A Scenario-based Predictive Control Approach to Building HVAC Management Systems

Alessandra Parisio, Marco Molinari, Damiano Varagnolo, Karl Henrik Johansson

***Abstract*— We present a Stochastic Model Predictive Control (SMPC) algorithm that maintains predefined comfort levels in building Heating, Ventilation and Air Conditioning (HVAC) systems while minimizing the overall energy use. The strategy uses the knowledge of the statistics of the building occupancy and ambient conditions forecasts errors and determines the optimal control inputs by solving a scenario-based stochastic optimization problem. Peculiarities of this strategy are that it does not make assumptions on the distribution of the uncertain variables, and that it allows dynamical learning of these statistics from true data through the use of *copulas*, i.e., opportune probabilistic description of random vectors. The scheme, investigated on a prototypical student laboratory, shows good performance and computational tractability.**

***Index Terms*— Model predictive control, thermal control, weather forecasts, building modeling, building occupancy, Copula**

I. Introduction

Buildings account for approximately 40% of the total energy use in industrialized countries [1]. To reduce this consumption while satisfying occupants comfort requirements it is possible to develop building control strategies that incorporate occupancy and weather forecasts, time-dependent energy costs, bounds for control actions, and comfort ranges for the controlled variables. A natural scheme to achieve systematic integration of all the aforementioned components is Model Predictive Control (MPC) [2], [3].

Several studies show that predictive control strategies can significantly decrease energy consumption when considering both real-time measurements and foreknowledge of upcoming weather conditions and occupancy [4], [5], [6], [7], [8], [9]. Experimental results on real buildings are also encouraging and suggest that MPC yields better control performance (in terms of energy use and comfort levels) than current practices [10], [11].

Nonetheless utilizing nominal deterministic forecasts, as in the MPC schemes proposed in the aforementioned studies, can lead to inadequate control actions. The amplitude and statistics of the unavoidable forecasts errors can in fact severely affect the performance of predictive controllers. To improve the control performance one can thus explicitly consider the probabilistic distribution of the plausible future evolutions of the system, and develop building controllers that account also for uncertainties in the forecasts.

Literature review: here we specifically review MPC control schemes for building temperature regulation which account for uncertainty. The approach described in [12] incorporates stochastic occupancy models within the control loop. The authors in [13] propose a stochastic predictive building temperature regulator where weather and load disturbances are modeled as Gaussian processes. The resultant nonlinear program is then solved with a tailored sequential quadratic programming which exploits the sparsity of the quadratic sub-problems.

Also in [14] stochastic MPCs and weather predictions are integrated. Here authors firstly compute the control action by solving a non-convex problem which exploits linearizations of the nonlinear system model around nominal trajectories, and then apply a disturbance feedback. We notice that in [14] the predictions of internal gains are assumed to be perfect, i.e., the realization is equal to the prediction. Thus the only considered uncertainty is in weather predictions. The approach proposed in [14] assumes Gaussianly distributed variates. Nonetheless this assumption does not generally hold in practice.

Statement of contributions: in this work we present a method to develop stochastic indoor climate controllers, where the control objective is to minimize the energy use while satisfying thermal comfort and air quality requirements.

We provide a control-oriented building model and a tractable formulation of a Heating, Ventilation and Air Conditioning (HVAC) Stochastic Model Predictive Control (SMPC) which addresses the uncertainty both in weather predictions and occupancy. The proposed strategy uses predictive knowledge of weather and occupancy and manages generic statistic of the weather and occupancy forecasts. Importantly, we do not assume the uncertain variables to be Gaussians, but rather allow every plausible distribution. Technically this is performed by the usage of copulas, see Section III, which allow either to exploit apriori information on the statistics of the forecasts or also to implement dynamical learning schemes from true data. This eventually allows the strategy to adapt to the environment and to self-tune parts of its parameters.

Organization of the paper: we start proposing a tailored building model in Section II, and outlining a learning scheme to continuously and dynamically infer the statistics of the forecasts errors from real data in Section III. We then build

All the authors are with the ACCESS Linnaeus Center and the Automatic Control Lab, School of Electrical Engineering, KTH Royal Institute of Technology, Sweden. Emails: parisio@kth.se, marco.molinari@byv.kth.se, damiano@kth.se, kallej@kth.se.

This work is supported by the European Institute of Technology (EIT) Information and Communication Technology (ICT) Labs, the Swedish Energy Agency, the Swedish Governmental Agency for Innovation Systems (VINNOVA) and the Knut and Alice Wallenberg Foundation.

our SMPC controller on top of these results and illustrate it in Section IV. Section V eventually provides simulation results and comparisons with other MPC schemes. We collect some concluding remarks and draw plausible future extensions in Section VI.

II. Physical modeling

A. Room model

To decrease the computational burden, MPC controllers need sufficiently simple models. Similarly to previous works in the field, [15], [16], [17], [18], [19], we base our MPC scheme on a simplified general building physical model that can be used for whole building simulation both in cooling and heating conditions. The model has been developed in Matlab and then verified against the results provided by IDA-ICE [20], a commercial software program for energy and comfort calculations in buildings. The model used in this work is based on the following main assumptions:

- no infiltrations are considered, so that the inlet airflow in the zone equals the outlet airflow;
- the zone is well mixed;
- the thermal effects of the vapor production are neglected.

The room temperature is calculated via the following energy balance of the zone, modeled as a lumped node:

$$\begin{aligned} m_{\text{air,zone}} c_{\text{pa}} \frac{dT_{\text{room}}}{dt} &= Q_{\text{vent}} + Q_{\text{int}} + \textstyle\sum_{\text{j}} Q_{\text{wall,j}} \\ &\quad + \textstyle\sum_{\text{j}} Q_{\text{win,j}} + Q_{\text{heating}} \\ &\quad + Q_{\text{cooling}}. \end{aligned} \tag{1}$$

In (1) the left-hand term represents the heat stored in the room air. Q_{vent} is the heat flow due to ventilation. Q_{int} are the internal gains, sum of the heat flows due to occupancy, equipment and lighting. $Q_{\text{wall,j}}$ and $Q_{\text{win,j}}$ represent the heat flows exchanged between walls and room and windows and room respectively. Q_{cooling} and Q_{heating} are the heating and cooling flows necessary to keep the room environment within thermally comfortable conditions.

Equation (1) can be manipulated to yield the following explicit dependence between room temperature variation and heat flows:

$$\begin{aligned} \frac{dT_{\text{room}}}{dt} &= \frac{\dot{m}_{\text{vent}} \Delta T_{\text{vent}}}{m_{\text{air,zone}}} + \\ &\quad + \sum_{\text{j}} \frac{h_{\text{i}} A^{\text{j}}_{\text{wall}} \left(T^{\text{j}}_{\text{wall,i}} - T_{\text{room}}\right)}{m_{\text{air,zone}} c_{\text{pa}}} \\ &\quad + \sum_{\text{j}} \frac{\left(T_{\text{amb}} - T_{\text{room}}\right)}{R^{\text{j}}_{\text{win}} m_{\text{air,zone}} c_{\text{pa}}} + \frac{c N_{\text{people}}}{m_{\text{air,zone}} c_{\text{pa}}} \\ &\quad + \frac{\sum_{\text{j}} G^{\text{j}} A^{\text{j}}_{\text{win}} I^{\text{j}}}{m_{\text{air,zone}} c_{\text{pa}}} + \frac{A_{\text{rad}} h_{\text{rad}} \Delta T_{\text{h,rad}}}{m_{\text{air,zone}} c_{\text{pa}}} \end{aligned} \tag{2}$$

where

$$\begin{aligned} Q_{\text{vent}} &= \dot{m}_{\text{vent}} c_{\text{pa}} \Delta T_{\text{vent}} = \dot{m}_{\text{vent}} c_{\text{pa}} \left(T_{\text{air,sa}} - T_{\text{room}}\right), \\ Q_{\text{int}} &= c N_{\text{people}}, \\ Q_{\text{heating}} &= A_{\text{rad}} h_{\text{rad}} \Delta T_{\text{h,rad}} = A_{\text{rad}} h_{\text{rad}} \left(T_{\text{mr}} - T_{\text{room}}\right). \end{aligned}$$

The parameters involved in (2) are described in Table I, reported in appendix and presenting the parameters in alphabetical order for reading convenience.

The indoor wall temperature $T^{\text{j}}_{\text{wall,i}}$ in the j-th surface is calculated by means of the following energy balance on the wall outdoor (3) and indoor surface (4). Walls are modeled as two capacitance and three resistance (2C3R) systems [16], [17]. The three resistances $1/h_{\text{o}}$, $R^{\text{j}}_{\text{wall}}$ and $1/h_{\text{i}}$ are placed between the equivalent temperature T^{j}_{ee}, and the temperatures $T^{\text{j}}_{\text{wall,o}}$, $T^{\text{j}}_{\text{wall,i}}$ and T_{room}.

$$\frac{dT^{\text{j}}_{\text{wall,o}}}{dt} = \frac{\left[h_o A^{\text{j}}_{\text{wall}} \left(T^{\text{j}}_{\text{ee}} - T^{\text{j}}_{\text{wall,o}}\right) + \dfrac{\left(T^{\text{j}}_{\text{wall,i}} - T^{\text{j}}_{\text{wall,o}}\right)}{R^{\text{j}}_{\text{wall}}}\right]}{C^{\text{j}}/2} \tag{3}$$

$$\frac{dT^{\text{j}}_{\text{wall,i}}}{dt} = \frac{\left[h_{\text{i}} A^{\text{j}}_{\text{wall}} \left(T_{\text{room}} - T^{\text{j}}_{\text{wall,i}}\right) + \dfrac{\left(T^{\text{j}}_{\text{wall,o}} - T^{\text{j}}_{\text{wall,i}}\right)}{R^{\text{j}}_{\text{wall}}}\right]}{C^{\text{j}}/2} \tag{4}$$

In (3) and (4), $R^{\text{j}}_{\text{wall}}$ [°C/W] and C^{j} [J/°C] are the thermal resistance and the thermal capacity of the j-th wall respectively. The thermal capacity C^{j} is calculated after the model of Active Heat Capacity proposed by [21]. The equivalent external temperature T^{j}_{ee} accounts for the different radiation heat exchange due to the orientation of the external walls. The outdoor temperature is modified by the effects of radiation on the j-th wall as (adapted from [22])

$$T_{\text{ee,j}} = T_{\text{amb}} + \frac{a I^{\text{j}}}{\alpha_{\text{e}}}. \tag{5}$$

The air mass flow for ventilation $\dot{m}_{\text{vent}}$ in (2) is determined by the CO_2 concentration in the room, calculated after the model proposed in [23] as

$$V \frac{dC_{\text{CO}_2}}{dt} = \left(\dot{m}_{\text{vent}} C_{\text{CO}_2,\text{i}} + g_{\text{CO}_2} N_{\text{people}}\right) - \dot{m}_{\text{vent}} C_{\text{CO}_2}. \tag{6}$$

The Matlab model has been validated for the Stockholm climate against results from simulations carried out in IDA, using as climate data its internal database. The comparison has been performed under the same conditions of ventilation, solar radiation, internal gains and occupancy. In both cases, thermal bridges and infiltrations have been neglected. To clearly display the effects of the thermal behavior of the room model, no heating and cooling systems have been simulated. In Figure 1 the room temperature calculated with the Matlab model and IDA is displayed for two months and shows good accordance between the two models.

B. Control oriented model

Nonlinearities in the dynamic equations (2) and (6) can lead to intractable problems. To address this issue we derive linear equivalent formulations of the CO_2 concentration model (in Section II-C) and of the room thermal model (in Section II-D).

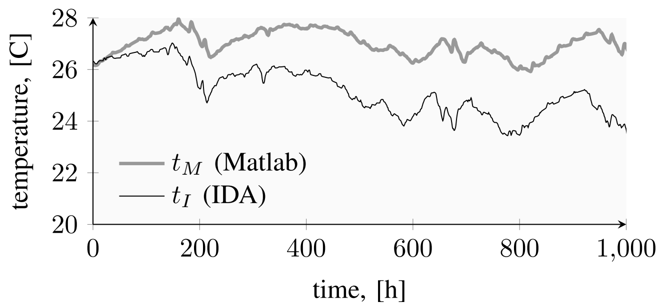

Fig. 1: Validation results. The thick gray / thin black lines represent the room temperatures calculated with respectively the Matlab model and IDA. The differences are induced mainly by the different modeling of the thermal capacities C^{j}, more accurate in IDA.

C. Linear formulation of the CO_2 concentration model

To linearize the CO_2 concentration dynamics (6) we replace the nonlinear term $\dot{m}_{\text{vent}} \cdot (C_{\text{CO}_2} - C_{\text{CO}_2,i})$ with u_{CO_2}, where $C_{\text{CO}_2,i}$ is a constant and $C_{\text{CO}_2} - C_{\text{CO}_2,i}$ is a nonnegative variable. The obtained linear continuous system is further discretized by the trapezoidal rule with a $\Delta T = 1$ hour sampling time.

To meet the physical bounds on the control input in the original nonlinear model, the following constraint on the input u_{CO_2} in the linear formulation must be satisfied at each time step k:

$$\begin{aligned} \dot{m}_{\text{vent}}^{\min}(k) \cdot \left(C_{\text{CO}_2}(k) - C_{\text{CO}_2,i}\right) \leq u_{CO_2}(k) \leq \\ \leq \dot{m}_{\text{vent}}^{\max} \cdot (C_{\text{CO}_2}(k) - C_{\text{CO}_2,i}). \end{aligned} \tag{7}$$

The original inputs can then be obtained as

$$\dot{m}_{\text{vent}}(k) = \frac{u}{(C_{\text{CO}_2}(k) - C_{\text{CO}_2,i})}.$$

Hence, the CO_2 concentration dynamics can be described by the discrete Linear Time Invariant (LTI) system

$$\begin{aligned} x_{CO_2}(k+1) &= a x_{CO_2}(k) + b u_{CO_2}(k) + e w_{CO_2}(k) \\ y_{CO_2}(k) &= x_{CO_2}(k), \end{aligned} \tag{8}$$

where $x_{CO_2}(k) = C_{\text{CO}_2}$ is the state and $w_{CO_2}(k) = N_{\text{people}}(k)$ is the disturbance at time step k, and a, b, e are appropriate scalars. Constraints (7) can then be rewritten as

$$\begin{aligned} g_{u,CO_2} u(k) + g_{x,CO_2} x(k) \leq g_{CO_2} \\ y_{CO_2}(k) \leq y_{CO_2}^{\max}, \end{aligned} \tag{9}$$

where the matrices g_{u,CO_2}, g_{x,CO_2} are derived from (7) and $y_{CO_2}^{\min}$ is upper bound on the CO_2 concentration.

D. Linear formulation of the room thermal model

Consider the room thermal model presented in Section II-A. The heat flow due to ventilation can be expressed as

$$\dot{m}_{\text{vent}} c_{\text{pa}} \Delta T_{\text{vent}} = \dot{m}_{\text{vent}} c_{\text{pa}} \big(\Delta T_h - \Delta T_c\big) = c_{\text{pa}}(u_h - u_c),$$

where the nonnegative variables ΔT_h and ΔT_c represent the temperature difference through the heating and cooling coils respectively. The obtained linear continuous system is then discretized by the trapezoidal rule with a $\Delta T = 1$ hour sampling time. Hence the inputs $u_h(k)$ and $u_c(k)$, multiplied by c_{pa}, model the portion of the ventilation heat flow due to heating and cooling respectively.

Hence, the room temperature dynamics can be described by the Linear Time Invariant (LTI) system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ew(k) \\ y(k) &= Cx(k), \end{aligned} \tag{10}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector containing the room temperature and the inner and outer temperatures of all the walls, $u(k) := \big(u_h(k), u_c(k), \Delta T_{h,\text{rad}}(k)\big) \in \mathbb{R}^{n_u}$ is the input vector, and $w(k) := \big(T_{\text{amb}}(k), I^1(k), \ldots, I^{n_{\text{wall}}}(k), N_{\text{people}}(k)\big) \in \mathbb{R}^{n_w}$ is the vector of random disturbances at time k, and the matrices A, B, E, C are of appropriate sizes. The output $y(k)$ is the room temperature at time k.

III. Management of the weather and room occupancy forecasts

The proposed MPC scheme uses statistics of the forecasts errors by means of so-called *scenarios*, i.e., independent extractions of the errors from their distribution. Thus the algorithm implicitly requires the knowledge of the joint distribution of these forecasts errors. Unfortunately, the forecasters generally exploited to predict the external temperature, the solar radiation and the room occupancy do not provide the users with the distributions of their errors.

We thus here propose the possibility of learning the statistics of the forecasts by means of *copulas*, i.e., opportune probabilistic description of random vectors, applied on real data. Here we describe the basics of this technology, aiming to allow the reader to implement our schemes.

Section III-A describes formally the concept of copulas. Section III-B then recalls how it is possible to estimate them from real data. Section III-C eventually describes how to generate the i.i.d. scenarios needed in our MPC schemes.

A. Copulas

Formally, copulas are particular probabilistic descriptions of random vectors. Here the marginal distributions of the components of the vectors and their joint moments are modeled independently. The relative theory is based on Sklar's representation theorem [24], that ensures that the Cumulative Distribution Function (CDF) of any T-uple of continuous r.v.'s $w(1), \ldots, w(T)$ can be written in terms of the marginal distributions $\mathbb{P}\left[w(1) \leq a_1\right], \ldots, \mathbb{P}\left[w(T) \leq a_T\right]$ and an opportune copula (i.e., a function $\mathbb{C} : [0,1]^T \mapsto [0,1]$) as

$$\begin{aligned} &\mathbb{P}\left[w(1) \leq a_1, \ldots, w(T) \leq a_T\right] = \\ &\quad = \mathbb{C}\Big(\mathbb{P}\left[w(1) \leq a_1\right], \ldots, \mathbb{P}\left[w(T) \leq a_T\right]\Big). \end{aligned} \tag{11}$$

Assume then the marginals $\mathbb{P}\left[w(t) \leq a_t\right]$ to be continuous. Then to reconstruct $\mathbb{P}\left[w(1) \leq a_1, \ldots, w(T) \leq a_T\right]$ it is sufficient to independently reconstruct the marginals of the

$w(t)$'s and the function $\mathbb{C}(\cdot)$. Let in fact $\mathbb{Q}_t(b_t)$ denote the quantile function of $w(t)$, i.e.,

$$\mathbb{Q}_t(b_t) := \inf_{a_t} \{a_t \mid \mathbb{P}[w(t) \leq a_t] \geq b_t\}. \tag{12}$$

Then it follows immediately from (11) that

$$\mathbb{C}\Big(b_1, \ldots, b_T\Big) = \mathbb{P}[w(1) \leq \mathbb{Q}_1(b_1), \ldots, w(T) \leq \mathbb{Q}_T(b_T)].$$

B. Estimation of copulas from real data

We now show how to learn $\mathbb{C}(\cdot)$ in (11) from real data using empirical methods[1]. Notice that we treat temperature, solar radiation and occupancy as independent processes. Thus each of these signals has its own $\mathbb{C}(\cdot)$, decoupled and learned independently of the other ones.

Let then the generic temperature / solar radiation / occupancy process be indicated with $w(k)$, where k is a discrete time index. Let its t-steps ahead predictor be $\widehat{w}(k+t|k)$, and the corresponding forecasting errors be $e(k+t|k) := w(k+t) - \widehat{w}(k+t|k)$. We assume then the errors $e(k+1|k), \ldots, e(k+T|k)$ to be independent of $w(0), \ldots, w(k)$, i.e.,

$$\begin{aligned} &\mathrm{p}\left(e(k+1|k), \ldots, e(k+T|k) \mid w(0), \ldots, w(k)\right) = \\ &\quad = \mathrm{p}\left(e(k+1|k), \ldots, e(k+T|k)\right). \end{aligned} \tag{13}$$

Moreover each $e(k+t|k)$ is a stationary ergodic random process in k.

These assumptions are simplificative but fundamental for our learning purposes[2]. Assume in fact to own a database $\mathcal{D}_t$ containing some $e(k+t|k)$'s for several k's and t's. Let for simplicity $\mathcal{D}_t = \{e(1+t|1), \ldots, e(K+t|K)\}$. Thanks to the previous assumptions, the marginal distributions of all the $e(k+t|k)$'s are all equal for different k's (not t's), i.e., $\mathbb{P}[e(1+t|1) \leq a] = \mathbb{P}[e(2+t|2) \leq a] = \ldots$ for all a's. We can thus approximate the marginals $\mathrm{p}(e(k+t|k))$ with the empirical marginals

$$\widehat{\mathbb{P}}[e(\kappa+t|\kappa) \leq a] := \frac{1}{K}\sum_{k=1}^{K} \mathbb{1}\{e(k+t|k) \leq a\} \tag{14}$$

where $\mathbb{1}\{\cdot\}$ is the indicator function and $e(\kappa+t|\kappa)$ is a r.v. (and not an element of $\mathcal{D}_t$).

Denoting the empirical marginals $\widehat{\mathbb{P}}[e(\kappa+t|\kappa) \leq a]$ with $\widehat{\mathbb{P}}_t(a)$, we can express the empirical copula $\widehat{\mathbb{C}}(\cdot)$ as

$$\begin{aligned} &\widehat{\mathbb{C}}(b_1, \ldots, b_T) \\ &\quad := \frac{1}{K}\sum_{k=1}^{K} \mathbb{1}\Big\{\widehat{\mathbb{P}}_1\big(e(k+1|k)\big) \leq b_1, \ldots, \\ &\qquad\qquad \ldots, \widehat{\mathbb{P}}_T\big(e(k+T|k)\big) \leq b_T\Big\}. \end{aligned} \tag{15}$$

[1] In this manuscript we focus on constructing empirical copulas rather than fitting datasets to existing types of copula. The latter approach in fact needs tailored analyses, far beyond the scope of this article.

[2] We notice that actually the solar radiation and room occupancy processes are highly heteroskedastic. E.g., usually there is neither sun nor people in the testbed at midnight. Here we addressed this issue by clustering the data in time zones, e.g., morning, afternoon, night, and by assuming (13) in each cluster. A more detailed analysis of this strategy is in our future works.

C. Generation of scenarios from copulas

We now show how to generate the scenarios exploited in the next Section IV. The algorithm for the generation of N_s scenarios can be summarized as

1) consider a point forecast $[\widehat{w}(k+1|k), \ldots, \widehat{w}(k+T|k)]^T$, provided by the temperature / solar radiation / occupancy forecasting algorithm;
2) consider $\widehat{\mathbb{C}}(\cdot)$ and $\widehat{\mathbb{P}}_t(\cdot)$, computed applying (15) and (14) on a database that does not contain the current point forecast $[\widehat{w}(k+1|k), \ldots, \widehat{w}(k+T|k)]^T$ (this implicitly states that $\widehat{\mathbb{C}}(\cdot)$ and $\widehat{\mathbb{P}}_t(\cdot)$ have been computed before generating the current scenarios);
3) generate N_s i.i.d. T-dimensional vectors $\left[b_{1,i}, \ldots, b_{T,i}\right]^T$, $i = 1, \ldots, N_s$ from $\widehat{\mathbb{C}}(\cdot)$;
4) transform these vectors by means of the marginals $\widehat{\mathbb{P}}_t(\cdot)$, and obtain the N_s i.i.d. T-dimensional vectors

$$\begin{bmatrix} e_{1,i} \\ \vdots \\ e_{T,i} \end{bmatrix} = \begin{bmatrix} \widehat{\mathbb{Q}}_1\big(b_{1,i}\big) \\ \vdots \\ \widehat{\mathbb{Q}}_T\big(b_{T,i}\big) \end{bmatrix}, \qquad i = 1, \ldots, N_s \tag{16}$$

   where $\widehat{\mathbb{Q}}_t(\cdot)$ is the empirical quantile function, i.e., the quantile function corresponding to $\widehat{\mathbb{P}}_t(\cdot)$ computed as in (12);
5) obtain the N_s scenarios by summing the $[e_{1,i}, \ldots, e_{T,i}]^T$'s to the point forecast $[\widehat{w}(k+1|k), \ldots, \widehat{w}(k+T|k)]^T$.

IV. Control Problem Formulation

We now present the main features of our SMPC approach, which aims at increasing energy efficiency in buildings. The strategy is formalized precisely in Sections IV-A and IV-B.

• The inputs of the control scheme are, at every time step, weather conditions and occupancy scenarios, and measurements of the current state of the system. The output is instead a heating, cooling and ventilation plan for the next N hours, where N is the prediction horizon. Notice that only the first step of this control plan is applied to the HVAC system. After that, the whole procedure is repeated. This introduces feedback into the system, since the optimal control problem is a function of the current state and of any disturbance acting on the building at the current time step.

• Building climate control leads *naturally* to probabilistic constraints, commonly called *chance constraints*. Consider also that current standards, e.g., [25], explicitly state that rooms temperatures should be kept within a comfort range *with a predefined probability*. To have a tractable SMPC problem, here the probabilistic constraints will be translated into a series of deterministic constraints.

• The control strategy decouples the control of the temperature and of the air quality in two separated subproblems. This is possible because the dynamics of the air quality are independent of the ones of the room temperature[3]. Formally thus we have 2 controllers in cascade: (*i*) the first SMPC aims

[3] Incidentally, we also notice that the controllers must satisfy above all the air quality requirements.

at satisfying the required air quality at a minimum energy usage, (*ii*) the second SMPC controls the indoor temperature control.

A. SMPC for Room Temperature Control

1) Constraints: let x_0 denote the current state. It follows from the linear model (10), that the room temperature dynamics over the prediction horizon N can be written as

$$x(k) = A^k x_0 + \sum_{i=0}^{k-1} A^{k-i-1} B u^l(i) + \sum_{i=0}^{k-1} A^{k-i-1} E w(i).$$

Then let

$$\begin{aligned}
\boldsymbol{Y} &:= \left[y_0^{\mathrm{T}}, \dots, y_{N-1}^{\mathrm{T}}\right]^{\mathrm{T}}, \qquad \boldsymbol{Y} \in \mathbb{R}^{n_y N} \\
\boldsymbol{U} &:= \left[u_0^{\mathrm{T}}, \dots, u_{N-1}^{\mathrm{T}}\right]^{\mathrm{T}}, \qquad \boldsymbol{U} \in \mathbb{R}^{n_u N} \\
\boldsymbol{W} &:= \left[w_0^{\mathrm{T}}, \dots, w_{N-1}^{\mathrm{T}}\right]^{\mathrm{T}}, \qquad \boldsymbol{W} \in \mathbb{R}^{n_w N} \\
\boldsymbol{A} &:= \left[(A)^{\mathrm{T}} \ \dots \ (A^N)^{\mathrm{T}}\right]^{\mathrm{T}} \\
\boldsymbol{B} &:= \begin{bmatrix} B & & \mathbf{0} \\ \vdots & \ddots & \\ A^{N-1}B & \dots & B \end{bmatrix} \\
\boldsymbol{E} &:= \begin{bmatrix} E & & \mathbf{0} \\ \vdots & \ddots & \\ A^{N-1}E & \dots & E \end{bmatrix} \\
\boldsymbol{C} &:= \mathrm{diag}(C, \dots, C) \\
\boldsymbol{G}_x &:= \left[\boldsymbol{CA}\right] \\
\boldsymbol{G}_u &:= \left[\boldsymbol{CB}\right] \\
\boldsymbol{G}_w &:= \left[\boldsymbol{CE}\right] \\
\widetilde{\boldsymbol{g}} &:= \left[-y_{\min}(k)^{\mathrm{T}} \cdots - y_{\min}(k)^{\mathrm{T}} \right. \\
&\qquad\qquad\qquad \left. y_{\max}(k)^{\mathrm{T}} \cdots y_{\max}(k)^{\mathrm{T}}\right]^{\mathrm{T}} \\
\boldsymbol{g} &:= \widetilde{\boldsymbol{g}} - \boldsymbol{G}_x x_0 \\
\boldsymbol{F} &:= \begin{bmatrix} -\boldsymbol{I}_{Nn_u} \\ \boldsymbol{I}_{Nn_u} \end{bmatrix} \\
\boldsymbol{f} &:= \left[-u_{\min}^{\mathrm{T}} \cdots - u_{\min}^{\mathrm{T}} \ u_{\max}^{\mathrm{T}} \cdots u_{\max}^{\mathrm{T}}\right]^{\mathrm{T}}
\end{aligned}$$

where $\mathbf{0}$ is a zero matrix with appropriate dimensions and $\boldsymbol{I}_{Nn_u} \in \mathbb{R}^{Nn_u \times Nn_u}$ is the identity matrix. Hence we can express the output $\boldsymbol{Y}$ over the whole prediction horizon, given the initial state x_0, as

$$\boldsymbol{Y} = \boldsymbol{C}(\boldsymbol{A}x_0 + \boldsymbol{BU} + \boldsymbol{EW}) \tag{17}$$

and the constraints on the output and the inputs over the whole prediction horizon N as

$$\begin{aligned} \boldsymbol{G}_u\boldsymbol{U} + \boldsymbol{G}_w\boldsymbol{W} &\le \boldsymbol{g} \\ \boldsymbol{FU} &\le \boldsymbol{f}. \end{aligned}$$

Notice that we consider time varying bounds on the room temperature, $y_{\min}(k)$ and $y_{\max}(k)$, which account for the occupancy.

2) Problem Formulation: the SMPC room temperature control problem can be formulated as to minimize the energy use while maintaining the predefined indoor comfort conditions. We remark that we neglect the electricity consumed by fans and circulation pumps, and account for the air reheating energy by means of an opportune penalty factor. This is translated into the following stochastic problem with joint chance constraints:

Problem 1 (SMPC for Temperature Control)

$$\begin{aligned} \min_{\boldsymbol{U}} \quad & \boldsymbol{EP}_{\text{room}}^{\mathrm{T}}\boldsymbol{U} \\ \text{s.t.} \quad & \mathbb{P}\left[\boldsymbol{G}_u\boldsymbol{U} + \boldsymbol{G}_w\boldsymbol{W} - \boldsymbol{g} \le 0\right] \ge 1 - \alpha \\ & \boldsymbol{FU} \le \boldsymbol{f} \end{aligned} \tag{18}$$

where $1-\alpha$ is the predefined probability level for constraint satisfaction and $\boldsymbol{EP}_{\text{room}}^{\mathrm{T}}\boldsymbol{U}$ is the energy use vector over the whole prediction horizon, $\boldsymbol{EP}_{\text{room}} \in \mathbb{R}^{n_u N}$ containing the specific heat of the dry air, c_{pa}, and the product $A_{\text{rad}}h_{\text{rad}}$ between the emission area and the heat transfer coefficient of the radiators.

Problem 1 has to be solved at each time step k. Moreover the initial state x_0 is updated at every step using current measurements from the field.

Probabilistic constraints require multi-dimensional integrations and generally induce non-convex feasibility regions. Chance constrained problems are thus generally intractable, especially if joint chance constraints are included. A general way to build computationally tractable approximations of these problems is the scenario-based approximation approach, where the *scenarios* are i.i.d. samples of the random variables. Nevertheless, this approximation is not necessarily conservative, meaning that a feasible solution of the approximation problem might be non feasible for the original one [26]. Hence, computing reliable solutions using scenario-based approximation approaches requires a large number of samples. This can eventually lead to computationally intractable problems.

A possible solution to address these difficulties is to formulate conservative, computationally tractable and convex approximations of the original problem [26]. Here we follow this scheme and apply the Conditional Value at Risk (CVaR) approach, one of the most widely used strategies. Hence, we approximate the joint chance constraint in Problem 1 as

$$\begin{aligned} \mathcal{E}(\alpha, \tau) &:= \mathrm{E}\left[\tau + \alpha^{-1}\left[\boldsymbol{G}_u\boldsymbol{U} + \boldsymbol{G}_w\boldsymbol{W} - \boldsymbol{g} - \tau\right]_+\right] \\ \text{CVaR}(\alpha) &:= \min_{\tau}\left(\mathcal{E}(\alpha, \tau) \le 0\right), \end{aligned} \tag{19}$$

where $\tau \in \mathbb{R}$ and $[a]_+ := \max\{0, a\}$.

The expected value constrained stochastic problems can be solved by resorting to a sample approximation problem. This means that the expectation in (19) is replaced with the empirical expectation obtained from random i.i.d. samples. Thus, assuming that N_s i.i.d. samples $\boldsymbol{W}^1, \dots, \boldsymbol{W}^{N_s}$ are provided, the non-convex Problem 1 can be approximated with the following deterministic linear problem [27]:

Problem 2 (CVaR SMPC for Temperature Control)

$$\begin{aligned}
\min_{\boldsymbol{U},\tau} \quad & \boldsymbol{E}\boldsymbol{P}_{\text{room}}^{\mathrm{T}}\boldsymbol{U} \\
\text{s.t.} \quad & \boldsymbol{F}\boldsymbol{U} \leq \boldsymbol{f} \\
& \tau + \alpha^{-1}\textstyle\sum_{i=1}^{N_s} N_s^{-1} z_i \leq 0 \\
& \boldsymbol{G}_u^j\boldsymbol{U} + \boldsymbol{G}_w^j\boldsymbol{W}_i - \boldsymbol{g}^j - \tau - y_i^j \leq 0 \\
& z_i \geq y_i^j \qquad y_i^j \geq 0 \qquad z_i \geq 0
\end{aligned} \tag{20}$$

where $i = 1, \ldots, N_s$ is the scenario index and $\boldsymbol{G}_u^j, \boldsymbol{G}_w^j, \boldsymbol{g}^j$ indicate the j^{th} row of the corresponding matrices.

We notice that exponential convergence results for the sample approximation methods of expected value constrained stochastic programs are available [28], [29].

B. SMPC for Air Quality Control

Analogously to Section IV-A, we express the CO_2 concentration dynamics over the whole prediction horizon as

$$\boldsymbol{Y}_{CO_2} = \boldsymbol{X}_{CO_2} = \boldsymbol{A}_{CO_2}x_{0,CO_2} + \boldsymbol{B}_{CO_2}\boldsymbol{U}_{CO_2} + \boldsymbol{E}_{CO_2}\boldsymbol{W}_{CO_2}.$$

and the constraints (9) over the whole prediction horizon N as

$$\boldsymbol{G}_{u,CO_2}\boldsymbol{U}_{CO_2} + \boldsymbol{G}_{w,CO_2}\boldsymbol{W}_{CO_2} \leq \boldsymbol{g}_{CO_2}. \tag{21}$$

where $\boldsymbol{g}_{CO_2}$ contains the upper bound on the CO_2 concentration. The SMPC problem for air quality control can then be initially formulated as

Problem 3 (SMPC for Air Quality Control)

$$\begin{aligned}
\min_{\boldsymbol{U}_{CO_2}} \quad & \|\boldsymbol{U}_{CO_2}\|_1 \\
\text{s.t.} \quad & \mathbb{P}\left[\boldsymbol{G}_{u,CO_2}\boldsymbol{U}_{CO_2} + \boldsymbol{G}_{w,CO_2}\boldsymbol{W}_{CO_2} \leq \boldsymbol{g}_{CO_2}\ \right] \geq \\
& \geq 1 - \alpha_{\text{vent}}
\end{aligned}$$

where $1 - \alpha_{\text{vent}}$ is the probability level.

Then Problem 3 can be cast as a deterministic problem by resorting to its scenario-based approximation:

Problem 4 (Scenario-based SMPC for A. Q. Control)

$$\begin{aligned}
\min_{\boldsymbol{U}_{CO_2}} \quad & \|\boldsymbol{U}_{CO_2}\|_1 \\
\text{s.t.} \quad & \boldsymbol{G}_{u,CO_2}\boldsymbol{U}_{CO_2} + \boldsymbol{G}_{w,CO_2}\boldsymbol{W}_{CO_2}^i \leq \boldsymbol{g}_{CO_2}
\end{aligned}$$

where $i = 1, \ldots, N_{\text{vent}}$ is the scenario index.

Remarkably, from α_{vent} it is possible to compute a N_{vent} that ensures (with high probability) the solution of the approximation problem to be feasible also for the original one [26]. Importantly, even if the so-computed N_{vent} is high, our air quality control problem remains computationally tractable.

We point out that the optimal control sequence

$$\boldsymbol{U}_{CO_2} = [\dot{m}_{vent}(0), \ldots, \dot{m}_{\text{vent}}(N-1)]^{\mathrm{T}},$$

computed in Problem 4, provides the lower bound on the air flow rate in Problems 1 and 2. Hence, the mass air flow rate and the supply air temperature at each k are easily computed from the obtained values of either $u_h(k)$ or $u_c(k)$ considering both the requirements on the air quality and the comfort requirements on the supply air temperature.

V. Simulation Results

We consider a laboratory room in a university building, used intermittently for lecturing and experiments. The room, pictured in Figure (2), has 9.4m $\times$ 9m footprint dimensions and south-east external aerated concrete walls (0.4m thick) while all the other walls are internal. The south-east external facade comprises 4 windows, totaling approximately 2.6 m^2 of glazed surface. The zone is heated by waterborne radiators and cooled via ventilation air. The balanced ventilation system is equipped with a rotary heat exchanger for heat recovery. The ventilation air temperature is controlled by cooling and heating coils.

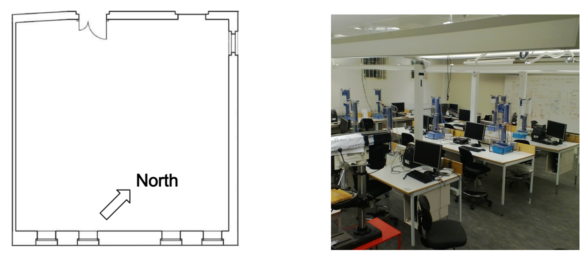

Fig. 2: Sketch and picture of the room considered in our simulations.

The copulas modeling the uncertainties of the solar radiation and outside temperature forecasts are based on the data collected from the NDFD database NDFD database, `http://www.nws.noaa.gov/ndfd/`. The same quantities, related to occupancy measurements, have instead been obtained from vision-based people counting devices mounted in our testbed[4].

We thus implement, in Matlab and CPLEX [30] on an Intel Core 2 Duo CPU 2 GHz, the three following MPC strategies:

Performance Bound (PB) MPC: ideal case endowed with error-free forecasts, used as a theoretical benchmark;

Certainty Equivalence (CE) MPC: common practice, it simply neglects the uncertainties in the forecasts;

Stochastic Model Predictive Control (SMPC): the MPC described in Problems 2 and 4 with inputs the copula-based scenarios.

In Problem 4 we set the confidence that the computed solution will be feasible to 0.99 and the constraint satisfaction level $1 - \alpha_{\text{vent}}$ to 0.91, leading to a number of required scenarios of $N_{\text{vent}} = 1223$. In Problem 2 we instead test various sample sizes from 30 to 120 and various constraint

[4] Scripts for downloading and processing the NDFD database can be found at `http://hvac.ee.kth.se/`.

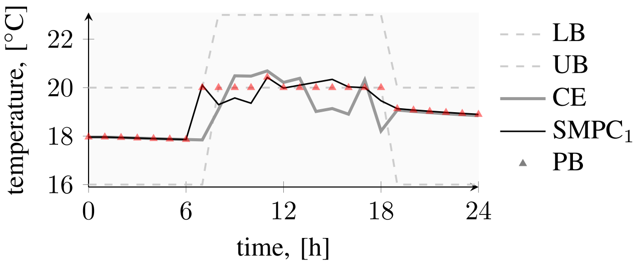

Fig. 3: Comparison of the room temperature obtained with different control strategies.

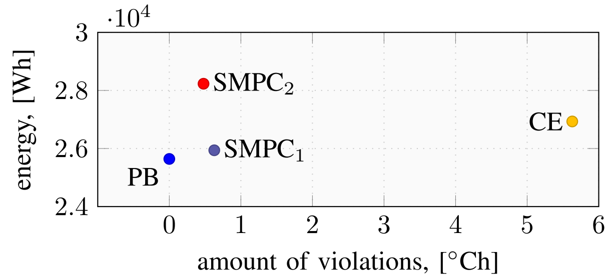

Fig. 4: Assessment of the performance of the controllers.

satisfaction levels from 90% to 95%. For sake of brevity we will show just the results for the representative cases:

- SMPC_1, with a constraint satisfaction level of 91% and 60 uncertainty scenarios;
- SMPC_2, with a constraint satisfaction level of 94% and 120 uncertainty scenarios.

A. Assessment Procedure

Wrong predictions can lead to constraints violations. Therefore, control performance is assessed in terms of both energy usage and constraint violation.

Figure 3 depicts the resulting room temperature profile through the whole day obtained using SMPC_1, CE MPC and PB MPC. It can be seen that our SMPC has a smaller amount of thermal comfort violations. This also indicates that the energy use can be still reduced with respect to the deterministic CE MPC controller. Further, notice that, in this simulation experiment, the resulting room temperature is significantly close to the theoretical benchmark.

Figure 4 shows the energy use versus the amount of violations for the two simulation cases for all the MPC controllers. The SMPC can be tuned by varying the parameter α, which describes the probability level of constraint violation. Further, increasing the number of scenarios yields more accurate results at the cost of a higher computational burden. Then, by changing both α and the number of scenarios, the SMPC can trade off energy use vs. probability of constraint violations, and solution reliability vs. computational effort. Using a higher constraint satisfaction probability, as in SMPC_2, provides less violations at the cost of a significant increase of the energy use. Thus, increasing the number of scenarios does not lead to meaningful improvements in the quality of the solution. We conclude that selecting a constraint satisfaction level of 91% and 60 scenarios can be enough for the SMPC to perform better than the deterministic controller and to be close to the benchmark. Simulation analysis can help finding a suitable controller in terms of energy use and occupant comfort while being sufficiently computationally tractable.

We point out that the CO_2 concentration is always kept within the comfort range (below 850 ppm).

We eventually notice that solving our optimization routines required on average ~ 18 seconds per iteration, making the problems affordable even with higher number of scenarios.

VI. Conclusions and Future Studies

To improve the building thermal control performance of Certainty Equivalence (CE) MPC schemes we proposed a Stochastic Model Predictive Control (SMPC) that accounts for the distribution of the weather and occupancy forecasts errors. This is performed by means of independent scenarios extracted from the copulas of the forecasts errors, i.e., opportune representations of their joint distributions. Importantly, these copulas can be learned in a on-line and continuous fashion, leading to a dynamically self-calibrating strategy.

Numerical experiments indicate that the resulting SMPC strategy leads to lower energy use than the CE scheme. The offered controller moreover performs closely to the Performance Bound (PB) MPC, a theoretically optimal scheme that exploits perfect knowledge of the future. The experiments, based on a room model that involves active heat capacities and that has been proved providing an accurate description of the behavior of buildings, indicate that it is eventually possible to figuratively convert information into energy savings at the cost of a more complex – but still feasible and solvable with normal hardware – problem.

This work thus motivates us to implement the proposed strategy on real testbeds, and evaluate performance improvements w.r.t. the current practice.

The manuscript suggests also that it is crucial to have an accurate knowledge of the statistics of the errors. Namely, poor descriptions of the uncertainties of the forecasts could lead the SMPC to perform worse than classic CE MPC schemes, or even worse than current practices. It is then necessary to understand which degree of knowledge eventually ensures a performance gain.

Another important question is whether the model and controller can be used for large systems, e.g., entire buildings or buildings communities, and still preserve their feasibility, implementability, and favorable performance w.r.t. CE MPC schemes.

Further, we are currently studying the persistent feasibility of our MPC scheme. It is well known that stability and feasibility are not easily ensured in MPC control schemes. To guarantee stability, the cost function is generally formulated as a Lyapunov function for the closed loop system. In practice, this requirement is relaxed for stable systems with slow dynamics, such as buildings. The main challenge here is how to guarantee the persistent feasibility of the proposed

MPC scheme, which is generally guaranteed by augmenting the system with a suitable terminal constraint set.

REFERENCES

[1] "Debate Europe-building on the experience of plan D for democracy, dialogue and debate," European Economic and Social Committee and the Committee of the Regions, COM 158/4, Brussels, 2008.

[2] J. Maciejowski, *Predictive control with constraints*. Prentice Hall, 2002.

[3] M. Morari, J. Lee, and C. Garcia, *Model Predictive Control*. Prentice Hall, 2001.

[4] V. Erickson, Y. Lin, A. Kamthe, R. Brahme, A. Surana, A. Cerpa, M. Sohn, and S.Narayanan, "Energy efficient building environment control strategies using real-time occupancy measurements," in *BuildSys2009*, November 2009, pp. 19–24.

[5] Y. Ma, F. Borrelli, B. Hencey, A. Packard, and S. Bortoff, "Model predictive control of thermal energy storage in building cooling systems," in *48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 2009.

[6] T. Salsbury, P. Mhaskar, and S. Qin, "Predictive control methods to improve energy efficiency and reduce demand in buildings," *Computers & Chemical Engineering*, vol. 51, pp. 77–85, August 2012.

[7] T. Nguyen and M. Aiello, "Energy intelligent buildings based on user activity: A survey," *Energy and Buildings*, no. 56, pp. 244–257, January 2013.

[8] S. Goyal, H. Ingley, and P. Barooah, "Zone-level control algorithms based on occupancy information for energy efficient buildings," in *American Control Conference*, June 2012, pp. 3063–3068.

[9] R. V. Andersen, B. Olesen, and J. Toftum, "Simulation of the effects of occupant behaviour on indoor climate and energy consumption," in *Proceedings of Clima 2007 WellBeing Indoors*, International Centre for Indoor Environment and Energy, Department of Mechanical Engineering, Technical University of Denmark, 2007.

[10] D. Sturzenegger, D. Gyalistras, M. Gwerder, C. Sagerschnig, M. Morari, and R. S. Smith, "Model Predictive Control of a Swiss office building," in *Clima-RHEVA World Congress*, June 2013.

[11] J.Široký, F. Oldewurtel, J. Ciglerc, and S. Prívarac, "Experimental analysis of model predictive control for an energy efficient building heating system," *Applied Energy*, vol. 88, no. 9, pp. 3079–3087, 2011.

[12] A. E. D. Mady, G. Provan, C. Ryan, and K. Brown, "Stochastic model predictive controller for the integration of building use and temperature regulation," in *Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence*, August 2011.

[13] Y. Ma and F. Borrelli, "Fast stochastic predictive control for building temperature regulation," in *American Control Conference*, June 2012, pp. 3075–3080.

[14] F. Oldewurtel, A. Parisio, C. Jones, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari, "Use of model predictive control and weather forecasts for energy efficient building climate control," *Energy and Buildings*, no. 45, pp. 15–27, February 2012.

[15] I. Hazyuk, C. Ghiaus, and D. Penhouet, "Optimal temperature control of intermittently heated buildings using model predictive control: Part i - building modeling," *Building and Environment*, vol. 51, pp. 379–387, 2012.

[16] B. Dong, K. P. Lam, and C. Neuman, "Integrated building control based on occupant behavior pattern detection and local weather forecasting," in *Building Simulation*, 2011, pp. 193–200.

[17] Z. O'Neill, S. Narayanan, and R. Brahme, "Model-based thermal load estimation in buildings," in *SimBuild*, 2010, pp. 474–481.

[18] Z. Liao and A. Dexter, "A simplified physical model for estimating the average air temperature in multi-zone heating systems," *Building and Environment*, vol. 39, no. 9, pp. 1013–1022, 2004.

[19] Y. Ma, A. Kelman, A. Daly, and F. Borrelli, "Predictive control for energy efficient buildings with thermal storage: Modeling, stimulation, and experiments," *Control Systems, IEEE*, vol. 32, no. 1, pp. 44–64, feb. 2012.

[20] "Equa Simulation AB, IDA ICE," www.equa-solutions.co.uk, February 2013.

[21] G. A. Johannesson, "Active Heat Capacity – Models and parameters for the thermal performance of buildings," Ph.D. dissertation, Lund Technical University, 1981.

[22] L. E. Nevander and B. Elmarsson, *Fukthandbok*. Svensk Byggtjänst, 1994.

[23] H. A. Aglan, "Predictive model for CO_2 generation and decay in building envelopes," *Journal of Applied Physics*, vol. 93, no. 2, pp. 1287–1290, 2003.

[24] A. Sklar, "Fonctions de répartition à n dimensions et leurs marges," *Publications de l'Institut de Statistique de L'Université de Paris*, vol. 8, pp. 229–231, 1959.

[25] BSI, "En 15251:2007: Indoor environmental input parameters for design and assessment of energy performance of buildings addressing indoor air quality, thermal environment, lighting and acoustics," British Standards Institute, Tech. Rep., 2008.

[26] A. Nemirovski, "On safe tractable approximations of chance constraints," *European Journal of Operational Research*, vol. 219, no. 3, pp. 707–718, 2012.

[27] P. Kall and J. Mayer, *Stochastic Linear Programming: Models, Theory, and Computation*. Springer-Verlag, 2005.

[28] H. Sun, H. Xu, and Y. Wang, "Asymptotic analysis of sample average approximation for stochastic optimization problems with joint chance constraints via conditional value at risk and difference of convex functions," *Journal of Optimization Theory and Applications*, pp. 1–28, 2012.

[29] A. Nemirovski and A. Shapiro, "Convex approximations of chance constrained programs," *SIAM Journal on Optimization*, vol. 17, no. 4, pp. 969–996, 2007.

[30] ILOG, *CPLEX 12.0 Users Manual*, 2010.

APPENDIX

α_{e}	$[\text{W/m}^2 {}^\circ\text{C}]$	external heat transfer coefficient
a	$[-]$	absorption factor for shortwave radiation
A_{rad}	$[\text{m}^2]$	emission area of the radiators
$A^{\text{j}}_{\text{wall}}$	$[\text{m}^2]$	wall area on the j-th surface
$A^{\text{j}}_{\text{win}}$	$[\text{m}^2]$	area of the window on the j-th surface
c	$[W]$	constant related to equipment and occupants activity
$C_{CO_2,\text{i}}$	[ppmV]	inlet air CO_2concentration, assumed equal to outdoor CO_2concentration
C_{CO_2}	[ppmV]	concentration of CO_2 within the room
c_{pa}	$[\text{J/kg}^\circ\text{C}]$	specific heat of the dry air
g_{CO_2}	$[\text{m}^3_{\text{CO}_2}/\text{pers.}]$	generation rate of CO_2 per person
G^{j}	$[-]$	G-value (SHGC) of the window on the j-th surface
h_{i}	$[\text{W/m}^2 {}^\circ\text{C}]$	indoor heat transfer coefficient
h_{o}	$[\text{W/m}^2 {}^\circ\text{C}]$	outdoor heat transfer coefficient
h_{rad}	$[\text{W/m}^2 {}^\circ\text{C}]$	heat transfer coefficient of the radiators
I^{j}	$[\text{W/m}^2]$	solar radiation on the j-th surface
$m_{\text{air,zone}}$	[kg]	air mass in the room
$\dot{m}_{\text{vent}}$	[kg/s]	ventilation mass flow
N_{people}	$[-]$	number of occupants in the room
N_s	$[-]$	number of scenarios for the temperature problem
N_{vent}	$[-]$	number of scenarios for the ventilation problem
$R^{\text{j}}_{\text{win}}$	$[^\circ\text{C/W}]$	thermal resistance of the window on the j-th surface
$T_{\text{air,sa}}$	$[^\circ\text{C}]$	supply air temperature
T_{amb}	$[^\circ\text{C}]$	outdoor temperature
T^{j}_{i}	$[^\circ\text{C}]$	indoor surface temperature of the wall on the j-th surface
T_{mr}	$[^\circ\text{C}]$	mean radiant temperature of the radiators
V	$[\text{m}^3]$	volume of the air inside the room

TABLE I: Summary of the parameters involved in the building model.