{n_0\times n_0}$, and n_0 is a positive integer.

3.2. Available information for agents

For the individual autonomous exosystem tracking, available output information for each agent $\nu_i \in \mathbf{V}$ is $y_{si} = C_{si} x_i + C_{wi} \omega_i$, where $y_{si} \in \mathbb{R}^{p_1}$, $C_{si} \in \mathbb{R}^{p_1\times n_i}$, $C_{wi} \in \mathbb{R}^{p_1\times q_i}$, and p_1 is a positive integer.

For the group autonomous exosystem tracking, only neighbor-based output information is available due to the constrained communication. This means that not all the agents have access to y_0. The available information is the neighbor-based sum of each agent's own output relative to that of its' neighbors, *i.e.,* $\zeta_i = \sum_{j=0}^{n} a_{ij}(t)(y_{di} - y_{dj})$ is available for each agent $\nu_i \in \mathbf{V}$, where $a_{ij}(t)$, $i = 0, 1, \ldots, n$, $j = 0, 1, \ldots, n$, is entry (i, j) of the adjacency matrix $\overline{A}_{\sigma(t)}$ associated with $\overline{G}_{\sigma(t)}$ defined in Section 2 at time t, $\zeta_i \in \mathbb{R}^{p_2}$, $i = 1, 2, \ldots, n$, y_{di} is represented by $y_{di} = C_{di}x_i$, $i = 1, 2, \ldots, n$ and $y_{d0} = C_0 x_0$, where $C_{di} \in \mathbb{R}^{p_2 \times n_i}$, $i = 1, 2, \ldots, n$, $C_0 \in \mathbb{R}^{p_2 \times n_0}$, $y_{di} \in \mathbb{R}^{p_2}$, $i = 0, 1, \ldots, n$, and p_2 is a positive integer. Also, the relative estimation information is available using the same communication topologies, *i.e.,* $\widehat{\zeta}_i = \sum_{j=0}^{n} a_{ij}(t)(\widehat{y}_i - \widehat{y}_j)$ is available for each agent $\nu_i \in \mathbf{V}$, where $\widehat{y}_i$ is an estimate produced internally by each agent $\nu_i \in \mathbf{V}$, $\widehat{\zeta}_i \in \mathbb{R}^{p_2}$, $i = 1, 2, \ldots, n$ and $\widehat{y}_i \in \mathbb{R}^{p_2}$, $i = 0, 1, \ldots, n$, which will be given explicitly in Section 4.

3.3. Switching topologies

For the communication topology set $\{\overline{G}_k\}_{k \in \Gamma}$, we assume that $\overline{G}_k$, $\forall k \in \Gamma_c$, is a graph containing a directed spanning tree with ν_0 rooted. Without loss of generality, we relabel $\Gamma_c := \{1, 2, \ldots, \delta_1\}$, $1 \le \delta_1 \le \delta$. The remaining graphs are labeled as $\overline{G}_k$, $\forall k \in \Gamma_d$, where $\Gamma_d := \{\delta_1 + 1, \delta_1 + 2, \ldots, \delta\}$. Denote the graph set $\overline{\mathbb{G}}_c = \{\overline{G}_k\}_{k \in \Gamma_c}$ and the graph set $\overline{\mathbb{G}}_d = \{\overline{G}_k\}_{k \in \Gamma_d}$, respectively. We also denote $T^d_{\bar{t}_0}(t)$ and $T^c_{\bar{t}_0}(t)$ as the total activation time when $\overline{G}_{\sigma(\varsigma)} \in \overline{\mathbb{G}}_d$ and total activation time when $\overline{G}_{\sigma(\varsigma)} \in \overline{\mathbb{G}}_c$, respectively, during $\varsigma \in [\bar{t}_0, t)$ for $\bar{t}_0 \ge t_0$.

Assumption 1. The dwell time τ_d is a positive constant.

Assumption 2. There exist positive constants κ and $\bar{t}_0 \ge t_0$ such that $T^c_{\bar{t}_0}(t) \ge \kappa T^d_{\bar{t}_0}(t)$ for all $t \ge \bar{t}_0$.

3.4. Control objective and control architecture

The control objective of each agent is to track a given trajectory determined by the combination of the group reference x_0 and the individual offset ω_i, $i = 1, 2, \ldots, n$. Such a combination is captured by the coordinated output regulation tracking error (*i.e.,* the total tracking error representing the combination of both individual tracking and group tracking of each agent):

$$e_i = D_{si}x_i + D_{wi}\omega_i + D_0 x_0, \tag{4}$$

where $D_{si} \in \mathbb{R}^{p_3 \times n_i}$, $D_{wi} \in \mathbb{R}^{p_3 \times q_i}$, $e_i \in \mathbb{R}^{p_3}$, $i = 1, 2, \ldots, n$, $D_0 \in \mathbb{R}^{p_3 \times n_0}$, and p_3 is a positive integer. Thus, our objective is to guarantee that $\lim_{t \to \infty} e_i(t) = 0$. One example of the overall control can correspond to a formation control problem, where ω_i encodes the relative position between each agent and the leader while the leader x_0 defines the overall motion of the group.

Our goal is to design an observer-based controller with available individual output information and neighbor-based group output information to solve this problem. The control of each agent is supposed to have the structure depicted in Fig. 1. In the next section, we will specify the design procedure.

4. Coordinated output regulation with unified observer design

As suggested by Fig. 1, the design procedure to solve the coordinated output regulation problem includes two main steps: the first one is the state feedback control design and the second one is the observer design for the group autonomous exosystem, the individual autonomous exosystem, and the internal state information for each agent.

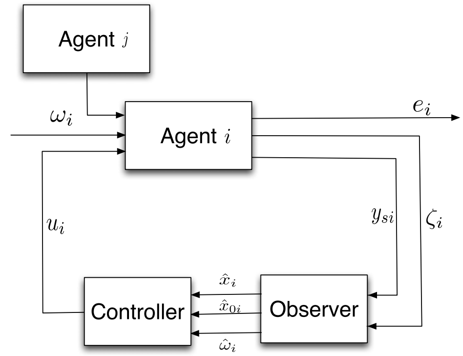

Fig. 1. Control architecture for agent ν_i.

4.1. Redundant modes

Before designing the state feedback control and distributed observer, we need first to remove the redundant modes that have no effect on y_{si} and $y_{di} - y_{d0}$. We impose the following assumptions on the structure of the systems.

Assumption 3.
- $\left(A_i, \begin{bmatrix} C_{si} \\ C_{di} \end{bmatrix}\right)$, $i = 1, 2, \ldots, n$ is observable.
- (S_i, C_{wi}), $i = 1, 2, \ldots, n$ is observable.
- (A_0, C_0), $i = 1, 2, \ldots, n$ is observable.

We write the state and output of each agent in the compact form: $\begin{bmatrix} \dot{x}_i \\ \dot{\omega}_i \\ \dot{x}_0 \end{bmatrix} = \begin{bmatrix} A_i & 0 & 0 \\ 0 & S_i & 0 \\ 0 & 0 & A_0 \end{bmatrix} \begin{bmatrix} x_i \\ \omega_i \\ x_0 \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix} u_i$, and $\begin{bmatrix} y_{si} \\ y_{di} - y_{d0} \end{bmatrix} = \begin{bmatrix} C_{si} & C_{wi} & 0 \\ C_{di} & 0 & -C_0 \end{bmatrix} \begin{bmatrix} x_i \\ \omega_i \\ x_0 \end{bmatrix}$.

Given that Assumption 3 is satisfied, we can perform the state transformation given in Step 1 of Grip et al. (2012) by considering ω_i and x_0 together. We construct a new state $\bar{x}_i = W_i \begin{bmatrix} x_i \\ \omega_i \\ x_0 \end{bmatrix}$ with the dynamics

$$\dot{\bar{x}}_i = \overline{A}_i \bar{x}_i + \overline{B}_i u_i = \begin{bmatrix} A_i & \overline{A}_{i12} \\ 0 & \overline{A}_{i22} \end{bmatrix} \bar{x}_i + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i, \tag{5a}$$

$$\begin{bmatrix} y_{si} \\ e_{di} \end{bmatrix} = \overline{C}_i \bar{x}_i = \begin{bmatrix} C_{si} & \overline{C}_{i21} \\ C_{di} & \overline{C}_{i22} \end{bmatrix} \bar{x}_i, \tag{5b}$$

where $e_{di} = y_{di} - y_{d0}$, and the details of $W_i, \overline{A}_i, \overline{B}_i, \overline{C}_i$ are given in Grip et al. (2012). It was shown that the pair $(\overline{A}_i, \overline{C}_i)$ is observable and the eigenvalues of $\overline{A}_{i22}$ are a subset of the eigenvalues of S_i and A_0, $i = 1, 2, \ldots, n$.

4.2. Regulated state feedback control law

We now design a controller to regulate e_i to zero for each agent based on the state information $\bar{x}_i = \left[\bar{x}_{i1}^{\mathrm{T}}, \bar{x}_{i2}^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\bar{x}_{i1} \in \mathbb{R}^{n_i}$. We impose the following assumptions on the structure of the systems.

Assumption 4.
- (A_i, B_i) is stabilizable, $i = 1, \ldots, n$.
- (A_i, B_i, D_{si}) is right-invertible, $i = 1, \ldots, n$.
- (A_i, B_i, D_{si}) has no invariant zeros in the closed right-half complex plane that coincide with the eigenvalues of S_i or A_0, $i = 1, \ldots, n$.

Lemma 1. *Let Assumption 4 hold. Then, the regulator equations (6) are solvable and the state-feedback controller* $u_i = F_i(\bar{x}_{i1} -$

$\Pi_i\bar{x}_{i2}) + \Gamma_i\bar{x}_{i2}$ ensures that $\lim_{t\to\infty} e_i(t) = 0$, $i = 1, 2\ldots, n$, where Π_i, Γ_i are the solutions of the equations

$$\Pi_i\bar{A}_{i22} = A_i\Pi_i + \bar{A}_{i12} + B_i\Gamma_i, \tag{6a}$$

$$0 = D_{si}\Pi_i + \begin{bmatrix} D_{wi} & D_0 \end{bmatrix}, \quad i = 1, 2, \ldots, n, \tag{6b}$$

and F_i is chosen such that $A_i + B_iF_i$ is Hurwitz.

Proof. Following from Corollary 2.5.1 of Saberi, Stoorvogel, and Sannuti (2000) and a similar analysis as in the proof of Lemma 3 in Grip et al. (2012), we can show that the regulator equations (6) are solvable given that Assumption 4 is satisfied. Then, by considering $\dot{\bar{x}}_{i2} = \bar{A}_{i22}\bar{x}_{i2}$ as the exosystem and $\dot{x}_i = A_ix_i + B_iu_i$ as the system to be regulated for the classic output regulation result (Francis, 1977), we know that $u_i = F_i(\bar{x}_{i1} - \Pi_i\bar{x}_{i2}) + \Gamma_i\bar{x}_{i2}$ ensures that $\lim_{t\to\infty} e_i(t) = 0$, $i = 1, 2\ldots, n$, where Π_i and Γ_i are the solutions of the regulator equations (6). ■

We next design an observer to estimate $\bar{x}_i$ based on output information y_{si} and ζ_i for each agent.

4.3. Pseudo-identical linear transformation

Note that the individual offset ω_i can be estimated from y_{si} and the group reference x_0 can be estimated from $\widehat{\zeta}_i$. In contrast, the internal state information x_i for each agent can be obtained from either y_{si} or $\widehat{\zeta}_i$. In this section, we use the combination of y_{si} and $\widehat{\zeta}_i$ to develop a unified observer design.

We define $\chi_i = T_i\bar{x}_i \in \mathbb{R}^{p\bar{n}}$, $i = 1, 2, \ldots, n$, where $\bar{n} = n_0 + \max_{i=1,2,\ldots,n}(n_i + q_i)$, $p = p_1 + p_2$, and $T_i = \begin{bmatrix} \bar{C}_i \\ \vdots \\ \bar{C}_i\bar{A}_i^{\bar{n}-1} \end{bmatrix}$. Note that T_i is full column rank since the pair $(\bar{A}_i, \bar{C}_i)$, $i = 1, 2, \ldots, n$ is observable. This implies that $T_i^{\mathrm{T}}T_i$ is nonsingular. Therefore, from (5) and above state transformation, we obtain

$$\dot{\chi}_i = (\mathscr{A} + \mathscr{L}_i)\chi_i + \mathscr{B}_iu_i, \tag{7a}$$

$$\begin{bmatrix} y_{si} \\ e_{di} \end{bmatrix} = \mathscr{C}\chi_i, \quad i = 1, 2, \ldots, n, \tag{7b}$$

where $\mathscr{A} = \begin{bmatrix} 0 & I_{p(\bar{n}-1)} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{p\bar{n}\times p\bar{n}}$, $\mathscr{L}_i = \begin{bmatrix} 0 \\ L_i \end{bmatrix}$, $\mathscr{B}_i = T_i\bar{B}_i$, $\mathscr{C} = \begin{bmatrix} I_p & 0 \end{bmatrix} \in \mathbb{R}^{p\times p\bar{n}}$ for some matrix $L_i \in \mathbb{R}^{p\times p\bar{n}}$.

4.4. Unified observer design

Motivated by Grip et al. (2012), based on the available output information y_{si} and the neighbor-based group output information ζ_i, the distributed observer for (7) is proposed to be

$$\dot{\widehat{\chi}}_i = (\mathscr{A} + \mathscr{L}_i)\widehat{\chi}_i + \mathscr{B}_iu_i + S(\varepsilon)\mathscr{P}\mathscr{C}^{\mathrm{T}} \times \left(\begin{bmatrix} y_{si} \\ \sum_{j=0}^{n} a_{ij}(t)(y_{di} - y_{dj}) \end{bmatrix} - \begin{bmatrix} \widehat{y}_{si} \\ \sum_{j=0}^{n} a_{ij}(t)(\widehat{y}_i - \widehat{y}_j) \end{bmatrix} \right), \quad i = 1, 2\ldots, n, \tag{8}$$

where $a_{ij}(t)$, $i = 0, 1, \ldots, n$, $j = 0, 1, \ldots, n$, is entry (i, j) of the adjacency matrix $\bar{A}_{\sigma(t)}$ associated with $\bar{G}_{\sigma(t)}$ defined in Section 2 at time t,

$$\widehat{y}_{si} = \mathscr{C}_1\widehat{\chi}_i, \quad i = 1, \ldots, n, \tag{9}$$

$$\widehat{y}_i = \mathscr{C}_2\widehat{\chi}_i, \quad i = 1, \ldots, n, \tag{10}$$

$\mathscr{C}_1$ is first p_1 rows of $\mathscr{C}$, $\mathscr{C}_2$ is the remaining p_2 rows of $\mathscr{C}$, and $\widehat{y}_0 = 0$. In addition, $S(\varepsilon) = \mathrm{diag}(I_p\varepsilon^{-1}, I_p\varepsilon^{-2}, \ldots, I_p\varepsilon^{-\bar{n}})$, where $\varepsilon \in (0, 1]$ is a positive constant to be determined, and $\mathscr{P} = \mathscr{P}^{\mathrm{T}}$ is a positive definite matrix satisfying

$$\mathscr{A}\mathscr{P} + \mathscr{P}\mathscr{A}^{\mathrm{T}} - 2\mathscr{P}\mathscr{C}^{\mathrm{T}} \begin{bmatrix} I_{p_1} & 0 \\ 0 & \theta I_{p_2} \end{bmatrix} \mathscr{C}\mathscr{P} + I_{p\bar{n}} = 0, \tag{11}$$

where $\theta = \min_{k\in\Gamma_c} \beta_k$, β_k is a positive constant satisfying $\beta_k < \min \Re\{\lambda(L_k)\}$, $k \in \Gamma_c$, and $\min \Re\{\lambda(L_k)\}$ denotes the minimum value of all the real parts of the eigenvalues of L_k. Note that the existence of $\mathscr{P}$ is due to the fact that $\left(\mathscr{A}, \begin{bmatrix} I_{p_1} & 0 \\ 0 & \sqrt{\theta}I_{p_2} \end{bmatrix}\mathscr{C}\right)$ is observable.

Lemma 2. • *All the eigenvalues of L_k are in the closed right-half plane and those on the imaginary axis are simple, where L_k is associated with $\bar{G}_k$ defined in Section 2, for some $\bar{G}_k \in \{\bar{G}_k\}_{k\in\Gamma}$.*
• *Furthermore, all the eigenvalues of L_k are in the open right-half plane for $\bar{G}_k \in \{\bar{G}_k\}_{k\in\Gamma_c}$.*

Proof. See Theorem 4.29 in Qu (2009) and Lemma 1.6 in Ren and Cao (2011). ■

Lemma 3. *Let Assumptions 1–3 hold and assume that $\kappa \geq \frac{\alpha + 4\max\{\theta,1\}\lambda^2_{\max}(\mathscr{P})}{1-\alpha}$, where $\alpha \in (0, 1)$, θ and $\mathscr{P}$ are given by (11). Then, there exists an $\varepsilon^* \in (0, 1]$ such that, if $\varepsilon \in (0, \varepsilon^*]$, $\lim_{t\to\infty}(\chi_i(t) - \widehat{\chi}_i(t)) = 0$, $i = 1, 2\ldots, n$, for systems (8).*

Proof. Note that for all $i = 1, 2, \ldots, n$, $\sum_{j=0}^{n} a_{ij}(t)(y_{di} - y_{dj}) = \sum_{j=1}^{n} l_{ij}(t)(y_{dj} - y_{d0}) = \sum_{j=1}^{n} l_{ij}(t)e_{dj}$. Define $\widetilde{\chi}_i = \chi_i - \widehat{\chi}_i$. It then follows from (7) and (8) that for all $i = 1, 2\ldots, n$,

$$\dot{\widetilde{\chi}}_i = (\mathscr{A} + \mathscr{L}_i)\widetilde{\chi}_i - S(\varepsilon)\mathscr{P}\mathscr{C}^{\mathrm{T}} \begin{bmatrix} y_{si} - \widehat{y}_{si} \\ \sum_{j=1}^{n} l_{ij}(t)(e_{dj} - \widehat{y}_j) \end{bmatrix},$$

where $l_{ij}(t)$, $i = 1, \ldots, n$, $j = 1, \ldots, n$, is the (i, j)th entry of the adjacency matrix $L_{\sigma(t)}$ associated with $\bar{G}_{\sigma(t)}$ defined in Section 2 at time t. It follows that $\dot{\widetilde{\chi}}_i = (\mathscr{A} + \mathscr{L}_i)\widetilde{\chi}_i - S(\varepsilon)\mathscr{P}\mathscr{C}^{\mathrm{T}} \begin{bmatrix} \mathscr{C}_1\widetilde{\chi}_i \\ \mathscr{C}_2 \sum_{j=1}^{n} l_{ij}(t)\widetilde{\chi}_j \end{bmatrix}$, $i = 1, 2\ldots, n$. By introducing $\xi_i = \varepsilon^{-1}S^{-1}(\varepsilon)\widetilde{\chi}_i$ and after some manipulations, we have that $\varepsilon\dot{\xi}_i = (\mathscr{A} + \mathscr{L}_{i\varepsilon})\xi_i - \mathscr{P}\mathscr{C}^{\mathrm{T}} \begin{bmatrix} \mathscr{C}_1\xi_i \\ \mathscr{C}_2 \sum_{j=1}^{n} l_{ij}(t)\xi_j \end{bmatrix}$, $i = 1, 2\ldots, n$, where $\mathscr{L}_{i\varepsilon} = \begin{bmatrix} 0 \\ \varepsilon^{\bar{n}+1}L_iS(\varepsilon) \end{bmatrix} = O(\varepsilon)$.

Note that $\begin{bmatrix} \mathscr{C}_1\xi_i \\ \mathscr{C}_2\xi_i \end{bmatrix} = \mathscr{C}\xi_i$, for all $i = 1, 2, \ldots, n$. The overall dynamics can be written as

$$\varepsilon\dot{\xi} = \left(I_n \otimes \mathscr{A} + \mathscr{L}_\varepsilon - (I_n \otimes \mathscr{P}\mathscr{C}^{\mathrm{T}}) \times \left(I_n \otimes \begin{bmatrix} I_{p_1} & 0 \\ 0 & 0 \end{bmatrix} + L_\sigma \otimes \begin{bmatrix} 0 & 0 \\ 0 & I_{p_2} \end{bmatrix} \right) (I_n \otimes \mathscr{C}) \right) \xi, \tag{12}$$

where $\xi = [\xi_1^{\mathrm{T}}, \xi_2^{\mathrm{T}}, \ldots, \xi_n^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathscr{L}_\varepsilon = \mathrm{diag}(\mathscr{L}_{1\varepsilon}, \mathscr{L}_{2\varepsilon}, \ldots, \mathscr{L}_{n\varepsilon})$.

Note that $-L_k$, $k \in \Gamma_c$ is a Hurwitz matrix according to Lemma 2. Therefore, we can always guarantee that $-L_k + \beta_kI_n$ is also a Hurwitz matrix by choosing β_k sufficiently small. In particular, we choose β_k as a positive constant satisfying $\beta_k < \min \Re\{\lambda(L_k)\}$, $k \in \Gamma_c$. Then, we define the piecewise Lyapunov function candidate $V_k = \varepsilon\xi^{\mathrm{T}}(P_k \otimes \mathscr{P}^{-1})\xi$, where P_k is a positive definite matrix satisfying

$$P_k(-L_k + \beta_kI_n) + (-L_k + \beta_kI_n)^{\mathrm{T}}P_k = -I_n < 0, \quad k \in \Gamma_c,$$
$$P_k(-L_k) + (-L_k)^{\mathrm{T}}P_k \leq 0, \quad k \in \Gamma_d,$$

where the second inequality is due to Lemma 2.

It then follows that for all $k \in \Gamma_c$,

$$\begin{aligned}
\dot{V}_k \le\ & 2\xi^{\mathrm{T}}\left(P_k \otimes \mathscr{P}^{-1}\mathscr{A}\right)\xi + 2\xi^{\mathrm{T}}\left(P_k \otimes \mathscr{P}^{-1}\right)\mathscr{L}_\varepsilon \xi \\
& - 2\xi^{\mathrm{T}}\left(P_k \otimes \left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} I_{p_1} & 0 \\ 0 & 0\end{bmatrix}\mathscr{C}\right)\right)\xi \\
& - 2\xi^{\mathrm{T}}\left(P_k L_k \otimes \left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} 0 & 0 \\ 0 & I_{p_2}\end{bmatrix}\mathscr{C}\right)\right)\xi \\
\le\ & \xi^{\mathrm{T}}\left(P_k \otimes \left(\mathscr{P}^{-1}\mathscr{A} + \mathscr{A}^{\mathrm{T}}\mathscr{P}^{-1} - 2\theta\mathscr{C}^{\mathrm{T}}\begin{bmatrix} 0 & 0 \\ 0 & I_{p_2}\end{bmatrix}\mathscr{C}\right.\right. \\
& \left.\left. -2\mathscr{C}^{\mathrm{T}}\begin{bmatrix} I_{p_1} & 0 \\ 0 & 0\end{bmatrix}\mathscr{C}\right)\right)\xi + 2\xi^{\mathrm{T}}\left(P_k \otimes \mathscr{P}^{-1}\right)\mathscr{L}_\varepsilon\xi \\
& - \xi^{\mathrm{T}}\left((2P_kL_k - 2\theta P_k) \otimes \left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} 0 & 0 \\ 0 & I_{p_2}\end{bmatrix}\mathscr{C}\right)\right)\xi \\
\le\ & \xi^{\mathrm{T}}\left(P_k \otimes \left(\mathscr{P}^{-1}\left(\mathscr{A}\mathscr{P} + \mathscr{P}\mathscr{A}^{\mathrm{T}}\right.\right.\right. \\
& \left.\left.\left. - 2\mathscr{P}\mathscr{C}^{\mathrm{T}}\begin{bmatrix} I_{p_1} & 0 \\ 0 & \theta I_{p_2}\end{bmatrix}\mathscr{C}\mathscr{P}\right)\mathscr{P}^{-1}\right)\right)\xi \\
& - \xi^{\mathrm{T}}\left(\left(P_kL_k + L_k^{\mathrm{T}}P_k - 2\beta_k P_k\right) \otimes \left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} 0 & 0 \\ 0 & I_{p_2}\end{bmatrix}\mathscr{C}\right)\right)\xi \\
& + 2\lambda_{\max}(P_k)\lambda_{\max}(\mathscr{P}^{-1})\|\mathscr{L}_\varepsilon\|\,\|\xi\|^2 \\
\le\ & -\xi^{\mathrm{T}}\left(P_k \otimes (\mathscr{P}^{-1}\mathscr{P}^{-1})\right)\xi - \xi^{\mathrm{T}}\left(I_n \otimes \left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} 0 & 0 \\ 0 & I_{p_2}\end{bmatrix}\mathscr{C}\right)\right)\xi \\
& + \frac{2\lambda_{\max}(P_k)\lambda_{\max}(\mathscr{P}^{-1})\|\mathscr{L}_\varepsilon\|}{\varepsilon\lambda_{\min}(P_k)\lambda_{\min}(\mathscr{P}^{-1})}V_k \\
\le\ & -\xi^{\mathrm{T}}\left(P_k \otimes (\mathscr{P}^{-1}\mathscr{P}^{-1})\right)\xi + \frac{2\lambda_{\max}(P_k)\lambda_{\max}(\mathscr{P}^{-1})\|\mathscr{L}_\varepsilon\|}{\varepsilon\lambda_{\min}(P_k)\lambda_{\min}(\mathscr{P}^{-1})}V_k \\
\le\ & -\left(\frac{\lambda_{\min}(\mathscr{P}^{-1})}{\varepsilon} - \frac{2\lambda_{\max}(P_k)\lambda_{\max}(\mathscr{P}^{-1})\|\mathscr{L}_\varepsilon\|}{\varepsilon\lambda_{\min}(P_k)\lambda_{\min}(\mathscr{P}^{-1})}\right)V_k,
\end{aligned}$$

where we have used (11) and the fact that $\theta \le \beta_k$, $k \in \Gamma_c$. It then follows that $\dot{V}_k \le -\frac{1}{\varepsilon}\lambda_k V_k$, $\forall k \in \Gamma_c$, if $\|\mathscr{L}_\varepsilon\| < \frac{\lambda_{\min}(P_k)\lambda_{\min}(\mathscr{P})}{4\lambda_{\max}(P_k)\lambda^2_{\max}(\mathscr{P})}$, where $\lambda_k = \frac{1}{2\lambda_{\max}(\mathscr{P})}$, $\forall k \in \Gamma_c$.

On the other hand, for all $k \in \Gamma_d$, we have that

$$\begin{aligned}
\dot{V}_k \le\ & 2\xi^{\mathrm{T}}\left(P_k \otimes (\mathscr{P}^{-1}\mathscr{A})\right)\xi + 2\xi^{\mathrm{T}}\left(P_k \otimes \mathscr{P}^{-1}\right)\mathscr{L}_\varepsilon\xi \\
& - 2\xi^{\mathrm{T}}\left(P_k \otimes \left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} I_{p_1} & 0 \\ 0 & 0\end{bmatrix}\mathscr{C}\right)\right)\xi \\
& - 2\xi^{\mathrm{T}}\left(P_kL_k \otimes \left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} 0 & 0 \\ 0 & I_{p_2}\end{bmatrix}\mathscr{C}\right)\right)\xi \\
\le\ & \xi^{\mathrm{T}}\left(P_k \otimes (\mathscr{P}^{-1}(\mathscr{A}\mathscr{P} + \mathscr{P}\mathscr{A}^{\mathrm{T}})\mathscr{P}^{-1})\right)\xi \\
& + 2\lambda_{\max}(P_k)\lambda_{\max}(\mathscr{P}^{-1})\|\mathscr{L}_\varepsilon\|\,\|\xi\|^2 \\
\le\ & 2\xi^{\mathrm{T}}\left(P_k \otimes \left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} I_{p_1} & 0 \\ 0 & \theta I_{p_2}\end{bmatrix}\mathscr{C}\right)\right)\xi - \frac{\lambda_{\min}(\mathscr{P}^{-1})}{\varepsilon}V_k \\
& + \frac{2\lambda_{\max}(P_k)\lambda_{\max}(\mathscr{P}^{-1})\|\mathscr{L}_\varepsilon\|}{\varepsilon\lambda_{\min}(P_k)\lambda_{\min}(\mathscr{P}^{-1})}V_k,
\end{aligned}$$

where we have used (11). Note that $\lambda_{\max}\left(\mathscr{C}^{\mathrm{T}}\begin{bmatrix} I_{p_1} & 0 \\ 0 & \theta I_{p_2}\end{bmatrix}\mathscr{C}\right) = \max\{\theta, 1\}$. It follows that $\dot{V}_k \le \frac{1}{\varepsilon}\lambda_k V_k$, $\forall k \in \Gamma_d$, if $\|\mathscr{L}_\varepsilon\| < \frac{\lambda_{\min}(P_k)\lambda_{\min}(\mathscr{P})}{2\lambda_{\max}(P_k)\lambda^2_{\max}(\mathscr{P})}$, where $\lambda_k = 2\max\{\theta, 1\}\lambda_{\max}(\mathscr{P})$, $\forall k \in \Gamma_d$.

Following the similar analysis as that in Liberzon and Morse (1999) and Zhai, Hu, Yasuda, and Michel (2000), we let $\sigma = p_j$ on $[t_{j-1}, t_j)$ for $p_j \in \Gamma$. Then, for any t satisfying $t_0 < t_1 < \cdots < t_\ell < t < t_{\ell+1}$, define $V = \varepsilon\xi^{\mathrm{T}}(P_{\sigma(t)} \otimes \mathscr{P}^{-1})\xi$ for (12). We have that, $\forall \zeta \in [t_{j-1}, t_j)$,

$$\begin{aligned}
V(\zeta) &\le e^{-\frac{1}{\varepsilon}\lambda_{p_j}(\zeta - t_{j-1})}V(t_{j-1}) \\
&\le e^{-\frac{1}{\varepsilon}\lambda^c(\zeta - t_{j-1})}V(t_{j-1}), \quad p_j \in \Gamma_c, \\
V(\zeta) &\le e^{\frac{1}{\varepsilon}\lambda_{p_j}(\zeta - t_{j-1})}V(t_{j-1}) \\
&\le e^{\frac{1}{\varepsilon}\lambda^d(\zeta - t_{j-1})}V(t_{j-1}), \quad p_j \in \Gamma_d,
\end{aligned}$$

where $\lambda^c = \min_{k\in\Gamma_c}\lambda_k = \frac{1}{2\lambda_{\max}(\mathscr{P})}$, $\lambda^d = \max_{k\in\Gamma_d}\lambda_k = 2\max\{\theta, 1\}\lambda_{\max}(\mathscr{P})$. Define $a = \frac{\lambda_{\max}(\mathscr{P})}{\lambda_{\min}(\mathscr{P})}\max_{k,j\in\Gamma}\frac{\lambda_{\max}(P_k)}{\lambda_{\min}(P_j)}$. We then know that $V(t_j) \le a\lim_{t\uparrow t_j}V(t)$. Thus, it follows that $V(t) \le a^\rho e^{\frac{1}{\varepsilon}\lambda^d T^d_{\bar{t}_0}(t) - \frac{1}{\varepsilon}\lambda^c T^c_{\bar{t}_0}(t)}V(\bar{t}_0)$, where ρ denotes times of switching during $[\bar{t}_0, t)$. Note that $\rho \le \frac{t-\bar{t}_0}{\tau_d}$. Given that $\kappa \ge \kappa^* = \frac{\lambda^d+\lambda}{\lambda^c-\lambda}$, for some $\lambda \in (0, \lambda^c)$, it follows from Assumption 2 that $T^c_{\bar{t}_0}(t) \ge \kappa^* T^d_{\bar{t}_0}(t)$ for all $t \ge \bar{t}_0$. This implies that $\lambda^d T^d_{\bar{t}_0}(t) - \lambda^c T^c_{\bar{t}_0}(t) \le -\lambda(T^d_{\bar{t}_0}(t) + T^c_{\bar{t}_0}(t))$, for all $t \ge \bar{t}_0$ and we therefore know that

$$\begin{aligned}
V(t) &\le a^\rho e^{-\frac{1}{\varepsilon}\lambda(t-\bar{t}_0)}V(\bar{t}_0) \\
&\le e^{\frac{t-\bar{t}_0}{\tau_d}\ln a - \frac{1}{\varepsilon}\lambda(t-\bar{t}_0)}V(\bar{t}_0) \\
&= e^{-\left(\frac{1}{\varepsilon}\lambda - \frac{\ln a}{\tau_d}\right)(t-\bar{t}_0)}V(\bar{t}_0).
\end{aligned}$$

Furthermore, set $\lambda = \alpha\lambda^c$, where some $\alpha \in (0, 1)$. We then have that $\kappa^* = \frac{\alpha + 4\max\{\theta,1\}\lambda^2_{\max}(\mathscr{P})}{1-\alpha}$, and

$$V(t) \le e^{-\left(\frac{\alpha}{2\varepsilon\lambda_{\max}(\mathscr{P})} - \frac{\ln a}{\tau_d}\right)(t-\bar{t}_0)}V(\bar{t}_0).$$

It follows that if $\varepsilon < \frac{\alpha\tau_d}{2\lambda_{\max}(\mathscr{P})\ln a}$, we have for (12) that $\|\xi(t)\| \le c^* e^{-\frac{1}{2}\left(\frac{\alpha}{2\varepsilon\lambda_{\max}(\mathscr{P})} - \frac{\ln a}{\tau_d}\right)(t-\bar{t}_0)}\|\xi(\bar{t}_0)\|$, where $c^* = \sqrt{\frac{\lambda_{\max}(\mathscr{P})\max_{k\in\Gamma}\lambda_{\max}(P_k)}{\lambda_{\min}(\mathscr{P})\min_{k\in\Gamma}\lambda_{\min}(P_k)}}$.

Therefore, we choose ε^* satisfying $\varepsilon^* < \frac{\alpha\tau_d}{2\lambda_{\max}(\mathscr{P})\ln a}$ and $\|\mathscr{L}_{\varepsilon^*}\| < \min_{k\in\Gamma}\frac{\lambda_{\min}(P_k)\lambda_{\min}(\mathscr{P})}{4\lambda_{\max}(P_k)\lambda^2_{\max}(\mathscr{P})}$. It then follows that $\lim_{t\to\infty}(\chi_i(t) - \widehat{\chi}_i(t)) = 0$, $i = 1, 2\ldots, n$. ■

From the unified observer design, we then have that

$$\widehat{\bar{x}}_i = (T_i^{\mathrm{T}}T_i)^{-1}T_i^{\mathrm{T}}\widehat{\chi}_i = [\widehat{\bar{x}}_{i1}^{\mathrm{T}}, \widehat{\bar{x}}_{i2}^{\mathrm{T}}]^{\mathrm{T}}, \quad i = 1, 2, \ldots, n, \tag{13}$$

which will be used in the control design.

4.5. Main results

In this section, we show that the observer architecture introduced in the previous sections provide an asymptotically stable closed-loop system, as presented in Theorem 1. The observer-based controller is

$$u_i = F_i\widehat{\bar{x}}_{i1} + (\Gamma_i - F_i\Pi_i)\widehat{\bar{x}}_{i2}, \tag{14}$$

where Π_i and Γ_i are the solutions of the regulator equations (6), and $\widehat{\bar{x}}_{i1}$ and $\widehat{\bar{x}}_{i2}$ can be obtained from (8) and (13).

Theorem 1. *Let Assumptions 1–4 hold and assume that $\kappa \ge \frac{\alpha+4\max\{\theta,1\}\lambda^2_{\max}(\mathscr{P})}{1-\alpha}$, where $\alpha \in (0, 1)$, θ and $\mathscr{P}$ are given by (11). Then, there exists $\varepsilon^* \in (0, 1]$ such that, if $\varepsilon \in (0, \varepsilon^*]$, (14) ensures that $\lim_{t\to\infty} e_i(t) = 0$, $i = 1, 2\ldots, n$, for the multi-agent system (1)–(4).*

Proof. Follows from Lemmas 1 and 3, and the separation principle. ■

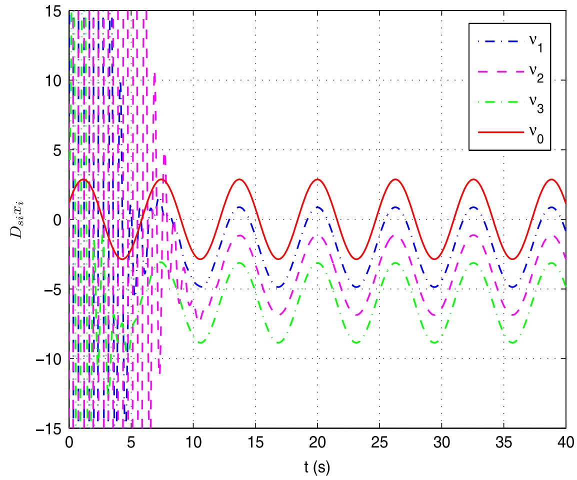

Fig. 2. Output convergence of system (1), (2), and (3) under the observer-based controller (14).

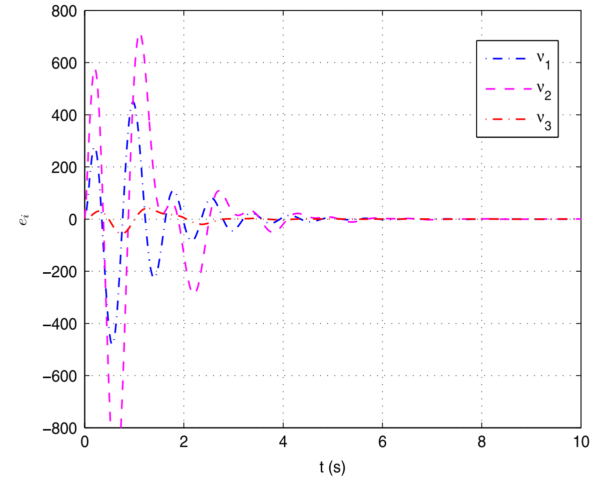

Fig. 3. Error convergence of system (1), (2), and (3) under the observer-based controller (14).

5. Simulation results

In this section, we illustrate the theoretical results. Consider a network of three agents. We assume that the adjacency matrix $\overline{A}_{\sigma(t)}$ associated with $\overline{G}_{\sigma(t)}$ is switching periodically. Denote $\ell = 0, 20, 40 \dots$. $\overline{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, when $t \in [\ell, \ell+6)$, $\overline{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, when $t \in [\ell+6, \ell+12)$, $\overline{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, when $t \in [\ell+12, \ell+18)$, $\overline{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, when $t \in [\ell+18, \ell+20)$.

The dynamics of the agents are described by $A_1 = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C_{s1} = C_{d1} = D_{s1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C_{s2} = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $C_{d2} = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $D_{s2} = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_{s3} = C_{d3} = D_{s3} = \begin{bmatrix} 1 & 0 \end{bmatrix}$. The dynamics of the individual autonomous exosystems are given by $S_i = 0$, $C_{wi} = D_{wi} = -1$, $i = 1, 2, 3$, and $\omega_1(0) = -2$, $\omega_2(0) = -4$, and $\omega_3(0) = -6$. The dynamics of the group autonomous exosystem are given by $A_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $C_0 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D_0 = -C_0$.

Following the design scheme proposed in Section 4, for the solutions of regulator equations (6), we have that $F_1 = \begin{bmatrix} -1 & -4.5 & -6 \end{bmatrix}$, $\Pi_1 = \begin{bmatrix} 1 & 1.0345 & -0.4138 \\ 0 & 0.1379 & 0.3448 \\ 0 & -0.1724 & 0.0690 \end{bmatrix}$, $\Gamma_1 = \begin{bmatrix} 0 & 0.0690 & 0.1724 \end{bmatrix}$ for agent ν_1, $F_2 = \begin{bmatrix} -2 & -6 \end{bmatrix}$, $\Pi_2 = \begin{bmatrix} 0 & 0.4 & -0.2 \\ 1 & 0.6 & 0.2 \end{bmatrix}$, $\Gamma_2 = \begin{bmatrix} 0 & -0.2 & 0.6 \end{bmatrix}$ for agent ν_2, $F_3 = \begin{bmatrix} 0 & -1 \end{bmatrix}$, $\Pi_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\Gamma_3 = \begin{bmatrix} 2 & 1 & 2 \end{bmatrix}$ for agent ν_3. We also have $\varepsilon = 0.2$ for (8) and $\theta = 0.1$ for (11).

Figs. 2 and 3 show, respectively, the state convergence and the error convergence of system (1), (2), and (3) under the observer-based controller (14). We see that coordinated output regulation is realized even when there exists multiple heterogeneous dynamics and the information interactions are switching. This agrees with the result in Theorem 1.

6. Conclusions

This paper studied the coordinated output regulation problem of multiple heterogeneous linear systems. We first formulated the coordinated output regulation problem and specified the information that is available for each agent. A high-gain based distributed observer and an individual observer were introduced for each agent and observer-based controllers were designed to solve the problem. The information interactions among the agents and the group autonomous exosystem were allowed to be switching over a finite set of networks containing both graphs having a spanning tree and graphs having not. Simulations were given to validate the theoretical results. Future directions include relaxing the dwell-time assumption.

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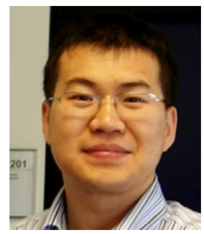

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