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Distributed Event-Triggered Estimation in Networked Systems ⋆

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Abstract: This paper examines distributed event-triggered estimation over wireless sensor networks. In such systems an efficient utilization of the wireless communications must be performed since energy consumption and communication bandwidth are limit resources. We pose a global event-triggered communication policy for state estimation that minimizes a weighted function of the network energy consumption and the number of transmissions subject to constraints on the estimator performance. The global communication policy determines when sensors transmit measurements to the central estimator using a sensor-to-estimator communication channel as well as when sensors received other sensors' measurements (which have been transmitted to the central estimator) using a estimator-to-sensor communication channel. A distributed 1-step greedy heuristic is introduced for the proposed global minimization problem such that sensors determine their respective communication policies using only the local information available at each sensor. Simulation results demonstrate that the number of sensor transmissions can be reduced at a potential increase in network energy consumption (number of sensor transmission and receptions) with the added benefit of reducing network congestion.

Keywords: Distributed Event-Triggered, State Estimation, Networked Systems

1. INTRODUCTION

In the last several decades we have seen great advances in computation, communication and control. The proliferation of tiny devices capable of performing computation, wireless communication, sensing and actuation has provided the means to create many intelligent complex networked systems. These systems are often geographically distributed, where individual subsystems exchange information over a shared wireless communication network. The wireless network is then a common resource, which cannot be disregarded when designing estimation and control algorithms. Additionally, wireless devices are often battery powered, which impose computation and communication constraints of the system design.

Recently, much research has been performed on identifying transmission policies that transmit information only when absolutely necessary. These event-triggered transmission protocols tend to greatly reduce the number of transmissions, but can result in increased network congestion in the presence of an unknown disturbance. While event-triggered transmission policies tend to reduce the number of transmissions, in networked systems this comes at the cost of requiring constant monitoring of the communication channel when not transmitting. Unfortunately this is not suitable for real deployments where the energy required to operate the radio is practically the same when both transmitting and receiving.

Many techniques have been proposed to address this issue. In Zhu et al. [2007], Weimer et al. [2011, 2008] sensor selection techniques are presented where it is identified which sensor (if any) should report a measurement at each periodic sampling instance. In Liu and Goldsmith [2004], Park et al. [2011] the authors propose a co-design of the MAC, sampling period and estimation and control algorithms. Instead of periodic transmissions, several researchers have proposed the use of aperiodic sampling techniques for control Åström and Bernhardsson [1999], Tabuada [2007] and estimation Yook et al. [2002], Xu and Hespanha [2004], Cogill [2009], Sijs and Lazar [2009], Li et al. [2010]. In this case, the transmission of data between sensors and controllers/estimators is performed only when required in order to achieve a certain desired level of performance. The distributed event-triggered problem has been addressed by Mazo Jr. and Tabuada [2010], Wang and Lemmon [2011], Guinaldo et al. [2011], Donkers and Heemels [2012] and by Li and Lemmon [2011], Trimpe and D'Andrea [2011] for the control and estimation cases, respectively. In these approaches, sensor nodes must broadcast its measurements to its neighbors whenever a triggering condition is violated, and continuously listen to the wireless channel. Unfortunately, in real deployment such requirement is not feasible for battery powered wireless devices, as continuously active radio would quickly drain its energy resource. In Mazo Jr. and Cao [2011] a method is proposed where these limitations are removed.

This paper addresses the problem of designing resource-aware distributed event-triggered estimation over large-scale systems communicating over a wireless network. Specifically, we aim at reducing sensor's energy consumption and wireless network congestion. Several sensor nodes measure parts of the plant state and transmit its measure-

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ments in an event-triggered fashion to a central estimator, which computes the estimate of the full plant state. We propose a distributed architecture where sensor broadcasts are not required in order to bound the central estimator mean squared error (MSE), thus removing the need for sensors to continuously listen to the wireless channel. By doing so, we provide large energy savings when compared to current approaches.

We address the wireless network congestion issue by allowing sensor nodes to decide to receive additional information from the central estimator. The reception of such information has the benefit of delaying transmission and avoiding unnecessary congestion in the case that sufficient information has already been provided by neighboring sensors to the central estimator. However, reception of information at the sensor nodes incurs in a certain energy cost. We formulate and solve a distributed constrained optimization problem whereby the energy consumption and network congestion of the wireless network are minimized.

The following section introduces notation and formulates an optimization problem that aims to minimize a weighted function of the network energy and congestion. Section 3 considers the optimization problem when only a single sensor exists, while Section 4 addresses networked systems. Simulation results and concluding remarks are provided in Sections 5 and 6, respectively.

2. PROBLEM FORMULATION

We consider the linear time-invariant stochastic system driven by noise,

$$x_{k+1} = Ax_k + w_k \tag{1}$$

where $x_k \in \mathbb{R}^N$ is the system state with the initial condition $x_0 = 0$, and $w_k \in \mathbb{R}^N$ is a Gaussian process noise with known mean and covariance,

$$\begin{aligned} \mathbb{E}\left[w_k\right] &= u_k \\ \mathbb{E}\left[(w_k - u_k)(w_k - u_k)^T\right] &= W \end{aligned} \tag{2}$$

A network of J sensors is employed to observe the state, where the measurement model for each sensor, $y_{k,j} \in \{y_{k,1}, \dots, y_{k,J}\}$, is

$$y_{k,j} = C_j x_k + v_{k,j} \tag{3}$$

assuming $v_{k,j}$ is a zero-mean Gaussian measurement noise and

$$\begin{aligned} \mathbb{E}\left[v_{k,j}v_{k',i}^T\right] &= \begin{cases} V_j, \; i = j, k = k' \\ 0, \quad \text{otherwise} \end{cases} \\ \mathbb{E}\left[v_{k,j}w_{k'}^T\right] &= 0 \;\; \forall k, k' \end{aligned} \tag{4}$$

In this work, we consider the general problem of performing state estimation using sensor measurements which are intermittently transmitted by the sensors to the central estimator using a wireless network. Specifically, we assume the system architecture illustrated in Fig. 2, which will be described in detail in the remainder of this section. As illustrated in Fig. 2, a two-channel wireless network is employed to both gather and distribute the sensor measurements. The first wireless channel is dedicated for sensor-to-estimator (S2E) communication, while the second is employed for estimator-to-sensor (E2S) communication. We assume the central estimator contains two

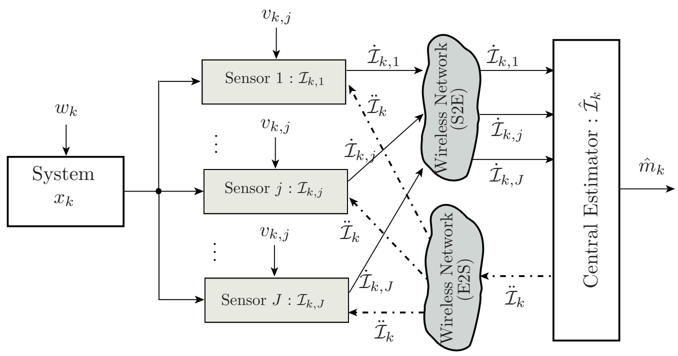

Fig. 1. Distributed event-triggered estimation system architecture.

wireless radios such that it can continuously access both communication channels, while the sensors have a single communication radio and must decide which channel (if any) to access at each time step. Each sensor decides whether to transmit or listen for measurements based on the measurements currently available at the sensor, $\mathcal{I}_{k,j}$. If a decision is made to transmit, then the S2E communication channel is accessed and the information set, $\dot{\mathcal{I}}_{k,j}$, containing all the new measurements of sensor j taken since the previous transmission and the last time sensor j accessed the E2S communication channel, is transmitted. Similarly, if a decision is made to receive, then the E2S communication channel is accessed and the information set, $\ddot{\mathcal{I}}_k$, which is a subset of the central estimator information set, $\hat{\mathcal{I}}_k$, containing all the measurements known by the central estimator that have not been distributed to the individual sensors, is received. Since when each sensor transmits its measurements to the central estimator, it also transmits the last time when it accessed the E2S communication channel, the central estimator can easily determine whether any particular measurement is known by all sensors. Before mathematically formulating the distributed event-triggered estimation problem, the following paragraphs introduce useful notation and mathematically define the information sets described above and in Fig. 2.

We denote the decision of sensor j to either transmit its measurements, receive the measurements known by the central estimator, or to turn off its radio and neither transmit or receive, using the test $\phi_j(k)$, such that

$$\phi_j(k) = \begin{cases} 1 \;\; \text{transmit (on S2E channel)} \\ 0 \qquad\qquad \text{radio off} \\ -1 \;\; \text{receive (on E2S channel)} \end{cases} \tag{5}$$

We write the latest time when sensor j transmits and receives as $\tau(k,j)$ and $\rho(k,j)$, respectively, where

$$\begin{aligned} \tau(k,j) &\triangleq \max\{k' | \phi_j(k') = 1, k' \leq k\} \\ \rho(k,j) &\triangleq \max\{k' | \phi_j(k') = -1, k' \leq k\}. \end{aligned} \tag{6}$$

We define the time, as known by the central estimator, that all the sensors have heard all the other sensors measurements as

$$\rho(k) = \min_{j,j' \in \{1,\dots,J\}} \rho(\tau(k,j), j') \tag{7}$$

and apply this time to mathematically define the information sets for S2E communication of sensor j, $\dot{\mathcal{I}}_{k,j}$, and E2S communication, $\ddot{\mathcal{I}}_k$ as

$$\begin{aligned}\dot{\mathcal{I}}_{k,j} &\triangleq \{y_{k',j}|k \geq k' > \tau(k-1,j)\} \cup \{\rho(k,j)\} \\ \ddot{\mathcal{I}}_{k} &\triangleq \{y_{k',j}|\tau(k-1,j) \geq k' > \rho(k-1)\},\end{aligned} \tag{8}$$

respectively. Additionally, we define the following measurement sets:

$$\begin{aligned}\bar{\mathcal{I}}_{k,j} &\triangleq \{y_{k',j}|k' \leq k\} \\ \mathcal{I}_k &\triangleq \bar{\mathcal{I}}_{k,1} \cup \cdots \cup \bar{\mathcal{I}}_{k,J} \\ \hat{\mathcal{I}}_k &\triangleq \mathcal{I}_k \backslash \{y_{k',j'}|k' > \tau(k,j'), 1 \leq j' \leq J\} \\ \hat{\mathcal{I}}_{k,j} &\triangleq \hat{\mathcal{I}}_k \backslash \{y_{k',j'}|k' > \tau(\rho(k,j),j'), 1 \leq j' \leq J\} \\ \mathcal{I}_{k,j} &\triangleq \hat{\mathcal{I}}_{k,j} \cup \bar{\mathcal{I}}_{k,j}\end{aligned} \tag{9}$$

where, $\bar{\mathcal{I}}_{k,j}$ is the set of sensor j measurements taken on or before time k, $\mathcal{I}_k$ is the set of all sensor measurements taken on or before time k, $\hat{\mathcal{I}}_k$ are the measurements known by the central estimator at time k, and $\hat{\mathcal{I}}_{k,j}$ are the measurements mutually known by the central estimator and sensor j. Additionally, we denote the minimum mean squared error (MMSE) estimate of x conditioned on the various measurement sets in (9) as

$$\begin{aligned}&\mathbb{E}[x_k|\mathcal{I}_k] = m_k, && \text{Cov}[x_k|\mathcal{I}_k] = P_k, \\ &\mathbb{E}\left[x_k|\hat{\mathcal{I}}_k\right] = \hat{m}_k, && \text{Cov}\left[x_k|\hat{\mathcal{I}}_k\right] = \hat{P}_k, \\ &\mathbb{E}[x_k|\mathcal{I}_{k,j}] = m_{k,j}, && \text{Cov}[x_k|\mathcal{I}_{k,j}] = P_{k,j}, \\ &\mathbb{E}\left[x_k|\hat{\mathcal{I}}_{k,j}\right] = \hat{m}_{k,j}, && \text{Cov}\left[x_k|\hat{\mathcal{I}}_{k,j}\right] = \hat{P}_{k,j},\end{aligned} \tag{10}$$

where $\mathbb{E}[\cdot]$ represents the expected value and Cov $[\cdot]$ is the associated covariance. The notation and information sets described above will be employed throughout the remainder of this work to formulate and solve the distributed event-triggered estimation problem.

As an estimation performance constraint, we require

$$\begin{aligned}&\forall j \;\; \mathbb{E}\left[\|F_j\left(x_k - \mathbb{E}\left[x_k|\hat{\mathcal{I}}_{k,j}\right]\right)\|_2^2|\mathcal{I}_{k,j}\right] \leq \eta_j \\ \Longleftrightarrow &\forall j \;\; \|F_j\left(m_{k,j} - \hat{m}_{k,j}\right)\|_2^2 + \text{Tr}\left[F_j P_{k,j} F_j^T\right] \leq \eta_j\end{aligned} \tag{11}$$

where the weights,$\{F_j\}_{j=1}^J$, are chosen to minimize the static distributed mean-squared error :

$$\begin{aligned}\{F_j\}_{j=1}^J = \arg \min_{\{F_j\}_{j=1}^J} \lim_{k\to\infty} \text{Tr}\left[\sum_{j=1}^J \text{Cov}\left[F_j x_k|\bar{\mathcal{I}}_{k,j}\right]\right] \\ \text{s.t} \;\; \sum_{j=1}^J F_j = I\end{aligned} \tag{12}$$

The constraint in (11) requires that a weighted expected euclidian distance squared between the state and the open-loop state estimate, conditioned on all the available measurements at each sensor, is bounded. Additionally, we assume that when $m_{k,j} = \hat{m}_{k,j}$ then the estimation constraint in (11) is always satisfied, namely

$$\text{Tr}\left[F_j P_{k,j} F_j^T\right] \leq \eta_j \quad \forall j,k \tag{13}$$

In wireless networking applications, a primary concern is minimizing the energy required to perform the task at hand. For estimation purposes, this requires minimizing the number of times the radio is operated for both transmitting and receiving measurements. In general, the energy consumed by transmitting a measurement is approximately equal to the energy consumed through reception Prayati et al. [2010], thus we write the energy consumed by the communication network at each time step as

$$\mathcal{E}\left(\phi_1(k),\ldots,\phi_J(k)\right) = \sum_{j=1}^J \text{I}\left[\phi_j(k) \neq 0\right] \tag{14}$$

where $\text{I}[z]$ is a binary indicator function that takes a value of one when z is true and a value of zero when z is false. Additionally, excessive transmission attempts has the added cost of creating congestion in the network which can restrict the flow of information from the sensors to the central estimator and result in increased energy consumption due to congestion resolution in the MAC. Thus, in this work, we define the network energy objective as a weighted sum of the network energy and the number of transmissions, namely

$$\begin{aligned}\mathcal{J}(\alpha,\phi_1(k),\ldots,\phi_J(k)) =& \alpha\mathcal{E}\left(\phi_1(k),\ldots,\phi_J(k)\right) \\ &+ \sum_{j=1}^J \text{I}\left[\phi_j(k) = 1\right]\end{aligned} \tag{15}$$

where α is the network energy weighting factor. By choosing α to be small, one effectively decides that transmissions cost more than the network energy consumption, and vice-versa. This is translated as a willingness to allow an increase in the network energy consumption in exchange for fewer sensor transmissions.

We formulate the constrained minimization problem which minimizes the network energy objective subject to the estimation performance constraint as

$$\begin{aligned}\min_{\Phi_1,\ldots,\Phi_J} \quad & \sum_{k=0}^\infty \mathcal{J}(\alpha,\phi_1(k),\ldots,\phi_J(k)) \\ \text{s.t} \quad & \|F_j\left(m_{k,j} - \hat{m}_{k,j}\right)\|_2^2 + \text{Tr}\left[F_j P_{k,j} F_j^T\right] \leq \eta_j \quad \forall k,j \\ & \phi_j(k) \in \{-1,0,1\} \; \forall k,j\end{aligned} \tag{16}$$

where,

$$\Phi_j = \{\phi_j(k)\}_{k=0}^\infty \tag{17}$$

The minimization problem in (16) requires identifying a communication schedule for the entire network for all time. In previous work, we addressed a similar problem using *a priori* communication scheduling in Weimer et al. [2011]. For wireless communication applications, *a priori* scheduling has the benefit that scheduling of sensors can be achieved such that congestion is reduced or eliminated. However, this approach has the drawback that sensors are required to transmit their measurements even when the information is not needed. Additionally, *a priori* communication scheduling is less responsive to disturbances, in general, since the sensor scheduling is performed without knowledge of the disturbance. Therefore, in this work, we consider an event-triggered scheduling solution to the minimization problem in (16) in the following sections.

3. SINGLE-SENSOR EVENT-TRIGGERED ESTIMATION

Before discussing a distributed event-triggered estimation framework in the next section, this section considers the special case when the network contains a single sensor ($J = 1$). The single-sensor framework and transmission policy discussed in this section follows closely the framework and measurement transmission policy introduced in Li et al. [2010] and serves as motivation for formulating a distributed event-triggered communication policy in the following section.

When only a single sensor exists, then it is clear from (12) that $F_1 = I$ and the minimization problem in (16) is equivalently written as

$$\begin{aligned} \min_{\Phi_1} \quad & \sum_{k=0}^{\infty} \mathcal{J}(\alpha, \phi_1(k)) \\ \text{s.t} \quad & \|m_k - \hat{m}_k\|_2^2 + \mathrm{Tr}\left[P_k\right] \leq \eta_1 \quad \forall k \\ & \phi_1(k) \in \{-1, 0, 1\} \; \forall k \end{aligned} \tag{18}$$

The above minimization problem is non-causal and requires future sensor measurements to calculate the expected value of the state estimate. Since future measurements are unknown, as a heuristic, we introduce a 1-step greedy approach for determining the decision at each time step based on the likelihood of future decisions. We introduce a greedy approximation of the energy objective function, $\hat{\mathcal{J}}(\alpha, \phi_1(k), \ldots, \phi_J(k))$, as

$$\hat{\mathcal{J}}(\alpha, \phi_j(k)) = \mathcal{J}(\alpha, \phi_j(k)) + (1+\alpha)\Pr\left[\phi_j(k+1) = 1\right], \tag{19}$$

where $\hat{\mathcal{J}}(\alpha, \phi_j(k))$ is the energy objective at time k plus the expected energy objective at time $k+1$ assuming the probability of E2S communication in the future is zero, $\Pr\left[\phi_j(k+1) = -1\right] = 0$. Employing the approximate energy objective, we formulate the 1-step greedy minimization problem for the minimization problem in (18) as

$$\begin{aligned} \min_{\phi_1(k)} \quad & \hat{\mathcal{J}}(\alpha, \phi_1(k)) \\ \text{s.t} \quad & \|m_k - \hat{m}_k\|_2^2 + \mathrm{Tr}\left[P_k\right] \leq \eta_1 \\ & \phi_1(k) \in \{-1, 0, 1\} \end{aligned} \tag{20}$$

The objective of the greedy minimization problem in (25) is the summation of the energy objective at the current time step with an approximation of the energy objective at the following time-step. When only a single sensor exists, the measurement sets in (9) have the following equivalences:

$$\mathcal{I}_k \equiv \mathcal{I}_{k,1} \equiv \bar{\mathcal{I}}_{k,1} \quad \text{and} \quad \hat{\mathcal{I}}_{k,1} \equiv \hat{\mathcal{I}}_k \tag{21}$$

These equivalences exist since the sensor has full knowledge of which measurements are known by the central estimator. Additionally, these equivalences render E2S communication irrelevant and thus the energy objective in (25) can take one of two values, namely

$$\begin{aligned} \hat{\mathcal{J}}(\alpha, 0) &= (1+\alpha)\Pr\left[\phi_1(k+1) = 1 | \phi_1(k) = 0\right] \\ \hat{\mathcal{J}}(\alpha, 1) &= (1+\alpha)\Pr\left[\phi_1(k+1) = 1 | \phi_1(k) = 1\right] + 1 + \alpha \end{aligned} \tag{22}$$

where it is clear that

$$\hat{\mathcal{J}}(\alpha, 1) \geq \hat{\mathcal{J}}(\alpha, 0) \tag{23}$$

and thus $\phi_1(k) = 0$ always minimizes the energy objective, but may not satisfy the estimation constraint. Recalling from the problem formulation that the estimation constraint is always satisfied if a transmission occurs, ($\phi_1(k) = 1$), we conclude that if the constraint on the estimator performance is satisfied, then the measurements are not transmitted, and vice-versa such that

$$\phi_1(k) = \begin{cases} 1 & \text{if } \|m_k - \hat{m}_k\|_2^2 + \mathrm{Tr}\left[P_k\right] > \eta_1 \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

The event-triggered estimation problem presented in this section assumes a single sensor exists, which has been shown to result in a communication policy that only requires S2E communication. In the following section, we consider the case with multiple sensors such that each sensor must not only decide when to transmit, but also when to receive measurements from the central estimator.

4. NETWORKED EVENT-TRIGGERED ESTIMATION

While the previous section discussed an event-triggered transmission policy for state estimation using a single sensor. In this section, we address the event-triggered estimation problem in (16) when multiple sensors exist. Using the same logic and reasoning as in the previous section, we assume a 1-step greedy approach to solving the minimization problem in (16) and write the resulting minimization problem as

$$\begin{aligned} \min_{\phi_1(k),\ldots,\phi_J(k)} \quad & \sum_{j=1}^{J} \hat{\mathcal{J}}(\alpha, \phi_j(k)) \\ \text{s.t} \quad & \|F_j\left(m_{k,j} - \hat{m}_{k,j}\right)\|_2^2 + \mathrm{Tr}\left[F_j P_{k,j} F_j^T\right] \leq \eta_j, \forall j \\ & \phi_j(k) \in \{-1, 0, 1\}, \; \forall j \end{aligned} \tag{25}$$

To solve the minimization problem in (25), requires globally determining which sensors transmit, receive, and are turned off. To avoid the inter-sensor communication overhead associated with solving the global minimization problem in (25), we propose, as a heuristic, a distributed approximation where each sensor decides its communication policy by solving a local optimization problem, namely

$$\begin{aligned} \min_{\phi_j(k)} \quad & \hat{\mathcal{J}}(\alpha, \phi_j(k)) \\ \text{s.t} \quad & \|F_j\left(m_{k,j} - \hat{m}_{k,j}\right)\|_2^2 + \mathrm{Tr}\left[F_j P_{k,j} F_j^T\right] \leq \eta_j, \\ & \phi_j(k) \in \{-1, 0, 1\} \end{aligned} \tag{26}$$

In the special case when each sensor observes a subset of states that are unobservable by the other sensors, solving the distributed minimization problem in (26) for each sensor is equivalent to solving the global minimization problem in (25). The sensor energy objective function in the distributed minimization problem can take the following values:

$$\begin{aligned} \hat{\mathcal{J}}(\alpha, -1) &= (1+\alpha)\Pr\left[\phi_j(k+1) = 1 | \phi_j(k) = -1\right] + \alpha \\ \hat{\mathcal{J}}(\alpha, 0) &= (1+\alpha)\Pr\left[\phi_j(k+1) = 1 | \phi_j(k) = 0\right] \\ \hat{\mathcal{J}}(\alpha, 1) &= (1+\alpha)\Pr\left[\phi_j(k+1) = 1 | \phi_j(k) = 1\right] + 1 + \alpha \end{aligned} \tag{27}$$

where consistent with the previous section, we observe that

$$\hat{\mathcal{J}}(\alpha, 1) \geq \hat{\mathcal{J}}(\alpha, 0). \tag{28}$$

Applying the same logic as the previous section, we conclude that sensor j transmits its measurements to the central estimator when the estimation constraint in (26) is not satisfied, namely

$$\phi_j(k) = 1 \Leftrightarrow \|F_j\left(m_{k,j} - \hat{m}_{k,j}\right)\|_2^2 + \mathrm{Tr}\left[F_j P_{k,j} F_j^T\right] > \eta_j. \tag{29}$$

To determine a communication policy for accessing the estimator-to-sensor communication channel, requires determining the probability of transmission at the next time step conditioned on the decision to access the estimator-to-sensor communication channel at the current time step. However, to calculate this probability requires knowing what measurements will be received by listening, which is unknown to the sensors. As a heuristic, we assume that upon sampling, any received sensor measurements will be equal to the mean value and approximate the probability of transmission by applying the Markov inequality such that

$$\begin{aligned}\hat{\mathcal{J}}(\alpha,-1) &\approx \frac{\|F_j A\left(m_{k,j}-\hat{m}_{k,j}\right)\|_2^2 + \mathrm{Tr}\left[F_j \hat{P}_{k+1} F_j^T\right]}{\frac{\eta_j}{1+\alpha}} + \alpha \\ \hat{\mathcal{J}}(\alpha,0) &\approx \frac{\|F_j A\left(m_{k,j}-\hat{m}_{k,j}\right)\|_2^2 + \mathrm{Tr}\left[F_j \hat{P}_{k+1} F_j^T\right]}{\frac{\eta_j}{1+\alpha}}.\end{aligned} \tag{30}$$

Since the covariance at the central estimator is unknown to the sensor before accessing the estimator-to-sensor communication channel, we approximate the central estimator covariance as a weighted sum of the sensor covariance and the central covariance assuming all measurements are received, written as

$$\hat{P}_k \approx (1+\beta_j)P_{k,j} + \beta_j P_k \tag{31}$$

The weighting factor at sensor j is chosen based on the likelihood that sensor j transmits and the most recent results from previous accesses of the estimator-to-sensor communication channel, written mathematically as

$$\beta_j = \begin{cases} 0 & \text{if } \ddot{\mathcal{I}}_{\rho(k,j)} \subseteq \hat{\mathcal{I}}_{\rho(k,j),j} \\ \hat{\beta}_j & \text{if } \tau(k,j) \geq \rho(k,j) \quad \vee \quad \ddot{\mathcal{I}}_{\rho(k,j)} \not\subseteq \hat{\mathcal{I}}_{\rho(k,j),j} \end{cases} \tag{32}$$

where

$$\hat{\beta}_j = \frac{\|F_j\left(m_{k,j}-\hat{m}_{k,j}\right)\|_2^2 + \mathrm{Tr}\left[F_j \hat{P}_{k,j} F_j^T\right]}{\eta_j} \tag{33}$$

The value of β_j varies according to the likelihood that sensor j transmits. The logic in this situation is that if sensor j is likely to transmit, then other sensors are likely to transmit as well. However, if sensor j attempts to access the communication channel and discovers that the measurement set provided through the estimator-to-sensor communication channel is already known locally, then the sensor no longer attempts to access the sensor-to-estimator communication channel until after a transmission occurs.

The sensor chooses to access the communication channel when the objective is minimized, namely when

$$\hat{\mathcal{J}}(\alpha,0) \geq \hat{\mathcal{J}}(\alpha,-1) \tag{34}$$

which by applying the approximations in (30), we conclude that

$$\phi_j(k) = -1 \Leftrightarrow \beta_j \mathrm{Tr}\left[F_j\left(\hat{P}_{k+1,j} - P_{k+1}\right)F_j^T\right] \geq \frac{\alpha}{1+\alpha}\eta_j \tag{35}$$

Thus, the communication policy for performing networked event-triggered state estimation is

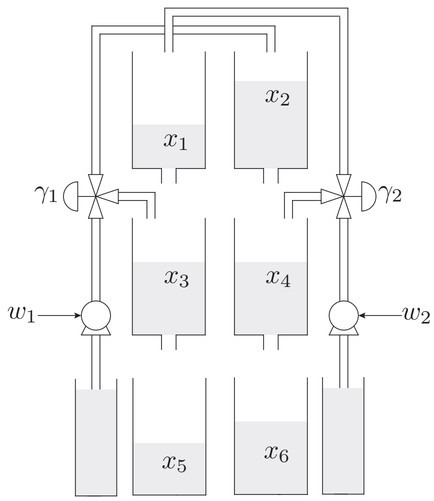

Fig. 2. The six-tank system. The water flows from tank 1 to 3 and from tank 2 to 4, and from tank 3 and 4 back to tank 5 and 6, which are reservoir tanks. The fraction of water pumped into each pair of tanks is regulated by the two valves γ_1 and γ_2. The water levels of the tanks are $x_i,\ i = 1,\ldots,6$. The water input is driven by process noises w_1 and w_2.

$$\phi_j(k) = \begin{cases} -1 \Leftrightarrow \beta_j \mathrm{Tr}\left[F_j\left(\hat{P}_{k+1,j} - P_{k+1}\right)F_j^T\right] \geq \dfrac{\alpha}{1+\alpha}\eta_j \\ 1 \Leftrightarrow \|F_j\left(m_{k,j}-\hat{m}_{k,j}\right)\|_2^2 + \mathrm{Tr}\left[F_j P_{k,j} F_j^T\right] > \eta_j \\ 0 \ \ \text{otherwise} \end{cases} \tag{36}$$

This section formulated a communication policy for networked event-triggered state estimation. The proposed strategy is a heuristic based the optimal distributed event-triggered state estimation communication policy when sensors have uncorrelated measurements. The communication policy introduced in this section is evaluated in the following section through simulation.

5. SIMULATION RESULTS

In this section, an example is used to evaluate the performance of the distributed event-triggered estimator proposed. We choose the six-tank system inspired from Johansson [2000], a multi-input multi-output nonlinear system consisting of two lower tanks (3 and 4), two upper tanks (1 and 2) and two reservoirs (5 and 6) and two pumps, as shown in Fig. 5. The water flows from tank 1 to 3 an from tank 2 to 4, and from tank 3 and 4 to tank 5 and 6, which are reservoir tanks. The pumps are connected so that pump 1 delivers water to tanks 1 and 4 and pump 2 delivers water to tanks 2 and 3. The fraction of water pumped into each pair of tanks is regulated by the valves γ_1 and γ_2. The state of the plant is composed by the water levels in all six tanks as $x_i,\ i = 1,\ldots,6$ and the input is driven by the noises w_1 and w_2. Two wireless sensors measure the water level in tank 5 (sensor 1) and 6 (sensor 2) and transmit the values to a central estimator. The goal of this example is to estimate the water level in the all of the six tanks based on measurements from the two wireless sensors.

The linearized continuous-time system dynamics around a working point x^0 are given by:

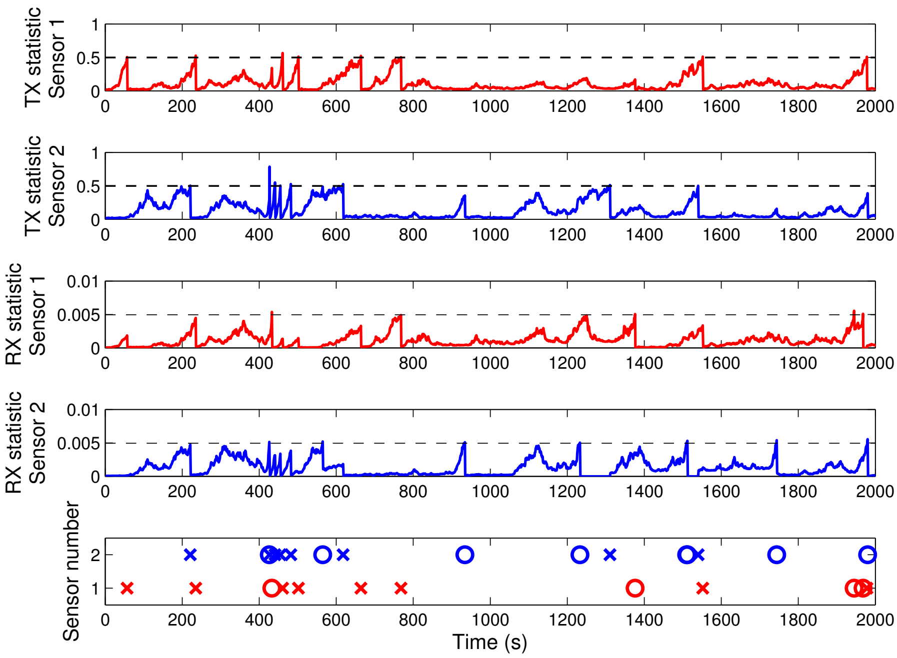

Fig. 3. Transmission and reception statistics for wireless sensor 1 and 2 in transient conditions; Transmissions is characterized by a cross (×) and receptions by a circle (∘). Tank 1 and tank 2 levels are shifted at time $t = 400$ s and at $t = 420$ s, sensor 1 avoids transmission since it receives information from the central estimator. The listening benefit is observed also for sensor 2 at time $t = 934$ s and $t = 1980$ s and for sensor 1 again at $t = 1374$ s.

$$\dot{x}(t) = A_c x(t) + B_c w(t). \tag{37}$$

The continuous-time system matrices are defined as:

$$A_c = \begin{pmatrix} -\tau_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\tau_2 & 0 & 0 & 0 & 0 \\ \tau_1 & 0 & -\tau_3 & 0 & 0 & 0 \\ 0 & \tau_2 & 0 & -\tau_4 & 0 & 0 \\ 0 & 0 & \tau_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_4 & 0 & 0 \end{pmatrix}, \quad B_c = \begin{pmatrix} \frac{\gamma_1}{k_1 \bar{A}} & 0 \\ 0 & \frac{\gamma_2}{k_2 \bar{A}} \\ 0 & \frac{(1-\gamma_1)}{k_2 \bar{A}} \\ \frac{(1-\gamma_1)}{k_1 \bar{A}} & 0 \end{pmatrix},$$

$$C_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \tau_i = \frac{a_i}{\bar{A}} \sqrt{\frac{g}{2x_i^0}}. \tag{38}$$

The water tank parameters a_i are the outflow diameter of the tanks, $\bar{A}$ is the diameter of the tank, g is the gravitational acceleration in cm/s^2, $\frac{1}{k_i}$ are the the pump motor constants. Discretizing system (37) with zero-order hold sampling and period T, we obtain the discrete-time system in (1).

We now validate the distributed event-triggered algorithm presented in Section 4 in transient and steady-state conditions. The process and measurement noise covariances are set to $W = diag(0.05, \ldots, 0.05)$ and $V = diag(0.2, 0.2)$, respectively. We define the transmission threshold, $\eta_j = 0.5$.

5.1 Transient conditions

The system is simulated for 2000 seconds and is affected by process and measurement noise. Additionally, at time $t = 400$s the levels of tank 1 and tank 2 are shifted by 3 cm and 5 cm, respectively. Figure 5 depicts the transmission (TX) and reception (RX) tests as defined in (36), as well as the history of transmissions and receptions for both wireless sensors. After the tank levels are shifted, both sensor 1 and sensor 2 have a consecutive rise of their transmission statistic. However, due to the listening feature of the proposed algorithm, at $t = 420$ s, sensor 1 avoids transmission since it receives information from the central estimator, thus, avoiding the risk of collision with packets transmitted by sensor 2. This benefit is high since sensor 2 had just performed a transmission and the sensors are correlated. The listening benefit is observed also for sensor 2 at time $t = 934$ s and $t = 1980$ s and for sensor 1 again at $t = 1374$ s. Receptions take place in other moments but its benefits are hardly noticeable since a long time has passed since the transmission of information from the neighboring sensor. The MSE of the simulation is shown in Figure 5.2 for completeness of the analysis. As it can be seen, the MSE stays bounded by $[-1, 1]$ using the proposed algorithm, which was the objective.

5.2 Steady-state conditions

Here we analyze the energy consumption of the wireless nodes, the estimation performance with respect to the average MSE as well as the probability of transmission and reception as a function of the network energy weighting α. We define the *energy consumption* as $E = \frac{1}{J*T} N_{TX} + N_{RX}$, where N_{TX} and N_{RX} are the number of transmissions and receptions, J the number of sensors and T is the simulation horizon. Furthermore, the probability of transmission and reception is defined as the average number of transmissions and receptions over the simulation horizon.

Figure 5.2 depicts the results obtained for 100 simulations of the system for an horizon of 1000 s under different values of α, for the case of no tank level shift and under a periodic tank level shift every 100 s. In the case of periodic tank level shifting, the results show that the number of network transmissions are able to be reduced if

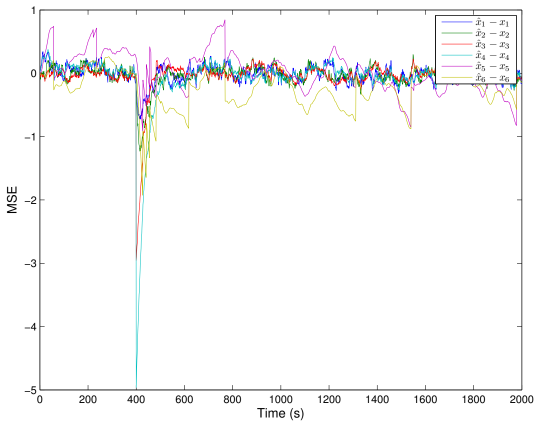

Fig. 4. Mean-Squared Error (MSE) for the six-tank process during a transient time-response.

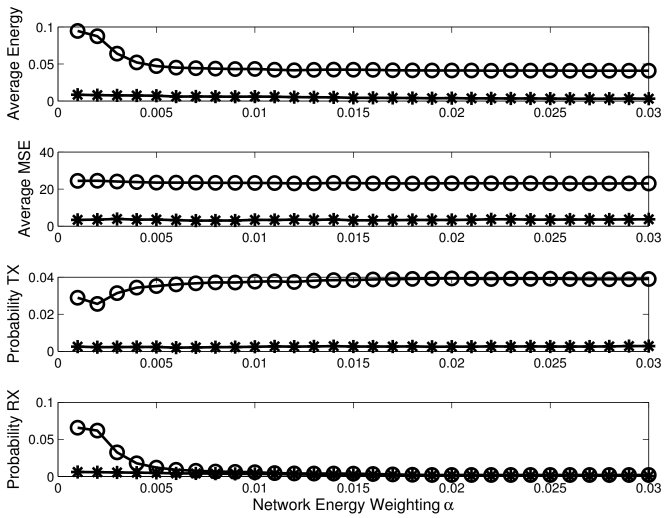

Fig. 5. Steady-state condition statistics in terms of average energy, MSE and probability of transmissions (TX) and reception (RX) for the case of: No tank level shift (Star ($\star$)) and with periodic tank level shifting every 100 s (Circle ($\circ$)).

receptions of information from the central estimator take place. This has the benefit of reducing the network traffic and device contention, possibly reducing packet collision, with the drawback of increasing the energy consumption of the network over the condition where no listening occurs. However, one should note that by reducing contention, benefits with respect to a reduced energy consumption are expected to be obtained. Note that the average MSE does not suffer any major change for different values of α. In the case of no tank level shift the number of transmission and receptions if small as expected. Moreover, there is no clear benefit of reception of information and the probability of transmission suffers a small reduction for a small increase of the probability of reception as α is decreased.

6. CONCLUSIONS AND FUTURE WORK

A distributed event-triggered estimation algorithm was developed for performing distributed estimation of networked systems when sensor measurements are transmitted over a wireless sensor network. The distributed event-triggered estimator employs a dual-channel architecture which allows for sensors to transmit their measurements to the central estimator, but also receive information from the central estimator in order to delay or avoid possible transmissions. This feature comes from the fact that the knowledge of previous transmissions of correlated measurements by neighboring sensors to the central estimator may allow a given sensor to stay open-loop for a longer period of time. Specifically, we pose a global event-triggered communication policy for state estimation that minimizes the weighted function of the network energy consumption and the number of transmissions, subject to constraints on the estimator performance. A distributed greedy heuristic is introduced for the proposed global minimization problem such that sensors determine their communication policies using local information available at each sensor. The distributed estimation algorithm is employed to perform the estimation of the water level in a six-tank system using two wireless sensor nodes. Results shown benefits on reducing the number of sensor transmissions and network congestion, with the potential increase of the network energy consumption.

Future extensions of this work include an evaluating the proposed mechanism in a real wireless sensor network system where we practically verify the impact of delaying or avoiding transmissions under high-traffic conditions. Additionally, investigating the inclusions of *a priori* scheduling for estimator-to-sensor communication as a means of ensuring data dissemination such that the probability of transmitting immediately after listening is reduced.

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