Event-Triggered Model Predictive Control With a Statistical Learning

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Abstract—The event-triggered model predictive control (MPC) reduces energy consumption for updating control sequences while maintaining the originality of the MPC, which copes with hard constraints on dynamical systems. In the presence of large uncertainties, however, the standard event-triggered MPC may generate too frequent event occurrences. To compensate for unknown uncertainties, this paper applies a statistical learning to event-triggered MPC. The stability and the feasibility of the proposed control system are analyzed in regard to the statistical learning influences, such as the number of training samples, model complexity, and learning parameters. Accordingly, the event-triggering policy is established to guarantee the stability. We evaluate the proposed algorithm for the tracking problem of a nonholonomic model perturbed by uncertainties. In comparison with the standard event-triggered control scheme, the simulation results of the proposed method show better tracking performance with less frequent event triggers.

Index Terms—Empirical risk minimization (ERM), event-triggered control, model predictive control (MPC), statistical learning.

I. INTRODUCTION

MODEL predictive control (MPC) is a form of optimal control method that derives its control action by solving a finite-horizon open-loop optimization problem. Due to its advantage to cope with hard constraints on state and control input in the online optimization, MPC has been employed for many applications, such as automated control systems, smart grids, networked control system, and communication technologies [1]–[4].

The MPC may require much computational time as the optimization operations are executed at every sampling instant. To improve computational efficiency without sacrificing control performance, event-triggered MPC methods as an extension of the MPC have been developed [2], [5]–[9]. While

maintaining the originality of MPC, it can significantly reduce computational cost by allowing control updates only when the events are triggered.

However, event-triggered MPC may suffer from a number of event-triggered occurrences when large uncertainty is presented in plant. In this case, uncertainty compensation for the event-triggered control may prevent deterioration of control performance [10], [11]. We herein propose an event-triggered MPC with a statistical machine learning that learns an uncertainty compensation model. In comparison to conventional control synthesis combined with learning techniques [12], [13], such as reinforcement learning [13]–[15] and supervised learning [16]–[18], we provide analysis of the stability of the closed-loop control system in terms of the learning influences, such as the number of training samples, model complexity, and learning parameters. This may help to develop and evaluate a realistic learning and control application with comprehension of the learning influences on a control object.

As a statistical machine learning algorithm, empirical risk minimization (ERM) is applied [19] to learn an uncertainty compensation model. This learning method provides a bound of the predictive learning error, and this bound can be used to derive the stability and the feasibility of the proposed control system. Here, the feasibility refers to the condition that there exists an MPC optimization solution satisfying all constraints on the states, the control input, and the learning error bound.

The main contributions of the proposed event-triggered MPC with a statistical learning can be summarized in the following.

1) The control system is adaptive to model uncertainty owing to the learning-based compensator, and is robust to state estimation error, which is bounded by the ERM learning. By the ERM learning, the proposed approach can relax restrictions on uncertainty, such as known maximum upper bound, which is a typical assumption in robust MPC [20].

2) Stability and feasibility are analyzed with respect to the learning influences while most of learning-based control methods do not address them in control theory perspective. To our best knowledge, this is the first work to apply ERM learning to event-triggered control.

The proposed algorithm is evaluated for a tracking problem of a nonholonomic robot subject to model uncertainty. The uncertainty is attenuated by ERM algorithm with training data samples obtained from repetitive control implementations. In comparison with event-triggered control scheme, the proposed
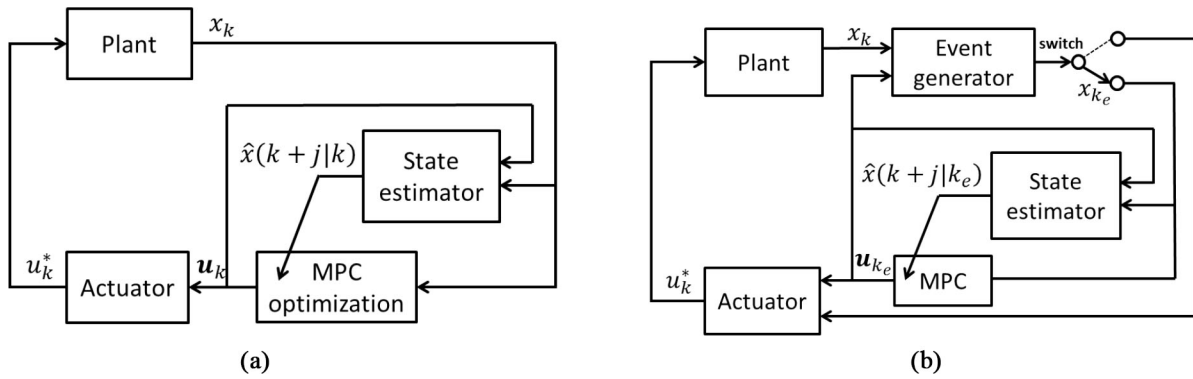
Fig. 1. Concepts of (a) original MPC and (b) event-triggered MPC are shown. The original MPC solves the control optimization at every time step and the actuator chooses the first element of the control input sequence $\mathbf{u}_k$. The event-triggered MPC solves optimization only when event-generator triggers an event. In the period of time between two events, it continues using some parts of the control input sequence $\mathbf{u}_{k_e}$, which was calculated at the last trigger moment k_e. In this paper, the design of the state estimator is supported by the ERM learning technique.

method shows better tracking performance and more efficient event-triggering mechanism.

The rest of this paper is organized as follows. Section II presents the primary of the proposed control system. Sections III and IV describe the event-triggered MPC combined with the statistical learning. Section V shows the simulation results and Section VI devotes to the concluding remarks.

II. PRIMARY RESULT

Fig. 1(a) illustrates the control system architecture of the normal MPC and Fig. 1(b) shows the event-triggered MPC mechanism. In the normal MPC strategy, the control optimization iterates at every time step to generate a new control sequence. In the event-triggered scheme as shown in Fig. 1(b), it is able to reduce the control updates by the event generator, which triggers MPC optimization only when it is required. The state estimator design is supported by the ERM learning to compensate for uncertainty.

Section II-A presents system description and introduces the learning-based estimator to compensate for uncertainty with description of training data configuration. Section II-B summarizes the event-triggered MPC combined with a learning approach and its contribution to reduce computational time.

A. System Description and Learning-Based Compensator

We consider the nonlinear deterministic discrete-time system

$$x_{k+1} = f(x_k, u_k) + w(x_k, u_k) \tag{1}$$

where $x_k \in \mathbb{R}^d$ is state at discrete time k, $u_k \in \mathbb{R}^m$ is control input, and $w(x_k, u_k) \in \mathbb{R}^d$ is model uncertainty. The dynamic model $f(x_k, u_k)$ is known and the uncertainty $w(x_k, u_k)$ is unknown. The state and control input belong to the compact sets

$$x_k \in \mathcal{X}, \; u_k \in \mathcal{U}. \tag{2}$$

To counteract the unknown disturbance $w(x_k, u_k)$, a machine learning technique is used to design the compensator (or estimator) $g(x_k, u_k)$ such that

$$\mathbb{E}[|w(x_k, u_k) - g(x_k, u_k)|] \leq \varepsilon \tag{3}$$

where $\varepsilon > 0$ is the upper bound of the prediction error and $|\cdot|$ denotes the Euclidean norm. The role of the machine learning is to design the uncertainty estimator $g(x_k, u_k)$ and to obtain the bound ε.

To model $g(x_k, u_k)$, the machine learning technique uses training set. In this paper, the training dataset is obtained by implementing repetitive control tasks. With the assumption that all states are measurable, a training dataset is given by

$$D = \{(X_i, Y_i)\}_{i=1}^n \tag{4}$$

with

$$X_i = \left[x_{k_i}^T, u_{k_i}^T\right]^T \tag{5}$$

$$Y_i = x_{k_i+1}^T - f(x_{k_i}, u_{k_i}) + \eta \tag{6}$$

where k_i are time indices when samples are collected and η is the random sampling noise. Based on the training set, the model $g(x_k, u_k)$ and the error bound ε are obtained by the ERM learning algorithm, which is going to be introduced in Section IV. We note that the applied learning technique does not refer to online learning that may learn and update $g(x_k, u_k)$ during a control operation. In this paper, we do not update $g(x_k, u_k)$ during the control operation. We leave the training set D as a random variable so that $g(x_k, u_k)(\equiv g(x_k, u_k; D))$ is the random function.

B. Event-Triggered MPC With Learning-Based Compensator

MPC solves an optimal control problem (OCP) to generate a predictive control input sequence. Suppose N is the prediction horizon length, then the states and control inputs at discrete time k are formed in vector format

$$\mathbf{x}_k = \left\{\hat{x}(k+j|k)\right\}_{j=0}^N, \; \mathbf{u}_k = \{u(k+j|k)\}_{j=0}^{N-1} \tag{7}$$

where the initial estimation $\hat{x}(k|k) = x_k$ is given.

The value function of OCP is defined as

$$J(\mathbf{x}_k, \mathbf{u}_k)$$
$$:= \mathbb{E}_{x_k}\left[\sum_{j=0}^{N-1} J_r\big(\hat{x}(k+j|k), u(k+j|k)\big) + J_v(\hat{x}(k+N|k))\right] \tag{8}$$

where $J_r : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ and $J_v : \mathbb{R}^n \to \mathbb{R}$ are the running and the terminal cost functions, respectively, and $\mathbb{E}_{x_k} = \mathbb{E}[\cdot \,|x_k]$ is the conditional expectation given the initial state x_k. Note that the estimation $\hat{x}(k+j|k)$ is random variable due to the random vector $g(x_k, u_k)$ so that the value function is defined as the expectation. Given the value function (8), the formulation of the optimization problem is given by

$$J^*(\mathbf{x}_k) = \min_{\mathbf{u}} J(\mathbf{x}_k, \mathbf{u}_k) \tag{9}$$

subject to

$$\hat{x}(k+j+1|k) = f\big(\hat{x}(k+j|k), u(k+j|k)\big) \\ + g\big(\hat{x}(k+j|k), u(k+j|k)\big) \tag{10}$$

$$u(k+j|k) \in \mathcal{U} \;\; \mathbb{E}_{x_k}[\hat{x}(k+N|k)] \in \mathcal{X}_f \tag{11}$$

$$\mathbb{E}_{x_k}[\hat{x}(k+j+1|k)] \in \mathcal{X}_{j+1} \\ \forall j = 0, \ldots, N-1 \tag{12}$$

where $J^*(\mathbf{x}_k)$ is the optimal value, and $\mathcal{U}$, $\mathcal{X}_{j+1}$, and $\mathcal{X}_f$ are the constraint sets. In (10), $g(x, u)$ counteracts the unknown uncertainty $w(x, u)$ introduced in (1) as the learning-based compensator function.

The original MPC solves the OCP (9) at every time step and uses the first control element of the sequence. Meanwhile, the event-triggered MPC solves the OCP (9) only when the event generator triggers an event, as described in Fig. 1(b).

The event-triggering condition is established to satisfy the stability of the control system. General approach to specify the event-triggering condition for nonlinear systems perturbed by uncertainties is based on input-to-state stability (ISS) [21]–[23]. They require the assumption for a known upper bound of the uncertainty, where this bound plays pivotal role in the control performance of the event-triggering policy. If the bound is large, the frequency of the event triggers increases. To improve the event-triggering efficiency, we apply a statistical machine learning technique to obtain a tight bound by compensation for uncertainty. By applying the statistical learning, our method can relax the restrictions that require known upper bound or structure of uncertainty, such as constant or harmonic, which is a common assumption in adaptive control and robust MPC [20].

III. EVENT-TRIGGERED CONTROL FORMULATION

This section devotes to deriving the proposed control law given the learning estimator in (3). Section III-A defines Assumptions 1–3 and Lemmas 1–3, which are required for the proposed control system. Based on those properties, Sections III-B and III-C present the feasibility and the stability analyses, respectively.

A. System Assumptions and Properties

Assumption 1: Given the system in (1), it is satisfied that

$$|f(x_1, u) - f(x_2, u)| \leq L_f |x_1 - x_2|. \tag{13}$$

The inequality in (13) refers to locally Lipschitz for $f(x, u)$ in $x \in \mathcal{X}$, $u \in \mathcal{U}$, where L_f is Lipschitz constant, and $\mathcal{X}$ and $\mathcal{U}$ are compact sets defined in (2), and $f(0, 0) = 0$.

Assumption 2: The cost function $J_r(x, u)$ in (8) is locally Lipschitz with Lipschitz constant L_r, and its expectation satisfies $\mathbb{E}[J_r(x, u)] \geq \alpha |(x, u)|^p$ with the positive integers $\alpha > 0$ and $p \geq 1$.

Assumption 3: A local stabilizing controller $h(x)$ for the terminal set $\mathcal{X}_f$ exists such that $\mathbb{E}[J_v(f(x, h(x))) - J_v(x)] \leq -\mathbb{E}[J_r(x, h(x))]$ for $\forall x \in \Phi$. The compact set Φ is given $\Phi = \{x \in \mathbb{R}^n : \mathbb{E}[J_v(x)] \leq \alpha_\Phi\}$ such that $\Phi \subseteq X_{N-m}$ for $m \in \{1, \ldots, N-1\}$, with the assumption that the cost function J_v in (8) is Lipschitz in Φ with Lipschitz parameter L_v. The terminal state region $\mathcal{X}_f$ in (11) is given $\mathcal{X}_f = \{x \in \mathbb{R}^n : \mathbb{E}[J_v(x)] \leq \alpha_v\}$ for $\forall x \in \Phi$ such that $f(x, h(x)) \in \mathcal{X}_f \subseteq \Phi$.

Assumption 3 refers to the computation of the terminal set $\mathcal{X}_f$ in (11) when the admissible positively invariant set Φ is obtained. Assumptions 1–3 are generally used in the MPC framework to guarantee the stability property under additive disturbances as in [21]–[23].

Suppose that e is Euclidean norm of the expected error such that

$$e(k+j|k) = \mathbb{E}\big[|x_{k+j} - \hat{x}(k+j|k)|\big] \tag{14}$$

where x_{k+j} is the true state and $\hat{x}(k+j|k)$ is the predicted state. Then, bound of an expected error can be calculated in Lemmas 1 and 2.

Lemma 1: The bound of $e(k+j|k)$ is given by

$$e(k+j|k) \leq \frac{L_f^j - 1}{L_f - 1} \varepsilon \;\; \forall j \geq 1. \tag{15}$$

Proof: See Appendix A. ∎

Lemma 2: The bound of the expected error between current time step k and previous time step $k-1$, for all $j \geq 0$, is given by

$$\mathbb{E}[|\hat{x}(k+j|k) - \hat{x}(k+j|k-1)|] \leq 2\frac{L_f^{j+1} - 1}{L_f - 1} \varepsilon \;\; \forall j \geq 0.$$

Proof: See Appendix B. ∎

The constraint set $\mathcal{X}_j$ in (12) is calculated to guarantee the invariant set for the closed-loop system, in which an OCP solution exists

$$\mathcal{X}_j = \mathcal{X} \ominus \mathcal{B}_j$$

$$\text{with } \mathcal{B}_j = \left\{ x \in \mathbb{R}^n : \mathbb{E}[|x|] \leq 2\frac{L_f^j - 1}{L_f - 1} \varepsilon \right\}. \tag{16}$$

By the Pontryagin difference operation $\ominus$, the set $\mathcal{X}_j$ is given by

$$\mathcal{X}_j = \big\{ x \in \mathbb{R}^n : x + x_b \in \mathcal{X} \;\; \forall x_b \in \mathcal{B}_j \big\}. \tag{17}$$

Lemma 3: If $\mathbb{E}[x] \in \mathcal{X}_j$ and $\mathbb{E}[y] \in \mathbb{R}^n$ satisfies

$$\mathbb{E}\big[|x - y|\big] \leq 2L_f^{j-m} \frac{L_f^m - 1}{L_f - 1} \varepsilon \;\; \text{for } 0 \leq m < j \tag{18}$$

then, $\mathbb{E}[y] \in \mathcal{X}_{j-m}$.

Proof: Let $z = x - y + e_{j-m}$, where $\mathbb{E}[e_{j-m}] \in \mathcal{B}_{j-m}$. It is clear that

$$\mathbb{E}[|z|] \leq \mathbb{E}\big[|x - y|\big] + \mathbb{E}\big[|e_{j-m}|\big]$$

$$\leq 2L_f^{j-m} \frac{L_f^m - 1}{L_f - 1} \varepsilon + 2\frac{L_f^{j-m} - 1}{L_f - 1} \varepsilon$$

[by (18) for the left term and (16) for the right]

$$= 2\frac{L_f^j - 1}{L_f - 1}\varepsilon.$$

Therefore, $z \in B_j$. Because $\mathbb{E}[y + e_{j-m}] = \mathbb{E}[z + x] \in \mathcal{X}$, we can have $\mathbb{E}[y] \in \mathcal{X}_{j-m}$. ∎

B. Event-Triggered Control Formulation and Feasibility

Assume that the optimal control sequence at the last time step $k - 1$ was calculated

$$\mathbf{U}^*(k-1) = \left\{u^*(k-1+i|k-1)\right\}_{i=0}^{N-1} \tag{19}$$

and we hold the corresponding value function $J^*(k-1)$. Given $\mathbf{U}^*(k-1)$, the future input set $\mathbf{U}(k+m) = \{\bar{u}(k+j|k+m)\}_{j=m}^{N-1}$ at the next possible event-triggered instants $k+m$ for $0 \le m \le N - 1$, is defined:

1) for $j = m, \ldots, N - 2$

$$\bar{u}(k + j|k + m) = u^*(k + j|k - 1) \tag{20}$$

2) for $j = N - 1, \ldots, m + N - 1$

$$\bar{u}(k + j|k + m) = h\big(\hat{x}(k + j|k + m)\big) \tag{21}$$

where $h(\cdot)$ is defined in Assumption 3. Note that without solving the OCP at $k+m$, we use $\mathbf{U}(k+m)$ not only to check the stability and the feasibility but also decide an event trigger.

The feasibility analysis for the MPC is needed to guarantee the existence of a solution satisfying every constraint of the OCP. In the event-triggered control formulation, the OCP at $k+m$ for $m \in \{0, 1, \ldots, N - 1\}$ is feasible if a solution exists, given the optimal solution $\mathbf{U}^*(k - 1)$ in (19).

Theorem 1: Given the system in (1) with Assumptions 1–3, the OCP is feasible if the prediction error in (3) is bounded such that

$$\varepsilon \le \frac{(L_f - 1)(\alpha_\Phi - \alpha_v)}{2L_v\left(L_f^N - 1\right)}. \tag{22}$$

Proof: See Appendix C. ∎

C. Stability and Triggering Condition

Stochastic ISS (SISS) is a general concept to analyze the stability of stochastic nonlinear control systems [24]–[26]. Some relevant function classes are defined as follows. A function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{K}$ if it is a continuous strictly increasing function with $\gamma(0) = 0$. On top of that it belongs to class $\mathcal{K}_\infty$ if $\gamma \in \mathcal{K}$ when $\gamma(r) \in \infty$ as $r \to \infty$. Furthermore, a function $\beta : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{KL}$ if $\beta(s, k)$ is of class $\mathcal{K}$ for each fixed k and it decreases to zero as $k \to \infty$ for each fixed $s \ge 0$.

Definition 1: For system (1), a continuous function $J(k) : \mathbb{R}^n \to \mathbb{R}_+$ is the SISS Lyapunov function if there exist $\bar{\alpha}, \underline{\alpha} \in \mathcal{K}_\infty$, $\varphi, \varrho \in \mathcal{KL}$ for all $x \in \mathbb{R}^n \setminus \{0\}$ such that

$$\underline{\alpha}(|x_k|) \le J(k) \le \bar{\alpha}(|x_k|)$$
$$\Delta J(k) \le -\varphi(|x_k|) + \varrho(\|w_k\|)$$

where $\Delta J(k) = \mathbb{E}_{x_k}[J(k+1)] - J(k)$.

Let us define $\bar{J}(k+\ell)$ for $\ell \in \{0, 1, \ldots, N - 1\}$ as the costs of the feasible sequence. Then, the difference between $\bar{J}(k+\ell)$ and the optimal cost at the last time step $J^*(k - 1)$ is given by

$$\Delta J_\ell = \mathbb{E}\big[\bar{J}(k + \ell) - J^*(k - 1)\big] = \mathbb{E}\big[\bar{J}(k + \ell)\big] - J^*(k - 1). \tag{23}$$

Theorem 2: Given the system in (1), Assumptions 1–3, and the control law from (20) and (21), ΔJ_ℓ is bounded such that:
1) for $\ell = 0$

$$\Delta J_0 \le L_{Z_l} \cdot e(k|k - 1) + L_{C_l} \cdot 2\varepsilon - \alpha|x_{k-1}|^p \tag{24}$$

2) for $1 \le \ell \le N - 1$

$$\Delta J_\ell \le \left\{L_{Z_j}\frac{L_f^j - 1}{L_f - 1} + 2 \cdot L_{C_j}\right\} \cdot \varepsilon - \alpha\sum_{i=0}^{\ell}|x_{k+\ell-i}|^p \tag{25}$$

where

$$L_{Z_\ell} = L_v L_f^{(N-1-\ell)} + L_r\frac{L_f^{(N-1-\ell)} - 1}{L_f - 1} \tag{26}$$

$$L_{C_\ell} = L_v\frac{L_f^{(N-1-\ell)} - 1}{L_f - 1}$$
$$+ \frac{L_r}{L_f - 1}\left\{\sum_{i=0}^{N-2-\ell} L_f^\ell - (N - 2 - \ell)\right\}. \tag{27}$$

The parameters L_f, L_r, and L_v are defined in Assumptions 1–3, respectively. According to SISS described in Definition 1, the optimal cost is the SISS-Lyapunov function so that the closed-loop system is stochastic stable.

Proof: See Appendix D. ∎

Finally, the event-triggering policy is constructed to keep the stability by subjecting ΔJ_ℓ to decrease such that

$$\Delta J_{\ell+1} \le \Delta J_\ell. \tag{28}$$

Under the rule of (28), the following theorem specifies the event-triggering condition.

Theorem 3: Given the system in (1), Assumptions 1–3, and the solution of OCP at the last time step $k - 1$, the event-triggering condition is as follows:
1) for $\ell = 0$

$$L_{Z_l} \cdot e(k|k - 1) + 2L_{C_l}\varepsilon \le \sigma \cdot \alpha|x_{k-1}|^p \tag{29}$$

2) for $1 \le \ell \le N - 1$

$$\left(L_{Z_\ell}\frac{L_f^\ell - 1}{L_f - 1} + 2L_{C_\ell}\right) \cdot \varepsilon \le \sigma \cdot \alpha\sum_{i=0}^{\ell}|x_{k-i+\ell}|^p \tag{30}$$

and

$$\left(L_{Z_\ell}\frac{L_f^\ell - 1}{L_f - 1} + 2L_{C_\ell} - L_{Z_{\ell-1}}\frac{L_f^{\ell-1} - 1}{L_f - 1} - 2L_{C_{\ell-1}}\right) \cdot \varepsilon$$
$$\le \sigma \cdot \alpha|x_k|^p \tag{31}$$

where L_{Z_ℓ} and L_{C_ℓ} are defined in (26) and (27), respectively, and $0 < \sigma < 1$.

Proof: See Appendix E. ∎

As a result, solving a new OCP is implemented at $k + \ell$ if (29) or (30) and (31) is violated at $k + \ell$. Otherwise, a new OCP is solved at $k + N$. Based on this event-triggered policy, the control system maintains the SISS.

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IV. STATISTICAL MACHINE LEARNING

This section describes a statistical learning algorithm to achieve two goals: 1) designing the disturbance predictor $g(x_k, u_k)$ in (3) and 2) obtaining the error bound ε in (3).

A. Learning Formulation

Equation (3) can be restated as follows:

$$
\begin{aligned}
\mathbb{E}\big[|w(x_k, u_k) - g(x_k, u_k)|\big] &\leq \sqrt{\mathbb{E}\big[|w(x_k, u_k) - g(x_k, u_k)|^2\big]} \\
&= \sqrt{\mathbb{E}\big[(w_{k1} - \hat{g}_{k1})^2\big] + \cdots + \mathbb{E}\big[(w_{kd} - \hat{g}_{kd})^2\big]} \\
&\leq \sqrt{b_1 + b_2 + \cdots + b_d} := \varepsilon
\end{aligned}
\tag{32}
$$

where $w(x_k, u_k) = [w_{k1}, \ldots, w_{kd}]^T$ and $g(x_k, u_k) = [\hat{g}_{k1}, \ldots, \hat{g}_{kd}]^T$.

Each prediction $\hat{g}_{kl}$ for $l = 1, \ldots, d$ is obtained by lth regressor, and corresponding error b_l can be obtained such that

$$
\mathbb{E}\big[(w_{kl} - \hat{g}_{kl})^2\big] \leq b_l.
\tag{33}
$$

It is noted that the prediction problem by this learning is categorized as a regression problem because $\hat{g}_{kl} \in \mathbb{R}$. Each regressor $\hat{g}_{kl}$ for $l = 1, \ldots, d$ are obtained independently. For simplicity, we omit all the subscripts k, l in (33) in the rest of this section.

Given training dataset $D = \{(X_i, Y_i)\}_{i=1}^n$ for $X \in \chi \subset \mathbb{R}^{d+m}, Y \in \mathcal{Y} \subset \mathbb{R}$ defined in (4), it aims to obtain a prediction model $g : \chi \to \mathcal{Y}$ by minimizing the empirical risk $\hat{R}(g)$ given by

$$
\hat{R}(g) := \mathbb{E}[l(g(X), Y)|g]
\tag{34}
$$

with a loss function $l : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}$. To obtain the prediction model, the penalized ERM is applied [19].

Definition 2 (Penalized Empirical Risk Minimizer): Suppose that $C(g, n, \delta)$ is a penalty function in which n is the number of training samples, $g \in \mathcal{G}$ is a candidate prediction model, and $\delta \in (0, 1)$. Then, the penalized ERM problem is defined by

$$
\hat{g} = \underset{g \in \mathcal{G}}{\operatorname{argmin}}\Big(\hat{R}(g) + C(g, n, \delta)\Big).
\tag{35}
$$

Definition 3 (Expected Risk Bound): Suppose that the loss function $l : \mathcal{Y} \times \mathcal{Y} \to [0, B]$ is bounded and $\inf_{g \in \mathcal{G}} R(g)$ is the Bayes risk for prediction models $g \in \mathcal{G}$. Then, the upper bound of the expected empirical is as follows:

$$
\begin{aligned}
\mathbb{E}[R(\hat{g})] - \inf_{g \in \mathcal{G}} R(g) &\leq \delta + C(g, n, \delta) \\
&\leq \delta + B\sqrt{\frac{\log(1/\delta) + c(g)}{2n}}
\end{aligned}
\tag{36}
$$

for $\delta \in (0, 1)$. The function $c(g)$ in (36) represents model complexity of $g(\cdot)$, which satisfies the condition $\sum_{g \in \mathcal{G}} \exp^{-c(g)} \leq 1$, which will be defined in (42).

From (36), we can ensure the bound of the prediction error after we obtain the predictor by (35).

Algorithm 1 Event-Triggered MPC With Statistical Learning

Require: Parameter configuration satisfying **Assumptions 1, 2 and 3**, given the control system in (1) and (9).

Learning:

Input: Training dataset $D = \{X_i, Y_i\}_{i=1}^n$ in (4)
  $\{X_i = [x_{k_i}^T, u_{k_i}^T]^T, Y_i = [x_{k_i+1}^T - f(x_{k_i}, u_{k_i}) + \eta]$ in (5)$\}$.

Output: $\hat{\omega}$ in (37) and ε in (32).

1: Compute $\hat{\omega}$ by solving (43).
2: Obtain the error bound ε by (46).

Event-triggered Control:

Input: State measurement x_k.

Output: Control signal u_k.

Initialize: Solve the initial MPC in (9), obtain the control sequence $\mathbf{U}^*(k)$ in (19), and apply the first control element of the sequence.

3: **repeat**
4:   Check triggering condition based on (29), (30) and (31).
5:   **if** the triggering condition is violated **then** compute (9), update a new control sequence (19), and apply the first control element of the sequence.
6:   **else if** time exceeds the prediction horizon $k + N$ **then** compute (9), update a new (19), and use the first control signal.
7:   **else** select the control input whose time index is matched with the current time, from the last calculated control sequence.
8:   **end if**
9: **until** end of control

B. ERM Learning

We define a prediction model such that

$$
g(X) = \mathbb{E}[Y|X, \omega] = \omega^T K_X
\tag{37}
$$

where the learning parameter $\omega \in \mathbb{R}^n$ is to be estimated and the Gaussian kernel vector $K_X \in \mathbb{R}^n$ is given by

$$
K_{X_i} = \exp\left(-\frac{(X_i - X)^2}{2\sigma_k^2}\right)
\tag{38}
$$

where $K_{X_i} \in \mathbb{R}$ is the ith element of K_X, and X_i is the ith sample in the training set defined in (4).

It is assumed that the likelihood of the prediction model (37) is the form of Gaussian distribution such that

$$
\mathbb{P}(Y|X, \omega) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(\omega^T K_X - Y)^2}{2\sigma_1^2}\right)
\tag{39}
$$

and the prior of ω follows Gaussian distribution:

$$
\mathbb{P}(\omega) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{\omega^T \omega}{2\sigma_2^2}\right).
\tag{40}
$$

Accordingly, the empirical risk in (34) is as follows:

$$
\begin{aligned}
\hat{R}(g) &= \frac{1}{n}\sum_{i=1}^n l(g(x_i), Y_i) = -\frac{1}{n}\sum_{i=1}^n \log \mathbb{P}(Y_i|X_i, \omega) \\
&= \frac{1}{2\sigma_1^2 n}\sum_{i=1}^n (\omega^T K_{X_i} - Y_i)^2 + \frac{1}{2}\log 2\pi\sigma_1^2.
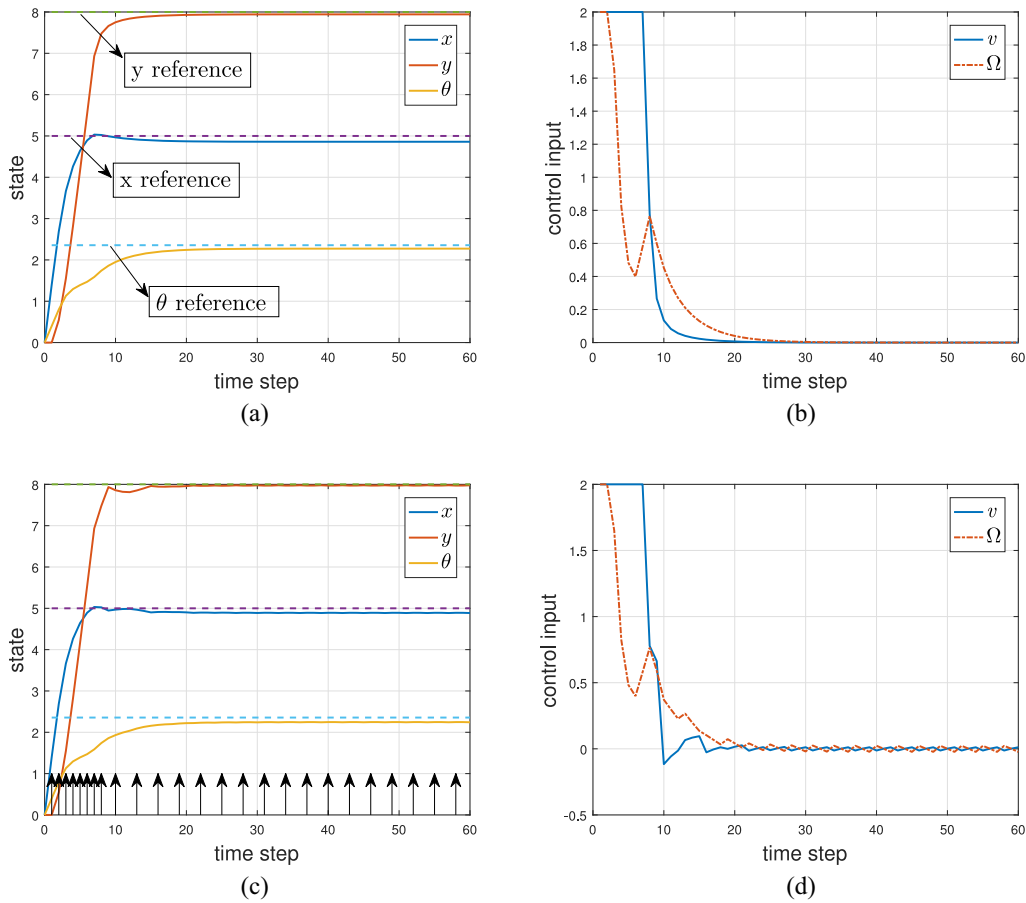\end{aligned}
\tag{41}
$$

Fig. 2. Comparison of MPC and event-triggered MPC without learning, where (a) and (c) tracking results, and (b) and (d) control inputs. In (c), the arrows denote the event-triggered instants to update control inputs.

When the model complexity $c(g)$ in (36) is defined by

$$c(g) = \frac{1}{2}\left(\log(2\pi\sigma_2^2) + \frac{\omega^T\omega}{\sigma_2^2}\right) \tag{42}$$

the optimization problem for the ERM in (35) is as follows:

$$\hat{\omega} = \underset{\omega}{\arg\min}\left(\frac{1}{2\sigma_1^2 n}\sum_{i=1}^{n}(\omega^T K_{X_i} - Y_i)^2 \right.$$
$$\left. + B\sqrt{\frac{\frac{1}{2}\log\left(2\pi\sigma_2^2\right) + \frac{1}{2}\frac{\omega^T\omega}{\sigma_2^2} + \log\left(1/\delta\right)}{2n}}\right). \tag{43}$$

To solve (43), a numerical optimization algorithm is required. In this paper, gradient descent method [27] is used.

By inserting (42) into (36), the error bound of the estimator ε in (32) can be calculated. The risk error becomes equivalent to the prediction error when the kernel function in (37) is applied, such that

$$\mathbb{E}[R(\hat{g})] - \inf_{g \in \mathcal{G}} R(g) = \mathbb{E}\left[(w - \hat{g})^2\right] \tag{44}$$

where w is the target of the predictor $\hat{g}$ in (32).

Herein, we summarize the learning procedure. Suppose that we have d learning models with the learned $\hat{\omega}_l$ for $l = 1, \ldots, d$. Then, given a test data $X_k = [x_k^T, u_k^T]^T$ in the control phase, the lth estimator computes

$$\hat{g}_{kl}(x_k, u_k; D) = \hat{\omega}_l^T K_{X_k}. \tag{45}$$

Also, the bound in (32) is calculated as

$$\mathbb{E}\big[|w(x_k, u_k) - g(x_k, u_k)|\big]$$
$$\leq \sqrt{\sum_{i=1}^{d}\left(B\sqrt{\frac{\frac{1}{2}\log\left(2\pi\sigma_2^2\right) + \frac{1}{2}\frac{\hat{\omega}_i^T\hat{\omega}_i}{\sigma_2^2} + \log\left(1/\delta\right)}{2n}} + \delta\right)}$$
$$:= \varepsilon \tag{46}$$

with $\delta = (1/\sqrt{n})$. Algorithm 1 shows the pseudocode of the proposed event-triggered MPC with the ERM learning.

V. SIMULATION RESULTS

We consider a tracking problem of the following nonholonomic system subject to model uncertainty:

$$x_{k+1} = x_k + (1 + C)v_k T \cos\theta_k \tag{47}$$
$$y_{k+1} = y_k + (1 + C)v_k T \sin\theta_k \tag{48}$$
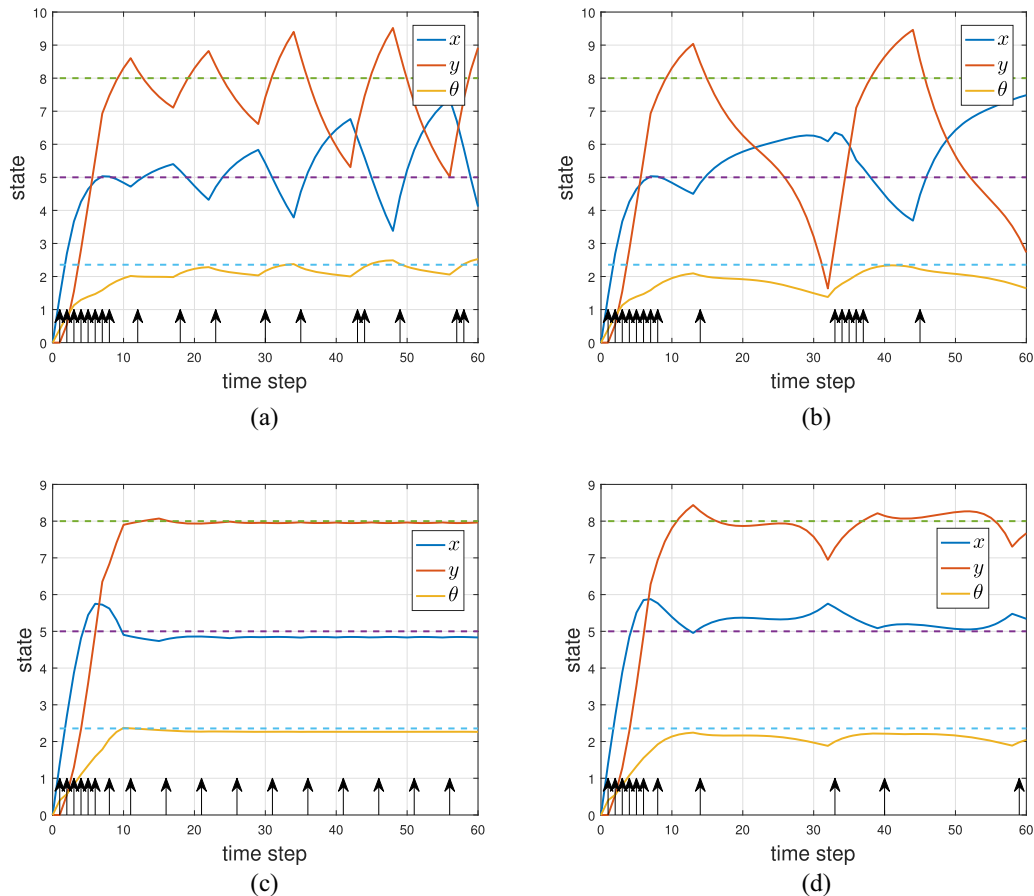$$\theta_{k+1} = \theta_k + \Omega_k T. \tag{49}$$

Fig. 3.   Simulation results of the (a), (b) standard event-triggered MPC and (c), (d) learning-based event-triggered MPC.

The uncertainty in (32) is defined as $w_{k1} = Cv_kT\cos\theta_k$ and $w_{k2} = Cv_kT\sin\theta_k$ with $C = 2.5$. The control input is $u_k = [v_k, \Omega_k]$ and the state variable is $x_k = [x_k, y_k, \theta_k]^T$, which is composed of the two-dimensional position of the robot (x_k, y_k) and the orientation θ_k. The state and control input are constrained by $|x_k|, |y_k| < 10$ and $|v_k|, |\Omega_k| < 2$, respectively. Given the reference $r = (5, 8, (3/4)\pi)$, the cost functions are defined as $J_r = (x - r)^T(x - r) + u^Tu$ and $J_v = x^Tx$, respectively. On top of that time steps $N = 25$, the time interval $T = 0.2$ s, the initial position $(x_0, y_0, \theta_0) = (0, 0, 0)$ are used. To obtain the training data, we implement repetitive control implementations with learning model updates. For example, Fig. 2(c) is one control implementation during 60 time steps, from which we collect one set of training data samples. We collect total seven training datasets, including 420 data points from the seven implementations for simulation study. At the initial learning, the control set is made by the event-triggered MPC without the learning. After that the proposed event-triggered MPC control implementations and the updates of the uncertainty compensator by the ERM learning are repeated to produce the training data until the seventh iteration.

Fig. 2(a) and (b) shows the control performance of the normal MPC, and Fig. 2(c) and (d) are the results of the event-triggered MPC. Their tracking performances are similar, but the event-triggered MPC uses only 25 control updates, which are illustrated by the arrows in Fig. 2(c). We can evaluate that the event-triggered method reduces the computational time to update control inputs without sacrificing tracking

performance. However, we found that the normal MPC and the event-triggered MPC do not compensate for the uncertainties at all. In both cases, the uncertainties are dismissed by updating the control inputs as frequent as the uncertainties do not deteriorate the tracking performances. We found that the uncertainties caused a significant tracking error of the standard event-triggered MPC, as shown in Fig. 3, when we change the parameter values to reduce the event-triggered period.

To investigate the performance variation with respect to the change in the event-triggered period, we vary the parameter α with the fixed $\sigma = 0.9$ in (29)–(31), where α and σ are the major components to adjust the event-triggered period. Fig. 2(c) is the result when $\alpha = 4.2$, which is the best tracking result when applying the standard event-triggered MPC. Fig. 3 shows the tracking results according to increment in α to hold a longer event period. The standard event-triggered MPC yields the results of unstable tracking performances by the uncertainties in both cases of $\alpha = 4.5$ in Fig. 3(a) and $\alpha = 4.7$ in Fig. 3(b). Meanwhile, the proposed learning-based event-triggered MPC method gives the accurate tracking result when $\alpha = 4.5$ in Fig. 3(c) as much as the best tracking performance of the normal MPC in Fig. 2(a). Even when $\alpha = 4.7$ in Fig. 3(d), it yields only the 11 trigger instants while maintaining a fine tracking performance. Consequently, the developed method achieves the outstanding improvement of the control performance as well as the reduction of trigger instants than the compared one owing to the learning capability.
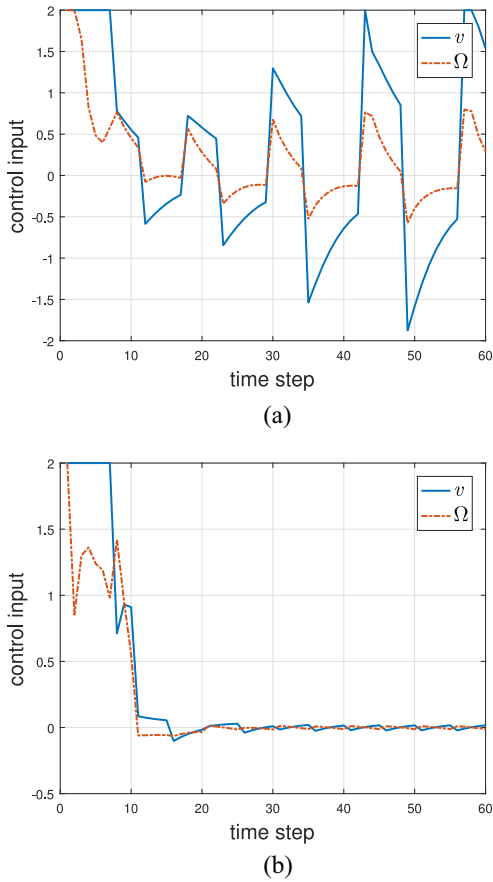
Fig. 4. Control input comparison between the event-triggered MPC and the proposed method. (a) Control input of the event-triggered MPC, which is generated from the simulation of Fig. 3(a). (b) Control input of the learning-based event-triggered MPC, used from Fig. 3(c).

Fig. 4(a) and (b) are the control inputs generated from the simulation results in Fig. 3(a) and (c), respectively. As shown, the standard event-triggered MPC generates the oscillatory control signals, while the proposed method shows the convergence of the control signals.

VI. CONCLUSION

We presented a new event-triggered MPC by applying a statistical learning method. ERM with kernel regression was used to predict the system state subject to model uncertainties. Owing to the learning, the control system became adaptive to the uncertainties and robust to state estimation errors. Further, the error bound analysis related the learning characteristics to the event-triggered policy, and the stability and feasibility of the control system were analyzed. From the simulation results, we validated the computational efficiency and accuracy of the proposed control algorithm.

APPENDIX A
PROOF OF LEMMA 1

Given Lipschitz assumption in Assumptions 1, the following recursion can be obtained:

$\cdot j = 1$
$$\mathbb{E}\big[|x_{k+1} - \hat{x}(k+1|k)|\big]$$

$$= \mathbb{E}\big[|x_{k+1} - f(x_k, u_k) - g(x_k, u_k)|\big] \quad (\text{since } \hat{x}(k|k) = x_k)$$
$$= \mathbb{E}\big[|w(x_k, u_k) - g(x_k, u_k)|\big] \leq \varepsilon \quad (\text{by (32)})$$

$\cdot j = 2$
$$\mathbb{E}\big[|x_{k+2} - \hat{x}(k+2|k)|\big]$$
$$\leq \mathbb{E}\big[|f(x_{k+1}, u_{k+1}) - f(\hat{x}(k+1|k), u_{k+1})|\big]$$
$$\quad + \mathbb{E}\big[|w(x_{k+1}, u_{k+1}) - g(\hat{x}(k+1|k), u_{k+1})|\big] \quad (\text{by (13)})$$
$$\leq L_f \cdot \mathbb{E}\big[|x_{k+1} - \hat{x}(k+1|k)|\big] + \varepsilon$$
$$\leq \varepsilon \cdot (L_f + 1)$$

$\cdot j$
$$\mathbb{E}\big[|x_{k+j} - \hat{x}(k+j|k)|\big]$$
$$\leq L_f^{j-1} \cdot \varepsilon + L_f^{j-2} \cdot \varepsilon + \cdots + L_f^0 \cdot \varepsilon$$
$$\leq L_f^{j-1} \cdot e(k+1|k) + \frac{L_f^{j-1} - 1}{L_f - 1} \cdot \varepsilon \leq \frac{L_f^j - 1}{L_f - 1} \cdot \varepsilon.$$

APPENDIX B
PROOF OF LEMMA 2

Similar to Lemma 1, we can obtain the following recursion:

$\cdot j = 0$ (50)
$$\mathbb{E}\big[|\hat{x}(k|k) - \hat{x}(k|k-1)\big] := e(k|k-1)$$

$\cdot j = 1$ (51)
$$\mathbb{E}\big[|\hat{x}(k+1|k) - \hat{x}(k+1|k-1)\big]$$
$$\leq L_f \cdot \mathbb{E}\big[|\hat{x}(k|k) - \hat{x}(k|k-1)|\big] (\text{by (13)})$$
$$\quad + 2\mathbb{E}\big[|g\big(\hat{x}(k|k), u_k\big) - w_k|\big]$$
$$\leq L_f \varepsilon + 2\varepsilon$$

$\cdot j = 2$ (52)
$$\mathbb{E}\big[|\hat{x}(k+2|k) - \hat{x}(k+2|k-1)|\big]$$
$$\leq L_f^2 \varepsilon + 2L_f^1 \varepsilon + 2L_f^0 \varepsilon$$

$\cdot j$ (53)
$$\mathbb{E}\big[|\hat{x}(k+j|k) - \hat{x}(k+j|k-1)|\big]$$
$$\leq L_f^j e(k|k-1) + 2\frac{L_f^j - 1}{L_f - 1}\varepsilon$$ (54)
$$\leq L_f^j \varepsilon + 2\frac{L_f^j - 1}{L_f - 1}\varepsilon \leq \left(L_f^j + \frac{L_f^j - 1}{L_f - 1}\right) \cdot 2\varepsilon$$
$$= \frac{L_f^{j+1} - 1}{L_f - 1} \cdot 2\varepsilon.$$ (55)

APPENDIX C
PROOF OF THEOREM 1

Suppose that we are given the optimal control input $\mathbf{U}^*(k-1)$ at the last time step $k-1$ in (19) and the cost $J^*(k-1)$. The next OCP is possibly determined at one of the instants $k+m$, with $0 \leq m < N-1$. The OCP can be determined feasible at any $k+m$ if there is a solution of (9) based on the control law in (20) and (21).

1) $\bar{u}(k+j|k+m) \in U$: This is clear condition from the control law in (20) and (21).

2) $\mathbb{E}[\bar{x}(k+j|k+m)] \in \mathcal{X}_{j-m}$ for $j = m+1, \ldots, N-1$: Since $\bar{u}(k+\ell|k+m) = u^*(k+\ell|k-1)$ for $\ell = m, \ldots, N-2$, and

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by Lemma 2, we can obtain the following inequality:

$$\mathbb{E}\big[|\bar{x}(k+j|k+m) - \hat{x}^*(k+j|k-1)|\big] \le L_f^{j-m}\frac{L_f^m - 1}{L_f - 1} \cdot 2\varepsilon. \tag{56}$$

Because $\mathbb{E}[\hat{x}^*(k+j|k-1)] \in \mathcal{X}_j$ in the left term of (56) and by Lemma 3, it is clear that $\mathbb{E}[\bar{x}(k+j|k+m)] \in \mathcal{X}_{j-m}$.

3) $\bar{x}(k+m+N|k+m) \in \mathcal{X}_f$: First, $\mathbb{E}[\bar{x}(k+N|k+m)] \in \Phi$ is going to be proved. Similar to (56), we obtain

$$\mathbb{E}\big[|\bar{x}(k+N|k+m) - \hat{x}^*(k+N|k-1)|\big] \le L_f^{N-m}\frac{L_f^m - 1}{L_f - 1} \cdot 2\varepsilon. \tag{57}$$

Also, from the Lipschitz assumption on the terminal cost function J_v in Assumption 3 and by (57), the following inequalities are given:

$$\mathbb{E}\big[|J_v(\bar{x}(k+N|k+m)) - J_v(\hat{x}^*(k+N|k-1))|\big]$$
$$\le L_v L_f^{N-m}\frac{L_f^m - 1}{L_f - 1} \cdot 2\varepsilon$$

and

$$\mathbb{E}[(J_v\bar{x}(k+N|k+m))]$$
$$\le \mathbb{E}\big[J : v(\hat{x}^*(k+N|k-1))\big] + L_v L_f^{N-m}\frac{L_f^m - 1}{L_f - 1} \cdot 2\varepsilon$$
$$\Big(\text{by the fact that } \mathbb{E}\big[|x - y|\big] = \mathbb{E}[x] - \mathbb{E}[y] \text{ for } x, y \in \mathbb{R} > 0\Big)$$
$$\le \alpha_v + L_v L_f^{N-m}\frac{L_f^m - 1}{L_f - 1} \cdot 2\varepsilon \text{ (by Assumption 3)}$$
$$\le \alpha_v + L_v\frac{L_f^N - 1}{L_f - 1} \cdot 2\varepsilon \le \alpha_\Phi \text{ (by Assumption 3).}$$

Therefore, $\mathbb{E}[\bar{x}(k+N|k+m)] \in \Phi$.

Second, we prove $\mathbb{E}[\bar{x}(k+N+m|k+m)] \in \Phi$ by recursion. It is clear that

$$\mathbb{E}[J_v(\bar{x}(k+N+1|k+m))] \le \mathbb{E}[J_v(\bar{x}(k+N|k+m))]. \tag{58}$$

Also, by Assumption 3, we can get

$$\mathbb{E}[J_v(\bar{x}(k+N|k+m))] \le \alpha_\Phi \tag{59}$$

which yields $\mathbb{E}[\bar{x}(k+N+1|k+m)] \in \Phi$. The recursion derives $\mathbb{E}[\bar{x}(k+N+m-1|k+m)] \in \Phi$, and thus $\mathbb{E}[\bar{x}(k+N+m|k+m)] \in \mathcal{X}_f$.

4) $\mathbb{E}[\bar{x}(k+j|k+m)] \in \mathcal{X}_{j-m}$ for $j = N, N+1, \ldots, N+m-1$: By Assumption 3, it can be confirmed that $\mathbb{E}[\bar{x}(k+N|k+m)] \in \Phi \subseteq \mathcal{X}_{N-m}$, $\mathbb{E}[\bar{x}(k+N+1|k+m)] \in \Phi \subseteq \mathcal{X}_{N-m+1}$, $\ldots$, $\mathbb{E}[\bar{x}(k+N+m-1|k+m)] \in \Phi \subseteq \mathcal{X}_{N-1}$. Therefore, $\mathbb{E}[\bar{x}(k+j|k+m)] \in \mathcal{X}_{j-m}$ for $j = N+1, \ldots, N+m-1$.

APPENDIX D
PROOF OF THEOREM 2

For $\ell = 0$, (23) is given by

$$\Delta J_0 = \mathbb{E}\big[\bar{J}(k) - J^*(k-1)\big]$$
$$= \mathbb{E}\bigg[\sum_{i=0}^{N-2}\{J_r(\bar{x}(k+i|k), \bar{u}(k+i|k)) \tag{60}$$
$$- J_r(\hat{x}(k+i|k-1), u^*(k+i|k-1))\} \tag{61}$$

$$+ J_r(\bar{x}(k+N-1|k), h(\bar{x}(k+N-1|k))) \tag{62}$$
$$+ J_v(\bar{x}(k+N|k)) - J_v(\bar{x}(k+N-1|k)) \tag{63}$$
$$+ J_v(\bar{x}(k+N-1|k)) - J_v(\hat{x}(k+N-1|k-1)) \tag{64}$$
$$- J_r(x_{k-1}, u_{k-1})\bigg]. \tag{65}$$

From the definition in (20) and (21), $\bar{u}(k+i|k) = u^*(k+i|k-1)$. By Lemma 2 and Assumption 2, the bound of (60) and (61) can be formulated

$$\mathbb{E}[J_r(\bar{x}(k+i|k), \bar{u}(k+i|k))$$
$$- J_r(\hat{x}(k+i|k-1), u^*(k+i|k-1))]$$
$$\le L_r\mathbb{E}[\bar{x}(k+i|k)] - \mathbb{E}[\hat{x}(k+i|k-1)|] \text{ (by Assumption 2)}$$
$$\le L_r L_f^i e(k|k-1) + 2\varepsilon\frac{L_f^i - 1}{L_f - 1} \text{ (by (54)).}$$

Consequently, the bound of summation of (60) and (61) is given by

$$\mathbb{E}\bigg[\sum_{i=0}^{N-1} J_r(\bar{x}(k+i|k), \bar{u}(k+i|k))$$
$$- J_r(\hat{x}(k+i|k-1), u^*(k+i|k-1))\bigg]$$
$$\le L_r \cdot \bigg[\frac{L_f^{N-1} - 1}{L_f - 1}e(k|k-1) + \frac{2\varepsilon}{L_f - 1}\bigg\{\sum_{i=0}^{N-2} L_f^i - (N-2)\bigg\}\bigg].$$

The bound of (63) and (64) is given by

$$\mathbb{E}\big[J_v(\bar{x}(k+N-1|k)) - J_v(\hat{x}(k+N-1|k-1))\big]$$
$$\le L_v \cdot \mathbb{E}[|\bar{x}(k+N-1|k)] - \hat{x}(k+N-1|k-1)|]$$
$$\text{(by Assumption 3)}$$
$$\le L_v \cdot \bigg\{L_f^{N-1}e(k|k-1) + 2\varepsilon\frac{L_f^{N-1} - 1}{L_f - 1}\bigg\} \text{ (by (54)).}$$

From Assumption 2, the bound of (65) is as follows:

$$\mathbb{E}\big[J_r(x_{k-1}, u_{k-1})\big] \ge \alpha|(x_{k-1}, u_{k-1})|^p \ge \alpha|x_{k-1}|^p.$$

And upper bound of (62) becomes zero due to Assumption 3. As a result, ΔJ_0 is bounded by

$$\Delta J_0 \le L_{Z_0} \cdot e(k|k-1) + L_{C_0} \cdot 2\varepsilon - \alpha|x_{k-1}|^w \tag{66}$$

with $L_{Z_0} = L_v \cdot L_f^{N-1} + L_r \cdot [(L_f^{(N-1)} - 1)/(L_f - 1)]$ and $L_{C_0} = L_v[(L_f^{(N-1)} - 1)/(L_f - 1)] + [L_r/(L_f - 1)]\{\sum_{i=0}^{N-2} L_f^i - (N-2)\}$.

For $\ell=1$, ΔJ_1 becomes

$$\Delta J_1 = \mathbb{E}\big[\bar{J}(k+1)\big] - J^*(k-1)$$
$$= \mathbb{E}\bigg[\sum_{i=0}^{N-3}\{J_r(\bar{x}(k+i+1|k), \bar{u}(k+i+1|k))$$
$$- J_r(\hat{x}(k+i+1|k-1), u^*(k+i+1|k-1))\}$$
$$+ J_r(\bar{x}(k+N-1|k+1), h(\bar{x}(k+N-1|k+1)))$$
$$+ J_v(\bar{x}(k+N|k+1)) - J_v(\bar{x}(k+N-1|k+1))$$
$$+ J_r(\bar{x}(k+N|k+1), h(\bar{x}(k+N|k+1)))$$

$$+ J_v(\bar{x}(k + N + 1|k + 1)) - J_v(\bar{x}(k + N|k + 1))$$
$$+ J_v(\bar{x}(k + N - 1|k + 1))$$
$$- J_v(\hat{x}(k + N - 1|k - 1))$$
$$\left. - J_r(x_{k-1}, u_{k-1}) - J_r(x_k, u_k) \right].$$

Similar to the case $\ell = 0$, we can obtain the bound of the cost terms. We omit the overlapped procedure. As a result, ΔJ_1 is bounded by

$$\Delta J_1 \leq L_{Z_1} \cdot e(k + 1|k - 1) + L_{C_1} \cdot 2\varepsilon - \alpha|x_{k-1}|^p - \alpha|x_k|^p \tag{67}$$

with $L_{Z_1} = L_v \cdot L_f^{N-2} + L_r \cdot [(L_f^{(N-2)} - 1)/(L_f - 1)]$ and $L_{C_1} = L_v[(L_f^{(N-2)} - 1)/(L_f - 1)] + [L_r/(L_f - 1)]\{\sum_{i=0}^{N-3} L_f^i - (N-3)\}$. Note that the error $e(k + 1|k - 1)$ is given by Lemma 1. By substituting (15) in Lemma 1 into (67), we can get

$$\Delta J_\ell \leq L_{Z_\ell} \cdot \frac{L_f^\ell - 1}{L_f - 1} \cdot \varepsilon + 2\varepsilon L_{C_\ell} - \alpha \sum_{i=0}^{\ell} |x_{k+\ell-i}|^p$$
$$= \left\{ L_{Z_\ell} \frac{L_f^\ell - 1}{L_f - 1} + 2L_{C_\ell} \right\} \cdot \varepsilon - \alpha \sum_{i=0}^{\ell} |x_{k+\ell-i}|^p \tag{68}$$

where

$$L_{Z_\ell} = L_v L_f^{(N-1-\ell)} + L_r \frac{L_f^{(N-1-\ell)} - 1}{L_f - 1}$$

$$L_{C_\ell} = L_v \frac{L_f^{(N-1-\ell)} - 1}{L_f - 1}$$
$$+ \frac{L_r}{L_f - 1} \left\{ \sum_{i=0}^{N-2-\ell} L_f^i - (N - 2 - \ell) \right\}.$$

APPENDIX E
PROOF OF THEOREM 3

The stability of the closed-loop control system is ensured by strictly decreasing ΔJ_ℓ given by

$$\Delta J_{\ell+1} \leq \Delta J_\ell. \tag{69}$$

To derive the condition (69), we can use bound property. First, bound of $\Delta J_{\ell+1}$ is given by

$$J_N^*(k + \ell + 1) - J_N^*(k - 1) \leq \Delta J_{\ell+1}$$
$$\leq L_{Z_\ell} e(k + \ell|k - 1) + 2L_{C_\ell}\varepsilon - \alpha \sum_{i=0}^{\ell} |x_{k-i+\ell}|^p \tag{70}$$
$$< L_{Z_\ell} e(k + \ell|k - 1) + 2L_{C_\ell}\varepsilon - \sigma \cdot \alpha \sum_{i=0}^{\ell} |x_{k-i+\ell}|^p \tag{71}$$

where (70) and (71) are obtained by (25) and introduction of new variable σ satisfying $0 < \sigma < 1$. Similarly, the bound of ΔJ_j is as follows:

$$J_N^*(k + \ell) - J_N^*(k - 1) \leq \Delta J_\ell$$
$$\leq L_{Z_{\ell-1}} e(k + \ell - 1|k - 1) + 2L_{C_{\ell-1}}\varepsilon - \alpha \sum_{i=0}^{\ell-1} |x_{k-i+\ell}|^p$$
$$< L_{Z_{\ell-1}} e(k + \ell - 1|k - 1) + 2L_{C_{\ell-1}}\varepsilon - \sigma \cdot \alpha \sum_{i=0}^{\ell-1} |x_{k-i+\ell}|^p.$$

Because $J_N^*(k + \ell + 1) \leq J_N^*(k + \ell)$, it is guaranteed that

$$J_N^*(k + \ell + 1) - J_N^*(k + \ell) \leq \Delta J_{\ell+1} - \Delta J_\ell. \tag{72}$$

Finally, in order to satisfy (69), the upper bound of $\Delta J_{\ell+1}$ should be smaller than the bound of ΔJ_ℓ, given by

$$L_{Z_\ell} e(k + \ell|k - 1) + 2L_{C_\ell}\varepsilon - \sigma \cdot \alpha \sum_{i=0}^{\ell} |x_{k-i+\ell}|^p$$
$$\leq L_{Z_{\ell-1}} e(k + \ell - 1|k - 1) + 2L_{C_{\ell-1}}\varepsilon - \sigma \cdot \alpha \sum_{i=0}^{\ell-1} |x_{k-i+\ell}|^p.$$

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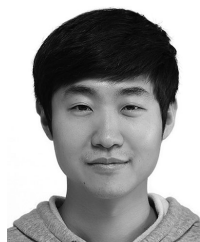
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