Decentralized symbolic control of interconnected systems with application to vehicle platooning ⋆

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Abstract: This work aims at extending some concepts of symbolic control design to decentralized control structures, with an approximate simulation approach. Symbolic models and controllers are based on abstractions of continuous dynamics where one symbol corresponds to an aggregate of continuous states. We consider a serial interconnection of continuous nonlinear systems and we address the decentralized design of local controllers to accomplish a given specification on the overall system. The results are applied to a vehicle platooning problem, where we jointly fulfill a safety constraint (collision avoidance) and reduce the fuel consumption.

Keywords: Decentralized control, automata, control applications.

1. INTRODUCTION

The use of formal methods and symbolic models for the analysis and control design of continuous and hybrid systems [Egerstedt et al. (2006)] is an emerging research area in the control systems and computer science communities. Symbolic models are abstract descriptions of continuous systems where each symbol corresponds to an "aggregate" of continuous states [Tabuada (2009)]. They also provide a formal approach to solve control problems in systems featuring a tight combination and coordination between computational elements and physical world (the so-called *Cyber-Physical Systems* [Lee (2006)]). Moreover, symbolic methods provide systematic techniques to address a wide spectrum of novel requirements that are difficult to enforce by means of classical control design paradigms, such as specifications expressed in linear temporal logic or automata.

Several classes of dynamical and control systems that admit equivalent symbolic models have been identified in the literature. These results are based on the notion of bisimulation [Milner (1989); Park (1981)] as a key ingredient to capture equivalence between dynamical and hybrid *infinite state* systems and the corresponding symbolic models. The notion of approximate bisimulation, a generalization of the notion of bisimulation to metric systems, introduced by [Girard and Pappas (2007)], inspired research to identify other classes of control systems admitting symbolic models, examples of which are nonlinear control systems with and without disturbances [Pola et al. (2008); Zamani et al. (2012); Pola and Tabuada (2009); Borri et al. (2012a)], switched systems [Girard et al. (2010)], time-delay systems [Pola et al. (2010a,b)], Networked Control Systems [Borri et al. (2012c,b)]. Building upon these finite models, symbolic control design problems, with specifications expressed in terms of automata, were formalized and solved (see e.g. [Tabuada (2008); Pola et al. (2012)]).

This work aims at extending some concepts of symbolic control design to decentralized settings, with several plants being serially interconnected, following an approximate simulation approach. We drew some inspiration from the work by [Karimadini and Lin (2011)], which considers purely discrete systems, but our main goal is *to manage systems including continuous dynamics*. Indeed, our viewpoint is that concrete applications most often include continuous processes along with discrete events/logics, hence an extension of purely discrete approaches to the aforementioned case is recommended. On the other hand, in the context of symbolic models and controllers, an extension to the distributed context is helpful, for the following reasons:

- *Computational complexity.* The size of a centralized symbolic controller [Pola et al. (2012)] scales exponentially with the number of interconnected plants

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(which is related to the dimension of the overall state space). This makes the centralized approach unfeasible, even for a relatively low number of interconnected plants.
- *Local controllers.* A centralized control of distributed agents by means of a remote symbolic controller requires taking into account the non-idealities of the resulting Networked Control System [Borri et al. (2012c,b)]. The design of local controllers (physically connected to each plant) allows us to neglect some of the communication non-idealities, such as packet dropouts and communication delays.
- *Identical agents.* A very common example of spatially distributed system is a network of identical agents. In this case, the symbolic model is the same for all the plants and the control design phase (based on the unique symbolic model) is faster and more efficient, even if the local specifications are different.

In the second part of the paper, we illustrate the application of our methods to the Heavy Duty Vehicle (HDV) Platooning Problem, see e.g. [Alam et al. (2011a)]. A nonlinear model of the vehicle motion is considered, where the dynamics of each agent is affected by the vehicle ahead, because of the aerodynamic drag which depends on their relative distance. Local cruise controllers are designed to proceed at the nominal platoon velocity and to keep a short inter-vehicle distance, leading to fuel reduction. The symbolic approach takes account natively of quantization effects and periodic sensing. On top of that, since the leading vehicle can change his velocity in consequence of external reasons (road obstructions, new speed limits, etc.), we design local symbolic controllers achieving a collision avoidance specification, while minimizing the incremental torque with respect to the nominal value.

The paper is organized as follows. In Section 2, we review some preliminary concepts and address the centralized symbolic control of serial-interconnected control systems. In Section 3, we illustrate the decentralized approach. In Section 4, we apply our results to the vehicle platooning application. Finally, Section 5 offers some concluding remarks. Some preliminary notions and the proofs of technical results are not included for lack of space. The interested reader is referred to [Borri et al. (2013)] for a full version of this work.

2. SYMBOLIC CONTROL OF SERIAL INTERCONNECTED CONTROL SYSTEMS

2.1 Preliminaries

The cardinality of a set A is denoted by $|A|$. The identity map on a set A is denoted by 1_A. The symbol χ_A denotes the characteristic function of a set A such that $\chi_A(a) = 1$ if $a \in A$ and $\chi_A(a) = 0$ if $a \notin A$. Given a set A we denote $A^2 = A \times A$ and $A^{n+1} = A \times A^n$ for any $n \in \mathbb{N}$. Given a pair of sets A and B and a relation $\mathcal{R} \subseteq A \times B$, the symbol $\mathcal{R}^{-1}$ denotes the inverse relation of $\mathcal{R}$, i.e. $\mathcal{R}^{-1} = \{(b, a) \in B \times A : (a, b) \in \mathcal{R}\}$.

The symbols $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{R}_0^+$ denote the set of natural, integer, real, positive real, and nonnegative real numbers, respectively. A *quasimetric* on a set A satisfies all axioms of a metric except symmetry, i.e. in general $d(x, y) \neq d(y, x)$, $x, y \in A$. Given a vector $x \in \mathbb{R}^n$ we denote by $\|x\|$ the infinity norm and by $\|x\|_2$ the Euclidean norm of x. Given $\mu \in \mathbb{R}^+$ and $A \subseteq \mathbb{R}^n$, we set $[A]_\mu = \mu\mathbb{Z}^n \cap A$; if $B = \bigcup_{i\in[1;N]} A^i$ then $[B]_\mu = \bigcup_{i\in[1;N]} ([A]_\mu)^i$. Consider a convex bounded set $A \subseteq \mathbb{R}^n$ with interior. Let $H = [a_1, b_1] \times \cdots \times [a_n, b_n]$ be the smallest hyperrectangle containing A and set $\hat{\mu}_A = \min_{i=1,2,\ldots,n}(b_i - a_i)$. It is readily seen that for any $\mu < \hat{\mu}_A$ and any $a \in A$ there always exists $b \in [A]_\mu$ such that $\|a - b\| \leq \mu$. Given $a \in A \subseteq \mathbb{R}^n$ and a precision $\mu \in \mathbb{R}^+$, the symbol $[a]_\mu$ denotes a vector in $\mu\,\mathbb{Z}^n$ such that $\|a - [a]_\mu\| \leq \mu$.

Given a measurable function $f : \mathbb{R}_0^+ \to \mathbb{R}^n$, the (essential) supremum of f is denoted by $\|f\|_\infty$. A continuous function $\gamma : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\gamma(0) = 0$; a function γ is said to belong to class $\mathcal{K}_\infty$ if $\gamma \in \mathcal{K}$ and $\gamma(r) \to \infty$ as $r \to \infty$.

We use the classical notion of (alternating) transition system [Alur et al. (1998)] to describe both control systems and symbolic models.

Definition 1. An *(alternating) transition system* T is a tuple

$$T = (Q, Q_0, L, \longrightarrow, O, H)$$

consisting of:

- a set of states Q;
- a set of initial states $Q_0 \subseteq Q$;
- a set of labels $L = A \times B$, where
  - A is the set of *control* labels;
  - B is the set of *disturbance* or *exogenous* labels;
- a transition relation $\longrightarrow \subseteq Q \times L \times Q$;
- a set of observations O;
- an observation function $H : Q \to O$.

When $Q_0 = Q$, we refer to the transition system T by means of the tuple $T = (Q, L, \longrightarrow, O, H)$. A transition $(q, l, q') \in \longrightarrow$ of T is denoted by $q \xrightarrow{l} q'$.

The transition system T is said to be:

- *countable*, if Q and L are countable sets;
- *symbolic* or *finite*, if Q and L are finite sets;
- *(quasi)metric*, if O is equipped with a (quasi)metric $d : O \times O \to \mathbb{R}_0^+$;
- *deterministic*, if for any $q \in Q$ and any $a \in A$ there exists at most one state $q' \in Q$ such that $q \xrightarrow{(a,b)} q'$ for some $b \in B$.

Definition 2. Given two transition systems $T_i = (Q_i, Q_{0,i}, L_i, \xrightarrow[i]{}, O_i, H_i)$ $(i = 1, 2)$, T_1 is a *sub-transition system* of T_2, denoted $T_1 \sqsubseteq T_2$, if $Q_1 \subseteq Q_2$, $Q_{0,1} \subseteq Q_{0,2}$, $L_1 \subseteq L_2$, $\xrightarrow[1]{} \subseteq \xrightarrow[2]{}$, $O_1 \subseteq O_2$, $H_1(q) = H_2(q)$ for any $q \in Q_1$.

In the sequel, we consider *(alternating) approximate (bi)simulation relations* to relate properties of transition systems. As discussed in [Pola and Tabuada (2009); Tabuada (2009)], this notion is a key ingredient when constructing symbolic models of systems affected by nondeterminism because it guarantees that control strategies synthesized on the symbolic models can be readily transferred to the original model.

2.2 Continuous dynamics and coupling

We consider N nonlinear control systems in the form:

$$P_i : \begin{cases} \dot{x}_i = \tilde{f}_i(x_i, u_i, w_i), \\ y_i = h_i(x_i), \end{cases} \tag{1}$$

where $x_i \in \mathbf{X}_i \subseteq \mathbb{R}^{n_i}$, $y_i \in \mathbf{Y}_i \subseteq \mathbb{R}^{q_i}$, $u_i(\cdot) : \mathbb{R}_0^+ \to \mathbf{U}_i \subseteq \mathbb{R}^{m_i}$, $w_i(\cdot) : \mathbb{R}_0^+ \to \mathbf{W}_i \subseteq \mathbb{R}^{s_i}$. The time dependence is omitted in (1). The quantities $x_i(t)$, $u_i(t)$, $w_i(t)$ and $y_i(t)$ denote state, control input, exogenous input and output of plant P_i at any time $t \in \mathbb{R}_0^+$. We assume $u_i(\cdot)$ to be a piecewise-constant function of time and $w_i(\cdot)$ to be a continuous function of time with bounded first-order derivative. We suppose that the sets $\mathbf{X}_i$, $\mathbf{U}_i$, $\mathbf{W}_i$, $\mathbf{Y}_i$ are convex, bounded and with interior. The state transition function $\tilde{f}_i$ is assumed to be Lipschitz on compact sets. The output function h_i is assumed to be continuously differentiable in the domain $\mathbf{X}_i$. In the sequel we denote by $\mathbf{x}_i(t, x_0, u, w)$ the state reached by (1) at time t from the initial state x_0 under the constant control input u and exogenous signal $w \in \mathcal{W}^i$, where $\mathcal{W}^i$ denotes the set of continuous functions defined on $\mathbb{R}_0^+$, taking values in $\mathbf{W}_i$ and satisfying a Lipschitz assumption of the form:

$$\|w(t_2) - w(t_1)\| \leq \kappa_{w_i} |t_2 - t_1|,$$

for some $k_{w_i} > 0$, any $w \in \mathcal{W}^i$ and any $t_1, t_2 \in \mathbb{R}_0^+$; the state $\mathbf{x}_i(t, x_0, u, w)$ is uniquely determined, since the assumption on $\tilde{f}_i$ ensures existence and uniqueness of trajectories. We assume that the control systems P_i are forward complete, namely that every trajectory is defined on an interval of the form $]a, \infty[$.

We assume a *serial interconnection* of plants P_i expressed by the following constraints:

$$w_i = y_{i-1} = h_{i-1}(x_{i-1}) \qquad i = 2, ..., N \tag{2}$$

and we denote $w_1 = w$. The overall plant P is depicted in Fig. 1 and has the form:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_N \end{bmatrix} = f(x, u, w) \tag{3}$$

$$= \begin{bmatrix} \tilde{f}_1(x_1, u_1, w) \\ \tilde{f}_2(x_2, u_2, h_1(x_1)) \\ \vdots \\ \tilde{f}_N(x_N, u_N, h_{N-1}(x_{N-1})) \end{bmatrix} = \begin{bmatrix} f_1(w, x_1, u_1) \\ f_2(x_1, x_2, u_2) \\ \vdots \\ f_N(x_{N-1}, x_N, u_N) \end{bmatrix} \tag{4}$$

where we properly defined functions f_i, which just depend on the local state x_i and on the state of plant P_{i-1}, according to the topological constraint (2). As a consequence, the local trajectory of plant P_i depends on all plants P_j with $j \leq i$ and on the exogenous signal w. Plant P_1 is called *leader* because it is not affected by plants $P_2, \ldots, P_N$, called *followers*. The spaces $\mathbf{X}$, $\mathbf{U}$, $\mathbf{Y}$ are naturally defined as cartesian products of the local spaces, i.e. $\mathbf{X} = \mathbf{X}_1 \times ... \times \mathbf{X}_N$, $\mathbf{U} = \mathbf{U}_1 \times ... \times \mathbf{U}_N$, $\mathbf{Y} = \mathbf{Y}_1 \times ... \times \mathbf{Y}_N$, while $\mathbf{W} = \mathbf{W}_1$. The quantity $\mathbf{x}(t, x_0, u, w)$ denotes the state reached by the plant P at time t from the initial state $x_0 = [x_{1,0}, \ldots, x_{N,0}]'$ under the constant control input $u = [u_1, \ldots, u_N]'$ and exogenous signal $w \in \mathcal{W} = \mathcal{W}^1$ with Lipschitz constant $k_w = k_{w_1}$.

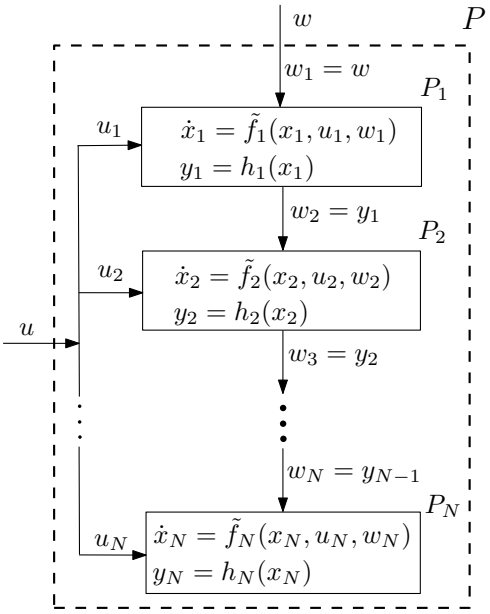

Fig. 1. Serial Interconnection of Control Systems

In the sequel, we make use of the following notion of *incremental forward completeness*.

Definition 3. [Zamani et al. (2012)] A control system P is incrementally forward complete (δ-FC) if it is forward complete and there exist continuous functions $\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$, $\gamma_u : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$, $\gamma_w : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$, such that:

- for any $s \in \mathbb{R}^+$, the function $\beta(\cdot, s)$ belongs to class $\mathcal{K}_\infty$;
- for any $x_1, x_2 \in \mathbf{X}$, any $t \in \mathbb{R}^+$, any $u_1, u_2 \in \mathbf{U}$ and any $w_1, w_2 \in \mathcal{W}$, the following inequality is satisfied:

$$\|\mathbf{x}(t, x_1, u_1, w_1) - \mathbf{x}(t, x_2, u_2, w_2)\| \leq \beta(\|x_1 - x_2\|, t) + \gamma_u(\|u_1 - u_2\|, t) + \gamma_w(\|w_1 - w_2\|_\infty, t).$$

The class of δ-FC control systems is rather large and includes also some subclasses of unstable control systems; for instance, unstable linear systems are δ-FC. The notion of δ-FC can be characterized in terms of Lyapunov-like functions.

Definition 4. [Zamani et al. (2012)] A control system P is δ–FC if there exists a smooth function $V : \mathbf{X} \times \mathbf{X} \to \mathbb{R}$, a number $\lambda \in \mathbb{R}$ and some $\mathcal{K}_\infty$ functions $\underline{\alpha}$, $\overline{\alpha}$, σ_u and σ_w such that for any $x_1, x_2 \in \mathbf{X}$, any $u_1, u_2 \in \mathbf{U}$ and any $w_1, w_2 \in \mathbf{W}$ the following conditions hold true:

(i) $\underline{\alpha}(\|x_1 - x_2\|) \leq V(x_1, x_2) \leq \overline{\alpha}(\|x_1 - x_2\|)$,
(ii) $\frac{\partial V}{\partial x_1} f(x_1, u_1, w_1) + \frac{\partial V}{\partial x_2} f(x_2, u_2, w_2) \leq \lambda V(x_1, x_2) + \sigma_u(\|u_1 - u_2\|) + \sigma_w(\|w_1 - w_2\|)$.

Function V is called a δ–FC Lyapunov function for P.

2.3 The symbolic control approach

According to the symbolic approach (see e.g. [Tabuada (2009)]), it is possible to build an infinite transition system $T_\tau(P)$ encoding all the trajectories of P of finite duration $\tau > 0$, as follows:

$$T_\tau(P) := (\mathbf{X}, \mathbf{U} \times \mathbf{W}, \xrightarrow[\tau]{}, 2^{\mathbf{X}}, 1_{\mathbf{X}}),$$

where $x \xrightarrow[\tau]{(u,w)} x'$ is a transition of $T_\tau(P)$ if $x' = \mathbf{x}(\tau, x, u, \bar{w})$ in P for some $\bar{w} \in \mathcal{W}$ s.t. $\bar{w}(0) = w$. We consider the metric induced by the infinity norm $\|\cdot\|$ on $\mathbf{X}$ and we assume the following quasimetric on the output set $2^{\mathbf{X}}$:

$$\mathbf{d}_{\mathbf{X}}(A, B) = \sup_{p \in A} \inf_{q \in B} \|p - q\|, \tag{5}$$

which corresponds to the *directed Hausdorff distance* from a set A to a set B.

We suppose the following assumptions on P:

(A1) There exists a δ–FC Lyapunov function satisfying the conditions in Definition 4 for some $\lambda \in \mathbb{R}$ and for some $\mathcal{K}_\infty$ functions $\underline{\alpha}$, $\overline{\alpha}$, σ_u and σ_w.

(A2) There exists a $\mathcal{K}_\infty$ function γ such that [1]:

$$V(x, x') - V(x, x'') \leq \gamma(\|x' - x''\|),$$

for every $x, x', x'' \in \mathbf{X}$.

In order to get a finite (symbolic) model, we consider finite quantization parameters $\mu_x > 0$, $\mu_u > 0$, $\mu_w > 0$, for state, control input and exogenous signal, respectively, and a design parameter $\eta > 0$, hence we build the finite transition system

$$T_*(P) := ([\mathbf{X}]_{\mu_x}, [\mathbf{U}]_{\mu_u} \times [\mathbf{W}]_{\mu_w}, \xrightarrow[*]{}, 2^{\mathbf{X}}, 1_{\mathbf{X}}),$$

where $x \xrightarrow[*]{(u,w)} x'$ is a transition of $T_*(P)$ if

$$V(\mathbf{x}(\tau, x, u, w \cdot \chi_{[0,\tau]}), x') \leq e^{\lambda\tau}\underline{\alpha}(\eta) + (1 - e^{\lambda\tau})\frac{\bar{\sigma}}{\lambda} + \gamma(\mu_x), \tag{6}$$

with $\bar{\sigma} = \max\{\sigma_u(\mu_u), \sigma_w(\mu_w + \kappa_w\tau)\}$. The δ-FC assumption on P ensures that, for any desired precision, there exists a choice of quantization parameters such that the symbolic model $T_*(P)$ is alternatingly approximately simulated by $T_\tau(P)$ (see [Borri et al. (2013)]).

The construction of a symbolic model is instrumental in solving control problems. Assume we want to solve a *symbolic control problem* in terms of approximate similarity game.

Problem 1. Let $S = (Q^s, Q_0^s, L^s, \xrightarrow[s]{}, O^s = 2^{\mathbf{X}}, H^s)$ be a specification, given in form of a transition system, defined over the state space $\mathbf{X}$ of the plant P. Then for any desired precision $\varepsilon \in \mathbb{R}^+$, find a sampling time $\tau \in \mathbb{R}^+$, a parameter $\theta \in \mathbb{R}^+$, a symbolic transition system (controller) C and a $A\theta A$ simulation relation $\mathcal{R}$ from C to $T_\tau(P)$ such that:

$$T_\tau(P) \times_\theta^{\mathcal{R}} C \preceq_\varepsilon S.$$

A graphical representation of Problem 1 is given in Fig. 2. This problem asks for the synthesis of a symbolic controller C such that the controlled plant implements the specification S up to a given approximation ε. This problem can be viewed as an approximate version of similarity games, as discussed in [Tabuada (2009)].

We now introduce the controller C^*, solving Problem 1, based on the symbolic model of P.

[1] Note that since V is smooth, one can choose $\gamma(\|w - z\|) = \left(\sup_{x,y \in X} \|\frac{\partial V}{\partial y}(x, y)\|\right) \|w - z\|$.

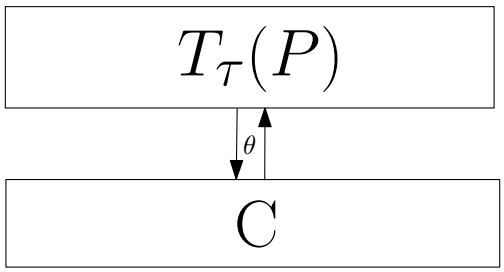

Fig. 2. Graphical representation of Problem 1

Definition 5. The symbolic controller C^* is the maximal sub-transition system [2] C of $T_*(P)$ such that $C \preceq_{\mu_x} S$ and $C \preceq_0^{\text{alt}} T_*(P)$.

The meaning of the conditions stated in Definition 5 is the following. Condition $C \preceq_{\mu_x} S$ requires that the transitions of S are approximated by the ones in C within the accuracy μ_x of the grid on the state space. Condition $C \preceq_0^{\text{alt}} T_*(P)$ ensures that the controller enforces the specification robustly with respect to the non-determinism of $T_*(P)$. The following result provides the solution of Problem 1.

Theorem 1. Consider a nonlinear control system P and a specification transition system S. Suppose that Assumptions (A1)–(A2) hold. For any desired precision $\varepsilon \in \mathbb{R}^+$, choose the parameters $\tau \in \mathbb{R}^+$, $\theta \in \mathbb{R}^+$, $\eta \in \mathbb{R}^+$, $\mu_u < \hat{\mu}_{\mathbf{U}}$, $\mu_w < \hat{\mu}_{\mathbf{W}}$, $\mu_x < \hat{\mu}_{\mathbf{X}}$, such that:

$$\mu_x + \theta \leq \varepsilon, \tag{7}$$

$$\mu_x \leq \overline{\alpha}^{-1}(\underline{\alpha}(\theta)) \leq \theta \leq \eta. \tag{8}$$

Then Problem 1 is solved with $C = C^*$, as in Definition 5, and with $\mathcal{R} = \mathcal{R}^*$, where $\mathcal{R}^*$ is a $A\theta A$ simulation relation from C^* to $T_\tau(P)$.

3. DESIGN OF DECENTRALIZED SYMBOLIC CONTROLLERS

The results in the previous section are based on the construction of the symbolic model of the overall plant P, which can be computationally demanding (the complexity scales exponentially with the number N of plants in P). In this section, we follow a decentralized approach to build local N low-dimensional symbolic controllers to accomplish a specification on P. The decentralized approach is particularly helpful in the case of networks of identical plants P_i, because one can use N replicas of the same reduced-order symbolic models and controllers, properly interconnected to one another.

We start building the time-discretization of each local plant P_i, for $i = 1, ..., N$, as follows:

$$T_\tau(P_i) := (\mathbf{X}_i, \mathbf{U}_i \times \mathbf{W}_i, \xrightarrow[\tau,i]{}, 2^{\mathbf{X}_i}, 1_{\mathbf{X}_i}),$$

where $x_i \xrightarrow[\tau,i]{(u_i,w_i)} x_i'$ is a transition of $T_\tau(P_i)$ if $x_i' = \mathbf{x}_i(\tau, x_i, u_i, w)$ in P_i for some $w \in \mathcal{W}^i$ s.t. $w(0) = w_i$.

We assume the incremental forward completeness of all the plants, namely for each $i = 1, ..., N$:

[2] Here maximality is defined with respect to the preorder induced by the notion of sub-transition system.

(A1′) There exists a δ–FC Lyapunov function V_i satisfying the conditions in Definition 4 for some $\lambda_i \in \mathbb{R}$ and for some $\mathcal{K}_\infty$ functions $\underline{\alpha}_i$, $\overline{\alpha}_i$, σ_{u_i} and σ_{w_i}.

(A2′) There exists a $\mathcal{K}_\infty$ function γ_i such that:

$$V_i(x, x') - V_i(x, x'') \leq \gamma_i(\|x' - x''\|),$$

for every $x, x', x'' \in \mathbf{X}_i$.

We assume the same quasimetric on $2^{\mathbf{X}_i}$ as in (5), and we can build a symbolic model for P_i, as follows:

$$T_*(P_i) := ([\mathbf{X}_i]_{\mu_x}, [\mathbf{U}_i]_{\mu_u} \times [\mathbf{W}_i]_{\mu_w}, \underset{*,i}{\longrightarrow}, 2^{\mathbf{X}_i}, 1_{\mathbf{X}_i}),$$

where $x_i \xrightarrow[*,i]{(u_i,w_i)} x_i'$ is a transition of $T_*(P_i)$ if

$$V_i(\mathbf{x}_i(\tau, x_i, u_i, w_i \cdot \chi_{[0,\tau]}), x_i') \leq e^{\lambda_i \tau}\underline{\alpha}_i(\eta) + (1 - e^{\lambda_i \tau})\frac{\bar{\sigma}_i}{\lambda_i} + \gamma_i(\mu_x),$$

with $\bar{\sigma}_i = \max\{\sigma_{u_i}(\mu_u), \sigma_{w_i}(\mu_w + \kappa_{w_i}\tau)\}$. Note that different local values of the design parameters μ_{x_i}, μ_{u_i}, μ_{w_i}, η_i could be defined, but we here adopt common values for an easier notation. The following concept of *serial composition* restates the serial coupling constraint in (2) in terms of interaction between a pair of transition systems.

Definition 6. Consider a pair of metric transition systems $T_i = (Q_i, Q_{0,i}, L_i = A_i \times B_i, \underset{i}{\longrightarrow}, O_i, H_i)$ $(i = 1, 2)$ and a coupling map $s : Q_1 \to 2^{B_2}$. The *serial composition* of T_1 and T_2 is the transition system

$$T = T_1 \|_s T_2 = (Q, Q_0, L = A \times B, \longrightarrow, O, H),$$

where

- $Q = Q_1 \times Q_2$
- $Q_0 = Q_{0,1} \times Q_{0,2}$;
- $A = A_1 \times A_2$;
- $B = B_1$;
- $(q_1, q_2) \xrightarrow{((a_1,a_2),b_1)} (q_1', q_2')$, if $q_1 \xrightarrow[1]{(a_1,b_1)} q_1'$, $q_2 \xrightarrow[2]{(a_2,b_2)} q_2'$ for some $b_2 \in B_2$ s.t. $b_2 \in s(q_1)$;
- $O = O_1 \times O_2$;
- $H(q_1, q_2) = H_1(q_1) \times H_2(q_2)$ for any $(q_1, q_2) \in Q$.

If $s(q_1) = B_2$, the coupling constraint $b_2 \in s(q_1)$ in the transition relation $\longrightarrow$ is always satisfied and the transition system $T_1 \|_s T_2$ is simply denoted as $T_1 \| T_2$.

It is readily seen that, for any coupling map s, one gets:

Lemma 1.

$$T_1 \|_s T_2 \preceq_0 T_1 \| T_2.$$

In the following, we write $T_1 \|_{s_1} T_2 \|_{s_2} T_3$ with the meaning $(T_1 \|_{s_1} T_2) \|_{s_2} T_3$. It follows from the previous definition that:

$$T_\tau(P) = T_\tau(P_1) \|_{s_1} T_\tau(P_2) \|_{s_2} \cdots \|_{s_{N-1}} T_\tau(P_N),$$

where $s_i : X_1 \times ... \times X_i \to W_{i+1}$ is defined as $s_i(x_1, ..., x_i) = h_i(x_i)$. According to the decomposition of $T_\tau(P)$ into N transition systems, the control problem can be reinterpreted as in Fig. 3 and can still be solved by means of a symbolic controller C^*, based on the symbolic model $T_*(P)$, as described in the previous section.

In the following, we develop a decentralized approach and construct *local symbolic controllers* $C_1^*, ..., C_N^*$ such that

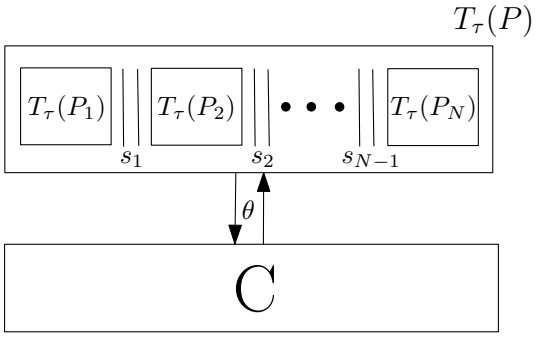

Fig. 3. Centralized Symbolic Control Approach.

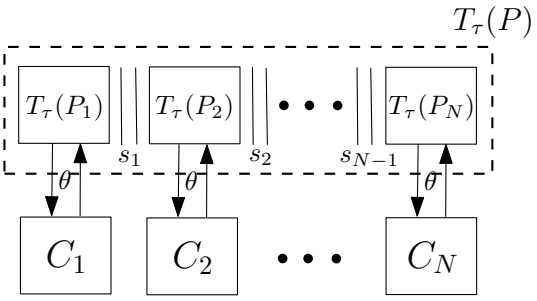

Fig. 4. Decentralized Symbolic Control Approach.

the closed-loop system still achieves the overall specification. The symbolic controller of plant P_i computes the control input u_i based on the knowledge of the state x_i and *robustly* with respect to the state of the plant P_{i-1}, which is considered as an exogenous signal w_i and is measured every τ seconds. This is expressed by means of the following control problem, which is illustrated in Fig. 4, where the role of the exogenous input is omitted.

Problem 2. Let $S = (Q^s, Q_0^s, L^s, \underset{s}{\longrightarrow}, O^s = 2^{\mathbf{X}}, H^s)$ be a specification, given in form of a transition system, defined over the state space $\mathbf{X}$ of the plant P, given by the interconnection of N plants $P_1, \ldots, P_N$. Then for any desired precision $\varepsilon \in \mathbb{R}^+$, find a sampling time $\tau \in \mathbb{R}^+$, a parameter $\theta \in \mathbb{R}^+$, some symbolic transition systems (controllers) C_i and some $A\theta A$ simulation relations $\mathcal{R}_i$ from C_i to $T_\tau(P_i)$, for all $i = 1, ..., N$, such that:

$$(T_\tau(P_1) \times_\theta^{\mathcal{R}_1} C_1) \|_{s_1} \cdots \|_{s_{N-1}} (T_\tau(P_N) \times_\theta^{\mathcal{R}_N} C_N) \preceq_\varepsilon S.$$

Note that the decentralized control problem above could be defined with different composition parameters θ_i, $i = 1, ..., N$.

In order to solve Problem 2, we consider the specification $S = (Q^s, Q_0^s, L^s, \underset{s}{\longrightarrow}, O^s = 2^{\mathbf{X}}, H^s)$ and introduce the local *projection maps* $H_i^s : Q^s \to 2^{\mathbf{X}_i}$ defined as $H_i^s(q_s) = F_i$, with $F_1 \times \cdots \times F_N$ being the maximal hyperrectangle contained in $H^s(q_s)$. According to this definition, we can define the *local specifications* $S_1, ..., S_N$ defined as:

$$S_i = (Q^s, Q_0^s, L^s, \underset{s}{\longrightarrow}, O_i^s = 2_i^{\mathbf{X}}, H_i^s) \tag{9}$$

for all i. It is readily seen that

$$S_1 \| \cdots \| S_N \preceq_0 S. \tag{10}$$

Remark 1. Formally, the local maps H_i^s defined above are used to *project* the overall specification, defined on the state space $\mathbf{X}$, onto local specifications defined on

the spaces $\mathbf{X}_1, \ldots, \mathbf{X}_N$. The joint fulfillment of the local specifications $S_1 \| \cdots \| S_N$ is not able, in general, to achieve the same *behavior* (in terms of trajectories in the overall state space) of the specification S, since there could be some part of the specification which might require strict cooperation among agents, hence it can be fulfilled just by means of a centralized controller. This is expressed by the similarity relation in (10). When (10) holds with the stronger property of bisimulation, the specification S is said to be *perfectly decoupable*.

The local symbolic controllers are defined next.

Definition 7. For any i, the symbolic controller C_i^* is the maximal sub-transition system C_i of $T_*(P_i)$ such that $C_i \preceq_{\mu_x} S_i$ and $C_i \preceq_0^{\text{alt}} T_*(P_i)$, where each local specification S_i is defined in (9).

The conditions in Definition 7 have the same meaning of the corresponding conditions given in Definition 5 with respect to the centralized controller. We are now ready to provide the decentralized solution to Problem 2, which concludes this section.

Theorem 2. Consider a nonlinear control system P and a specification transition system S. Assume that P is the serial interconnection of N plants $P_1, \ldots, P_N$, according to the constraint (2). Suppose that Assumptions (A1′)–(A2′) hold. Then for any desired precision $\varepsilon \in \mathbb{R}^+$, choose the parameters $\tau \in \mathbb{R}^+$, $\theta \in \mathbb{R}^+$, $\eta \in \mathbb{R}^+$, $\mu_u < \min_{i=1,\ldots,N} \hat{\mu}_{\mathbf{U}_i}$, $\mu_w < \min_{i=1,\ldots,N} \hat{\mu}_{\mathbf{W}_i}$, $\mu_x < \min_{i=1,\ldots,N} \hat{\mu}_{\mathbf{X}_i}$ such that:

$$\mu_x + \theta \leq \varepsilon \tag{11}$$

$$\mu_x \leq \min_{i=1,\ldots,N} \overline{\alpha}_i^{-1}(\underline{\alpha}_i(\theta)) \leq \theta \leq \eta. \tag{12}$$

Then Problem 2 is solved with $C_i = C_i^*$, as in Definition 7, and with $\mathcal{R}_i = \mathcal{R}_i^*$, where $\mathcal{R}_i^*$ is a $A\theta A$ simulation relation from C_i^* to $T_\tau(P_i)$, for all i.

4. THE HEAVY DUTY VEHICLE (HDV) PLATOONING APPLICATION

4.1 Platooning modeling

Nowadays, the Heavy Duty Vehicle (HDV) platooning is considered a balanced solution to the increasing transport intensity, traffic safety and fuel reduction (see e.g. [Alam et al. (2011a)] for more information). Each vehicle in the platoon is modeled by means of a nonlinear control system, describing the motion dynamics [Sahlholm and Johansson (2010)]. The model takes into account the actions of the powertrain and of the braking system, the aerodynamic drag, the rolling resistance and the gravitational force, as follows:

$$\begin{aligned} m\dot{v} &= F_{engine} - F_{brake} - F_{airdrag} - F_{roll} - F_{gravity} \\ &= k_e T - F_{brake} - k_D(d)v^2 - k_{f_r}\cos(\alpha) - k_g \sin(\alpha), \end{aligned}$$

where m is the accelerated HDV mass, v is the vehicle velocity, T is the net engine torque, d is the longitudinal distance from the vehicle ahead, α is the road incline, k_e, k_{f_r}, k_g are coefficients taking account of the vehicle engine, road friction and gravitation effects, and $k_D(d)$ is a least square approximation (within a relevant operating range) of the air drag function defined as:

$$k_D(d) = \begin{cases} a_{lsq} d + b_{lsq} & d < d_{\max}, \\ a_{lsq} d_{\max} + b_{lsq} & d \geq d_{\max}, \end{cases} \tag{13}$$

where $d_{\max}$ is the value of relative distance such that the air drag reduction caused by the presence of the vehicle ahead is negligible.

The engine torque T takes the role of control input for the HDV model. For simplicity, in the following we neglect the braking system, so we can reduce the speed of the single vehicle by reducing the torque.

The platoon of N vehicles can be regarded as a serial-interconnected system. We assume identical vehicles and denote as $\bar{F} := k_{f_r}\cos(\alpha) + k_g \sin(\alpha)$ the whole constant force depending on the road slope, which is assumed to be constant for an easier notation, but this is not restrictive. We refer to each vehicle by means of the index $i \in \{1, \ldots, N\}$, where the leader of the platoon has index 1. The relative distance $d_{(i-1)(i)}$ between two vehicles $i-1$ and i affects the dynamics of the follower vehicle i and is formally defined as:

$$d_{(i-1)(i)} = \begin{cases} d_{\max} & i = 1 \\ s_{i-1} - s_i & i \in \{2, ..., N\}, \end{cases} \tag{14}$$

where s_i is the absolute position of vehicle i in a fixed reference frame. As a consequence of the previous definition, the air drag term is assumed constant for the leader vehicle, whose dynamics is:

$$\dot{v_1} = -\frac{k_D(d_{\max})}{m} v_1^2 - \frac{\bar{F}}{m} + \frac{k_e}{m} T_1 + w, \tag{15}$$

where w is a disturbance modeling a possible deviation in the nominal leader velocity, due to road obstructions or new speed limits. For the following vehicles $i = 2, ..., N$, from (14), we get:

$$\begin{aligned} \dot{d}_{(i-1)(i)} &= \dot{s}_{i-1} - \dot{s}_i = v_{i-1} - v_i, \\ \dot{v_i} &= -\frac{k_D(d_{(i-1)(i)})}{m} v_i^2 - \frac{\bar{F}}{m} + \frac{k_e}{m} T_i. \end{aligned} \tag{16}$$

We now consider a nominal platoon velocity v_0 (with $w = 0$) and a nominal relative distance $d_0 \leq d_{\max}$ between each pair of HDVs. By imposing $\dot{v}_i = 0$ in (15)–(16), we compute the equilibrium torques ($\tilde{T}_0$ for leader and $T_0 \leq \tilde{T}_0$ for the followers) maintaining the velocity v_0. In order to write a simpler nonlinear dynamics with equilibrium point at the origin, we define the quantities

$$\begin{aligned} \Delta s_i &= d_{(i-1)(i)} - d_0, \\ \Delta v_i &= v_i - v_0, \\ \Delta T_i &= \begin{cases} T_i - \tilde{T}_0 & i = 1, \\ T_i - T_0 & i \in \{2, ..., N\}, \end{cases} \end{aligned}$$

as a perturbation with respect to the nominal quantities d_0, v_0, $\tilde{T}_0$, T_0 at the equilibrium. The leader dynamics in (15) is simply rewritten as:

$$\begin{aligned} \dot{\Delta v_1} = &- \frac{k_D(d_{\max})}{m} (\Delta v_1)^2 - \frac{2 v_0 k_D(d_{\max})}{m} \Delta v_1 \\ &+ \frac{k_e}{m} \Delta T_1 + w, \end{aligned} \tag{17}$$

and the follower dynamics in (16) are rewritten as:

$$\begin{aligned}\dot{\Delta s_i} &= \Delta v_{i-1} - \Delta v_i,\\ \dot{\Delta v_i} &= -\frac{k_D(d_0+\Delta s_i)}{m}(\Delta v_i)^2 - \frac{2v_0 k_D(d_0+\Delta s_i)}{m}\Delta v_i\\ &\quad -\frac{v_0^2}{m}(k_D(d_0+\Delta s_i)-k_D(d_0)) + \frac{k_e}{m}\Delta T_i, \qquad (18)\end{aligned}$$

for $i = 2, ..., N$. Note that the dynamics of state i just depends on the states $i-1$ and i and is nonlinear because of the piecewise-linear air drag function $K_D(\cdot)$ in (13) and of the quadratic terms $(\Delta v_i)^2$.

We define the state x_i of each vehicle in the platoon as $x_1 = \Delta s_1$, $x_i = [\Delta s_i, \Delta v_i]'$, $i \in \{2, ..., N\}$, and the input as $u_i = \Delta T_i$ for all i. We are now ready to restate each plant as in (1), with $P_1 = P_l$ (leader) and $P_i = P_f$ (follower) for $i = 2, ..., N$. We get:

$$P_l : \begin{cases} \dot{x}_1 = \tilde{f}_l(x_1, u_1, w_1) = -2v_0\bar{a}x_1 - \bar{a}x_1^2 + \frac{k_e}{m}u_1 + w_1,\\ y_1 = h_l(x_1) = x_1,\end{cases}$$

$$P_f : \begin{cases} \dot{x}_i = \begin{bmatrix}\dot{x}_{i,1}\\ \dot{x}_{i,2}\end{bmatrix} = \tilde{f}_f(x_i, u_i, w_i)\\ \quad = \begin{bmatrix}-x_{i,2} + w_i\\ a(x_{i,1})(-2v_0 x_{i,2} - x_{i,2}^2 - v_0^2) + v_0^2 a(0) + \frac{k_e}{m}u_i\end{bmatrix},\\ y_i = h_f(x_i) = x_{i,2},\end{cases} \qquad (19)$$

where we set $a(s) = \frac{k_D(d_0+s)}{m}$ and $\bar{a} = a(d_{\max} - d_0)$. The coupling constraint in (2) takes the form:

$$w_i = x_{i-1,1} \qquad i = 2, ..., N$$

and the overall state, input and exogenous signal of the vehicle formation are $x = [x_1, ..., x_N]' \in \mathbb{R}^{2N+1}$, $u = [u_1, ..., u_N]' \in \mathbb{R}^N$, $w = w_1 \in \mathbb{R}$, respectively.

4.2 Decentralized symbolic control

A popular approach in the control of a platoon of vehicles (see e.g. [Alam et al. (2011a)]) consists in linearizing the dynamics in (19) around the equilibrium point $x = 0$ and applying a suboptimal decentralized LQR feedback to achieve the fuel efficiency. The safety problem was instead investigated in [Alam et al. (2011b)] as a continuous pursuit-evasion, by means of a game-theoretical approach. Most often (see e.g. [Stankovic et al. (2000); Dunbar and Murray (2006)] and references therein), the available solutions to the platooning problem are based on continuous-time sensing and/or do not consider explicitly possible state and input quantizations.

In the following, we specialize the symbolic approach illustrated in the first part of the paper to the platooning framework. We are able to take into account the fully nonlinear model of the HDV platoon, we consider natively a periodical time sampling, and use sensors and actuators with finite precision. Finally, we formalize in the automata approach heterogenous requirements (safety and fuel efficiency), and manage robustly bounded disturbances (e.g. speed reduction of the leading vehicle) without affecting the achievement of the specifications.

Safety problem. We start by addressing the safety specification S, which is depicted in Fig. 5. The safe condition is formalized by means of the state q^{safe}, and the safe set is $H^s(q^{safe}) = \{x \in \mathbf{X} : x_j + d_0 \geq d_{\min}, j = 2n, n = 1, ..., N\}$, where $d_{\min}$ is the minimum distance allowed between two vehicles. Note that the overall specification S is perfectly decoupable, i.e. it is possible to define local specifications S_i (see Fig. 6) such that $S_1 \| \cdots \| S_N \cong_0 S$ (see Section 3). Note that the specifications for the followers are identical $S_f = S_2 = \cdots = S_N$, while the leader local safety requirement $S_1 = S_l$ is trivially satisfied. We formalize the safety problem as in Problem 2, where we choose $\varepsilon < d_{\min}$, to avoid collision. It is possible to prove that Assumptions (A1′)–(A2′) hold for P_l and P_f. The expressions of the design functions and the numerical values of the parameters depend on the vehicle and on the environment and are here omitted. We solve Problem 2 by means of Theorem 2. Since the models of the $N-1$ followers are assumed to be identical, we just need to compute two symbolic controllers C_l^* and C_f^*, and then we set $C_1^* = C_l^*$ and $C_i^* = C_f^*$ for $i = 2, ..., N$. We obtain each relation $\mathcal{R}_i^*$ solving the decentralized control problem by computing the maximal $A0A$-simulation relation from C_i^* to $T_*(P_i)$. For a more efficient control design, we adopt an integrated technique which does not require the preliminary computation of the whole symbolic models. Useful insights about this topic are discussed in [Pola et al. (2012)].

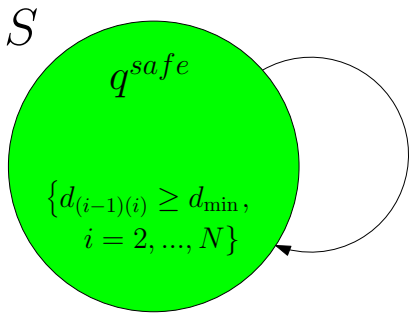

Fig. 5. Overall safety specification in terms of transition system

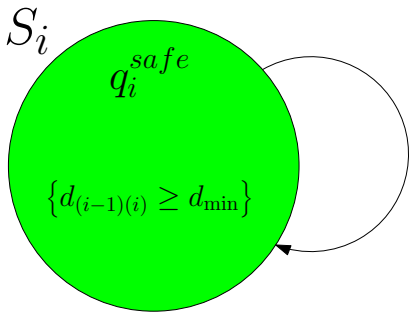

Fig. 6. Local safety specification (for the follower) in terms of transition system

Fuel efficiency. We address the fuel efficiency problem as a *refinement problem*, i.e. starting from each safe controller C_i^*, we design controllers $C_i^{**} \sqsubseteq C_i^*$ (still ensuring safety), by defining the transition relation $\xrightarrow[C_i^{**}]{}$ as:

$$x \xrightarrow[C_i^{**}]{(\bar{u},w)} x' \text{ if } \|\bar{u}\| = \min\left\{\|u\| : x \xrightarrow[C_i^*]{(u,w)} x'\right\}.$$

Since the definition of C_i^* allows for multiple transitions between each pair of states (x, x'), the previous requirement selects the *minimum-energy control* to achieve each transition. More complex cost functions could be consid-

ered for the minimization, also weighing the final state x' of each transition. Moreover, since the local feedback composition $T_\tau(P_i) \times_\theta^{\mathcal{R}_i^*} C_i^{**}$ is in general non-deterministic (more than one *good* control label is available, even for a given sample of the exogenous signal), we can further refine C_i^{**} to obtain a deterministic control behavior. This case is not formalized here. The control design phase is completed by computing the refined $A0A$-simulation relation $\mathcal{R}_i^{**}$ from C_i^{**} to $T_*(P_i)$.

5. DISCUSSION AND OPEN ISSUES

In this work, we introduced a symbolic approach to the decentralized control of a serial interconnection of nonlinear plants. Our results were applied to a vehicle platooning application.

Future work will explore some possible directions of extension of the results here presented. From the theoretical point of view, different local quantization values can be adopted, to build (computationally) smaller controllers. Moreover, techniques of structural decomposition can be exploited to reduce the role of non-determinism in the symbolic control. As regards the platooning application, other non-idealities (communication delays, time-varying slope) can be included in the framework, as well as the brake actuation and non-identical vehicles. Finally the fuel minimization can be generalized to optimize the cost along multiple sampling intervals, with the purpose of improving efficiency.

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