2021 60th IEEE Conference on Decision and Control (CDC)
December 13-15, 2021. Austin, Texas

# Distributed Event-Triggered Learning-Based Control for Nonlinear Multi-Agent Systems

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2021 60th IEEE Conference on Decision and Control (CDC) | 978-1-6654-3659-5/21/$31.00 ©2021 IEEE | DOI: 10.1109/CDC45484.2021.9683215

***Abstract*— This paper studies event-triggered consensus control for heterogenous nonlinear multi-agent systems. We present a new distributed nonlinear event-triggered control algorithm integrating basic radial basis function neural network with event-based control. We show that it can handle any unknown dynamics linear in the control input, achieving practical consensus without Zeno behaviour. A numerical example is provided to highlight the effectiveness of the proposed algorithm in terms of learning the unknown nonlinear dynamics.**

## I. INTRODUCTION

The consensus problem [1, 2], where a group of agents in the same network collectively seek to reach a mutually agreeable state, is not robust to nonlinearity, such as malicious agents or external disturbances [3, 4]. To overcome this, Rehan *et al.* [5] presented a modified consensus control protocol under one-sided Lipchitz nonlinearity. Ma *et al.* [6] adopted a feedback controller to handle stochastic nonlinear disturbances. These algorithms, however, require at least partial, if not full, information about the nonlinearity, such as Lipchitz constants and boundedness. It could be a strong assumption in physical systems as we often do not have sufficient knowledge or exact model of the dynamics. This motivates the use of learning techniques, specifically neural networks, to compensate the influence of the nonlinearity which has shown promising capabilities of estimating unknown dynamics [7–11]. In particular, Zhang *et al.* [12] proposed a continuous-time consensus algorithm with radial basis function neural network as an estimator of the unknown dynamics.

The learning-based consensus algorithms, similar to classical consensus algorithms [13], require continuous information exchange between the neighbours, which is often impractical in reality. In view of this, self-triggered and event-triggered control were introduced to mitigate such connection in control systems in general [14, 15]. Event-triggered consensus control problems have also been widely investigated [16–20]. Yi *et al.* [21] proposed a dynamic event-triggering law that drastically reduces communication amongst agents without the need for a heavy computation overhead. Tsang *et al.* [20] designed a stochastic trigger based on existing deterministic counterparts to achieve even lower communication rate. Exponential convegence to consensus can be achieved and require much less communication than time-triggered approaches.

This work is supported by the Swedish Strategic Research Foundation, the Swedish Research Council, and the Knut and Alice Wallenberg Foundation.

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In this paper, we consider the event-triggered consensus problem for nonlinear multi-agent systems linear in the control input. The main contributions are as follows: we introduce a novel nonlinear event-based distributed control law, integrating radial basis function neural networks with event-triggered control. We only assume that the nonlinear agent dynamics is given by nonlinear continuous function without any further knowledge. We prove that the closed-loop multi-agent system executing the proposed algorithm is able to reach practical consensus with bounded consensus error, while excluding the existence of Zeno behaviour. In addition, we show through numerical example that the algorithm is able to learn and compensate the unknown nonlinearities in the agents' dynamics. It outperforms the standard consensus algorithm (without adaptive control) significantly regarding both consensus performance and communication rate as well as some state-of-the-art consensus algorithms.

The rest of this paper is organised as follows. Section II defines the problem and objective. Section III presents the proposed algorithm to solve the problem. Section IV contains the main result on consensus and Zeno behaviour. Section V provides a numerical simulation to illustrate the effectiveness of the proposed algorithm and compare it with other research in consensus control algorithms. Section VI concludes the paper at last with potential future directions.

**Notation.** *The norm $||\cdot||_p$ denotes the $p$-norm for any vector or the induced $p$-norm for any matrix. Unless otherwise specified, $||\cdot||$ represents the 2-norm for vectors and Frobenius norm for matrices. A vector of dimension $n$ with all entries being 1 is denoted by $\mathbf{1}_n$. The $i$-th eigenvalue of a matrix $M$ in ascending order is $\lambda_i(M)$.*

## II. PROBLEM FORMULATION

Consider a multi-agent system with $N$ agents interacting over an undirected connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Each agent $i \in \mathcal{V}$ has the following nonlinear dynamics:

$$\dot{x}_i(t) = f_i(x_i(t)) + u_i(t) \tag{1}$$

where $x_i(t) \in \mathbb{R}^n, u_i(t) \in \mathbb{R}^n$ are the state and control input of agent $i$ respectively, and $f_i\colon \mathbb{R}^m \mapsto \mathbb{R}^m$ is an unknown continuous function. Let $\mathcal{N}_i$ denote the neghbours of agent $i$: $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$, for $i = 1, \dots, N$.

Each agent is equipped with an event-triggered learning-based controller, as shown in Fig. 1. It consists of a control law (C), neural network (NN) and event-trigger (ET). They will all be introduced in next section. The dashed lines represent communication with other agents. Here $\gamma_i(t)$ is an

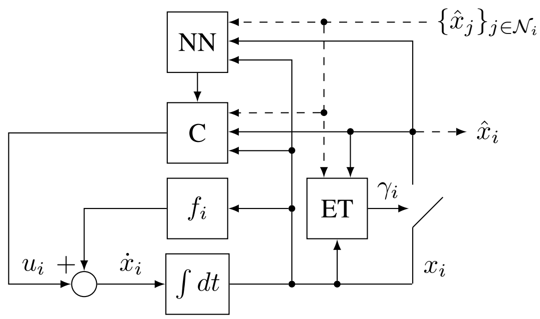



Fig. 1: Block diagram of the control system of agent $i$

impulse train representing the event that agent $i$ broadcasts its state to its neighbours at time $t$, i.e.,

$$\gamma_i(t) = \begin{cases} 1, & \text{Agent } i \text{ broadcasts to neighbours} \\ 0, & \text{Otherwise} \end{cases}.$$

Let $K_N = I_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T$ and the global consensus error $\varepsilon(t) = \frac{1}{N}x(t)^T(K_N \otimes I_n)x(t) = \frac{1}{N}\sum_{i=1}^N ||x_i(t) - \bar{x}(t)||^2$. We adopt the following definition of practical consensus as in [22]:

**Definition 1.** *The multi-agent system reaches practical consensus if there exists $\chi \geq 0$ such that*

$$\lim_{t\to\infty} \varepsilon(t) \leq \chi.$$

The objective of this work is to design the event-triggered learning-based controller in Fig. 1 such that the multi-agent system reaches practical consensus without exhibiting Zeno behaviour.

## III. Distributed Event-Triggered Learning-Based Control Algorithm

The control system of each agent in Fig. 1 will be described in detail in this section. We start with the control law followed by the event-triggering mechanism, and the neural network.

### A. Control Law

Each agent employs the following control law:

$$u_i(t) = -\delta_i(t) - W_i(t)^T\phi_i(x_i(t)) \tag{2}$$

$$\delta_i(t) = \sum_{j\in\mathcal{N}_i} L_{ij}\hat{x}_j(t) \tag{3}$$

where

$$\hat{x}_i(t) = \begin{cases} x_i(t), & \gamma_i(t) = 1 \\ x_i(\tau_i(t)), & \gamma_i(t) = 0 \end{cases}, \tag{4}$$

$$\tau_i(t) = \max\{k < t : \gamma_i(k) = 1\}. \tag{5}$$

The second term of the control law is the output of the neural network described below estimating the nonlinearity $f_i(x_i(t))$. The events $\gamma_i(t)$ are given by the event-trigger. The first term is the standard event-triggered consensus control proposed in [21].

### B. Event-Trigger

The event-triggered broadcast of agent $i$ is given by

$$\gamma_i(t) = \begin{cases} 1, & \rho_i(t) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

with the threshold function

$$\rho_i(t) = e_i(t)^T e_i(t) - \frac{\alpha_i}{L_{ii}}\left(\exp(-\beta_i t) + \epsilon_0\right). \tag{7}$$

where $e_i(t) = \hat{x}_i(t) - x_i(t)$ is the difference between the last broadcast state and its current value. The scalars $\alpha_i, \beta_i, \epsilon_0 > 0$ are design parameters. This event-triggering law is inspired by [14, 20, 21].

### C. Neural Network

The neural network in the control architecture in Fig. 1 is a radial basis function neural network, that is, a neural network with precisely one hidden layer. It is used to approximate the unknown function $f_i$. The network, with $n$-dimensional input and $m$-dimensional output, is given by

$$\hat{f}_i(z) = W_i^T\phi_i(z),$$

where $W_i \in \mathbb{R}^{p\times n}$ is the weight matrix, $p$ the number of neurons and $\phi_i(z)$ the radial basis function vector with $l$-th entry being

$$\phi_i^l(z) = \exp\left(-\frac{(z-\mu^l)^T(z-\mu^l)}{2{\sigma^l}^2}\right)$$

where $\mu^l \in \mathbb{R}^n, \sigma^l \in \mathbb{R}$, $l = 1, \ldots, p$, are the centres and widths of the Gaussian radial basis functions, respectively. The neural network can arbitrarily well compensate the effect of the nonlinear term $f_i(x_i(t))$, because of the following classical result.

**Lemma 1** (Universal Approximation Theorem [23])**.** *Given any continuous function $f_i(z)$ and positive constant $\epsilon_{f_i}$, there exists a neural network $f_i^*(z) = {W_i^*}^T\phi_i(z)$ that can approximate $f_i(z)$ arbitrarily well on a compact domain $\mathcal{X}_i \subset \mathbb{R}^n$ such that $||\epsilon_i(z)|| \leq \epsilon_{f_i}, \forall z \in \mathcal{X}_i$, where $\epsilon_i(z) = f_i(z) - f_i^*(z)$.*

In Lemma 1, $W_i^*$ is referred to as the optimal weight matrix, because it minimises the approximation error. Furthermore, we pose the following optimisation problem:

$$\max_{W_i} \quad \underbrace{\left(\hat{f}_i(x_i(t)) - f_i(x_i(t))\right)^T\delta_i(t)}_{\text{Correlation}} - \underbrace{\frac{\sigma_i}{2}||W_i||_F^2}_{\text{Regularisation}}. \tag{8}$$

The first term can be interpreted as the correlation between the estimation error of $f_i$ and the local consensus error $\delta_i$, while the second term is a regularisation term to avoid overfitting. By taking the gradient of (8) with respect to $W_i$, we have the update rule

$$\dot{W}_i(t) = \eta\left(\phi_i(t)\delta_i(t)^T - \sigma_i W_i(t)\right), \tag{9}$$

which is inspired by Zhang *et al.* [12].

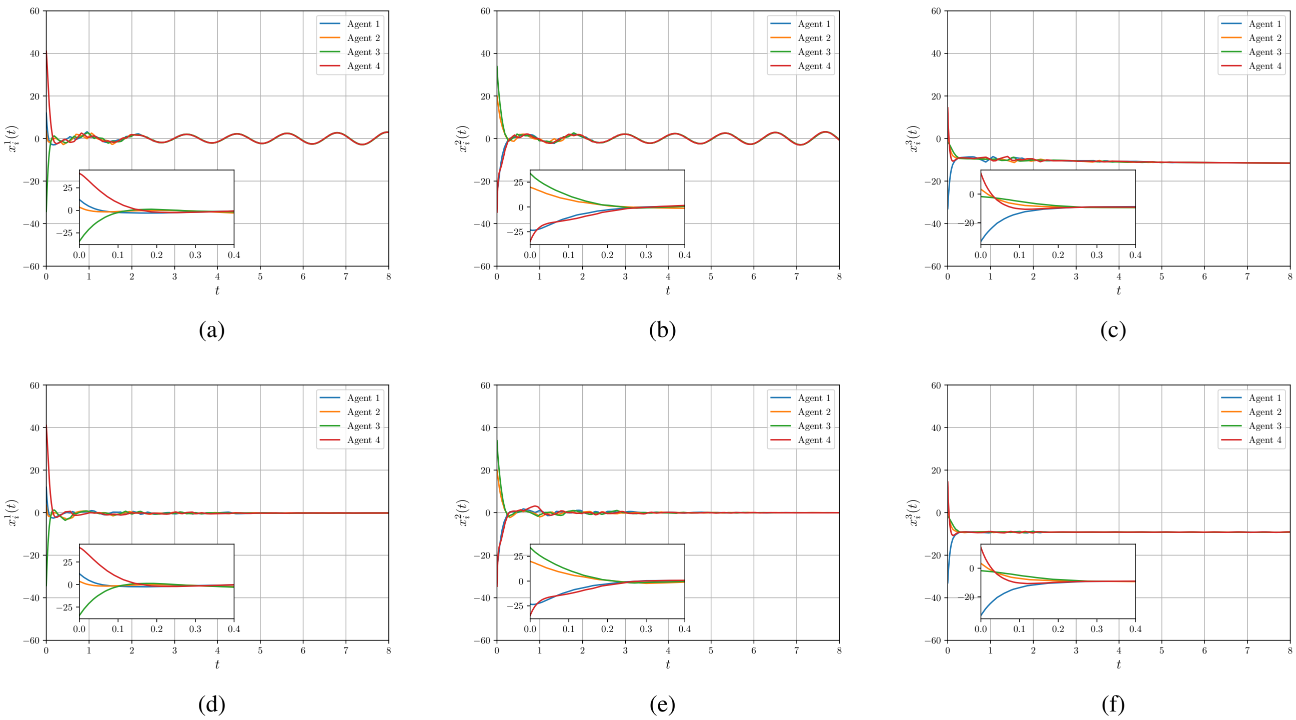



Fig. 2: State evolution for the standard consensus algorithm (A3) (a-c) and the proposed algorithm (d-f)

## IV. Main Results

In this section, we show that the distributed event-triggered learning-based control algorithm in Section III is able to drive the multi-agent system to practical consensus without exhibiting Zeno behaviour. The following theorem is the main result of this paper.

**Theorem 1.** *Consider a multi-agent system with dynamics* (1) *and the event-triggered learning-based control* (2)-(7) *and* (9)*. This closed-loop system does not exhibit Zeno behaviour and achieves practical consensus with the following asymptotic upper bound on the consensus error:*

$$\lim_{t\to\infty} \varepsilon(t) \leq \sum_{i=1}^{N} k_1 \sigma_i \left( ||W_i^*||_F^2 - ||\bar{W}_i||_F^2 \right) + k_2 \bar{\epsilon}_f^2 + k_3 \epsilon_0$$

*for some constants* $k_1, k_2, k_3 > 0$, $\bar{\epsilon}_f = \sqrt{\sum_{i=1}^{N} \epsilon_{f_i}^2}$ *and*

$$||\bar{W}_i||_F^2 = \limsup_{t\to\infty} ||W_i(t)||_F^2.$$

*Proof.* See Appendix B. □

Note that Theorem 1, together with Lemma 1, implies that there exists a set of neural networks $\{\hat{f}_i(z) : i \in \mathcal{V}\}$ such that the asymptotic consensus error can be made arbitrarily small because $\sigma_i$, $\bar{\epsilon}_f$ and $\epsilon_0$ can all be arbitrarily small, as follows from the proof.

## V. Simulation Results

Consider a group of $N = 4$ agents over a complete undirected graph network, each controlling the angular velocity of a rigid body rotating in a weightless environment with air resistance by providing an external torque. The objective is for all agents to synchronise the angular velocities. The dynamics of each agent is represented by the model:

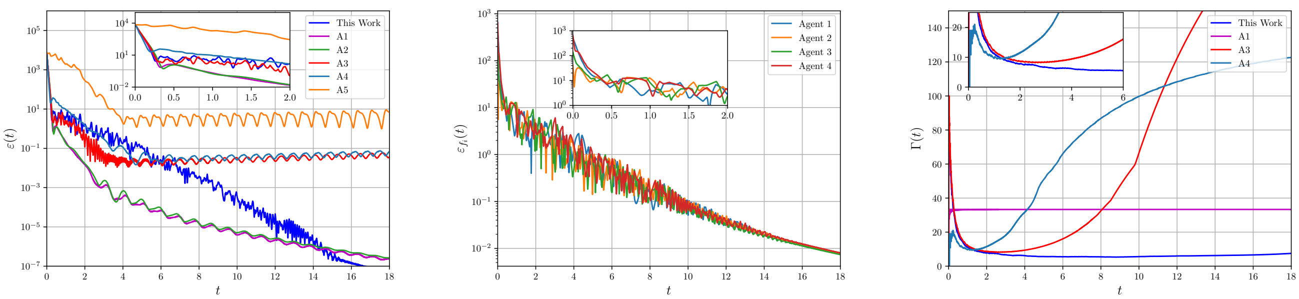
$$\dot{x}_i(t) = \underbrace{\begin{bmatrix} a_i x_i^2(t) x_i^3(t) \\ b_i x_i^1(t) x_i^3(t) \\ c_i x_i^1(t) x_i^2(t) \end{bmatrix}}_{f_i(x_i(t))} + u_i(t)$$

where $x_i^j$ is the $j$-th element of $x_i$ and represents the angular velocty along the $j$-th principle axis. The values of $a_i, b_i$ and $c_i$ are listed in Table I.

TABLE I: Dynamics parameters for each agent

| $i$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $a_i$ | −0.737 | −0.728 | −0.5875 | −0.700 |
| $b_i$ | 0.667 | 0.764 | 0.767 | 0.936 |
| $c_i$ | 0.138 | 0.193 | 0.138 | 0.344 |

The design parameters of the event-triggering algorithm are chosen as follows: $\alpha_i = 21L_{ii}$, $\beta_i = 2.5$, $\epsilon_0 = 10^{-4}$, $\sigma_i = 10^{-2}$, $\eta = 10$. The neural network of each agent has $11^3 = 1331$ neurons. The centres $\mu_i^l$ of the Gaussian basis functions are evenly spaced in $[-25, 30] \times [-25, 20] \times [-25, 30]$ with widths of $\sigma_i^l = 3$. The initial state of the agents are randomly sampled from $\mathcal{U}(-50, 50)$. The simulation was executed with a sampling time of $5 \times 10^{-3}$. The following algorithms are used as a comparison with our algorithm:




(a) Consensus Error $\varepsilon(t)$ (b) Dynamics Estimation Error $\varepsilon_{f_i}(t)$ (c) Average Communication Rate $\Gamma(t)$

Fig. 3: Performance metrics for the proposed adaptive algorithm versus state-of-the-art algorithms

1) Time-Triggered Adaptive Control

$$u_i(t) = -\delta_i(t) - W_i(t)^T \phi_i(x_i(t))$$
$$\gamma_i(t) = \begin{cases} 1, & t = kT \\ 0, & \text{otherwise} \end{cases} \tag{A1}$$

for $k \in \mathbb{N}_0$ and period $T = 0.03$.

2) Non-Triggered Adaptive Control [12]

$$u_i(t) = -\delta_i(t) - W_i(t)^T \phi_i(x_i(t))$$
$$\gamma_i(t) = 1, \quad \forall t \geq 0 \tag{A2}$$

3) Event-Triggered Consensus Control without Adaptive Control [21]

$$u_i(t) = -\delta_i(t)$$
$$\gamma_i(t) = \begin{cases} 1, & \rho_i(t) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{A3}$$

4) Event-Triggered Proportional-Integral Consensus Control without Adaptive Control [24]

$$u_i(t) = -a\delta_i(t) + v_i(t)$$
$$\dot{v}_i(t) = -b\delta_i(t)$$
$$\gamma_i(t) = \begin{cases} 1, & \tilde{\rho}_i(t) \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{A4}$$
$$\tilde{\rho}_i(t) = ||e_i(t)||^2 + c_i \sum_{j=1}^{N} L_{ij} ||\hat{x}_j(t) - \hat{x}_i(t)||^2$$

with $a = b = 1$ and $c_i = 0.24 L_{ii}^{-1}$ for all $i \in \mathcal{V}$.

5) Finite-Time Second-Order Consensus Algorithm without Event-Triggered and Adaptive Control [25]

$$u_i(t) = v_i(t)$$
$$\dot{v}_i(t) = -ag(\delta_i(t) - b\sum_{j=1}^{N}(v_i(t) - v_j(t))) - \frac{1}{b}v_i(t)$$
$$g(x) = [|x_1|^c \text{sign}(x_1), \ldots, |x_n|^c \text{sign}(x_n)]^T$$
$$\gamma_i(t) = 1, \quad \forall t \geq 0 \tag{A5}$$

with $a = 5$, $b = 0.2$ and $c = 0.5$.

Apart from the consensus error $\varepsilon(t)$, we introduce two more evaluation metrics: the local neural network estimation error

$$\varepsilon_{f_i}(t) = ||f_i(x_i(t)) - \hat{f}_i(x_i(t))|| \tag{10}$$

and the average communication rate

$$\Gamma(t) = \frac{1}{Nt}\int_0^t \sum_{i=1}^{N} \gamma_i(t)\, dt, \quad t > 0 \tag{11}$$

with the convention $\Gamma(0) = 0$.

The state evolutions of the agents for the event-triggered algorithms with and without adaptive control are shown in Fig. 2. It can be observed that from Fig. 2a-c that the system failed to reach equilibrium, despite the seemingly practical consensus, while the proposed algorithm successfully stabilised and reached practical consensus with negligible error at $t = 8$ in Fig. 2d-f. The neural network is able to estimate and compensate the unstable nonlinearity in the agent dynamics accurately.

To allow more detailed insights into the simulation results, the overall consensus error $\varepsilon(t)$ is plotted in Fig. 3a in semi-log scale. The multi-agent system with the proposed algorithm converged in $\varepsilon(t)$ asymptotically with exponential rate. It can be more readily observed from Fig. 3a that the systems executing consensus algorithms without adaptive control (A3-A5) were in fact gradually diverging while the proposed algorithm continue to converge within the finite time horizon. The remaining two algorithms also showed exponential convergence in $\varepsilon(t)$ as well. Inspite of the error gap between the proposed algorithm and the time-triggered or the non-triggered variants for $t \leq 15$, the proposed method has a more consistent convergence rate and outperforms the variants at $t = 15$.

The dynamics estimation errors $\varepsilon_{f_i}(t)$ for each agent are plotted in Fig. 3b in semilog scale. The convergence resembles piecewise linear function with noticeable flutuations across all agents. Nonetheless, the fluctuations gradually diminishes as time goes without compromising the convergence rate. The error for all agents diminishes to below $10^{-2}$ for $t \geq 17$ and seem to continue to convergence. This shows that the proposed algorithm is able to learn the unknown system dynamics well.

The average communication rate $\Gamma(t)$ is shown in Fig. 3c. Compared with the standard consensus algorithm without adaptive control, the proposed algorithm in this paper consistently required significantly lower average communication rate for $t \geq 2$. The asymptotic average communication for the proposed algorithm is approximately $79.7\%$ lower than the time-triggered variant.

The simulation results showed that the proposed algorithm is able to learn the unknown nonlinear dynamics of the agent exponentially fast, and is capable of achieving practical consensus with performance comparable with other variants of the learning-based adaptive control law at a significantly lower average communication rate.

## VI. Conclusion and Future Work

In this paper, we presented an adaptive event-triggered control algorithm with radial basis function neural network for the multi-agent consensus problem under unknown nonlinear state-dependent disturbances. It was proved that arbitrarily small consensus error can be achieved asymptotically without Zeno behaviour. The simulation results showed that the algorithm is able to estimate the nonlinear functions $f_i$ well enough to stabilise an otherwise unstable system while reaching consensus.

The most paramount future works include the hyperparameters design of the neural network, particularly on the size of hidden layer, as well as the application on other multi-agent control systems. In addition, experimental evaluation would also be conducive to showing the effectiveness of the proposed algorithm in reality.

## Appendix

### A. Useful Lemmas

**Lemma 2** ([21])**.** *For an undirected graph $\mathcal{G}$,*

$$0 \leq \lambda_2 K_N \otimes I_n \leq L \otimes I_n.$$

**Lemma 3.** *For any matrix $P \in \mathbb{R}^{p\times m}$,*

$$\mathrm{Tr}\left[\phi_i(t)^T P P^T \phi_i(t)\right] \leq p\mathrm{Tr}\left[P^T P\right].$$

*Proof.* From [26], we have

$$\begin{aligned}
&\mathrm{Tr}\left[\phi_i(t)^T P P^T \phi_i(t)\right] \\
&= \mathrm{Tr}\left[P P^T \phi_i(t)\phi_i(t)^T\right] \\
&\leq \lambda_N\left(\phi_i(t)\phi_i(t)^T\right)\mathrm{Tr}\left[P^T P\right] \\
&= \max_{v\in\mathbb{R}^p}\left\{v^T\phi_i(t)\phi_i(t)^T v\colon ||v|| = 1\right\}\mathrm{Tr}\left[P^T P\right] \\
&\leq \max_{v\in\mathbb{R}^p}\left\{\left(\mathbf{1}^T v\right)^2 : ||v|| = 1\right\}\mathrm{Tr}\left[P^T P\right] \\
&= p\mathrm{Tr}\left[P^T P\right].
\end{aligned}$$

The last inequality holds because $\phi_i(t) \in [0,1]^p$. □

### B. Proof of Theorem 1

Let $\tilde{W}_i(t) = W_i(t) - W_i^*$. Consider the following Lyapunov candidate:

$$V(t) = \underbrace{\frac{1}{2}x(t)^T(L\otimes I_n)x(t)}_{V_1(t)} + \underbrace{\frac{1}{2\eta}\sum_{i=1}^N \mathrm{Tr}\left[\tilde{W}_i(t)^T\tilde{W}_i(t)\right]}_{V_2(t)}.$$

Due to the event-triggering law, $x_i(t)$ and $\dot{x}_i(t)$ are not necessarily Lipchitz continuous. However, $x_i(t)$ is still continuous and differentiable while $\dot{x}_i(t)$ is Riemann integrable and piecewise in $t$, albeit possibly discontinuous. Therefore $\dot{V}_1(t)$ is well-defined as follows:

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^N\sum_{j=1}^N L_{ij}x_j(t)^T\dot{x}_i(t) \\
&= \sum_{i=1}^N\sum_{j=1}^N L_{ij}(\hat{x}_j(t) - e_j(t))^T\dot{x}_i(t) \\
&= \sum_{i=1}^N \delta_i(t)^T\Big(W_i^{*T}\phi_i(t) + \epsilon_i(x_i(t)) - \delta_i(t) \\
&\quad - W_i(t)^T\phi_i(t)\Big) - \sum_{i=1}^N\sum_{j=1}^N L_{ij}e_j(t)^T\Big(W_i^{*T}\phi_i(t) \\
&\quad + \epsilon_i(x_i(t)) - \delta_i(t) - W_i(t)^T\phi_i(t)\Big) \\
&\leq \sum_{i=1}^N \delta_i(t)^T\Big(W_i^{*T}\phi_i(t) + \epsilon_i(x_i(t)) - \delta_i(t) \\
&\quad - W_i(t)^T\phi_i(t)\Big) + \sum_{i=1}^N\sum_{j=1}^N L_{ij}e_j(t)^T\delta_i(t) \\
&\quad + \sum_{i=1}^N\sum_{j=1}^N L_{ij}e_j(t)^T\left(\tilde{W}_i(t)^T\phi_i(t)\right) \\
&\quad - \sum_{i=1}^N\sum_{j=1}^N L_{ij}e_j(t)^T\epsilon_i(x_i(t))
\end{aligned}$$

By Young's inequality, for some $\nu_1, \nu_2, \nu_3, \nu_4 > 0$,

$$\begin{aligned}
\dot{V}_1(t) &\leq \sum_{i=1}^N \delta_i(t)^T\Big(W_i^{*T}\phi_i(t) + \epsilon_i(x_i(t)) - \delta_i(t) \\
&\quad - W_i(t)^T\phi_i(t)\Big) + \frac{1}{2\nu_1}\sum_{i=1}^N L_{ii}\delta_i(t)^T\delta_i(t) \\
&\quad + \frac{1}{2\nu_2}\sum_{i=1}^N L_{ii}\phi_i(t)^T\tilde{W}_i(t)\tilde{W}_i(t)^T\phi_i(t) \\
&\quad + \frac{1}{2\nu_3}\sum_{i=1}^N L_{ii}\epsilon_i(x_i(t))^T\epsilon_i(x_i(t)) \\
&\quad + \frac{\nu_1+\nu_2+\nu_3}{2}\sum_{i=1}^N L_{ii}e_i(t)^T e_i(t) \\
&\leq \sum_{i=1}^N \delta_i(t)^T\Big(W_i^{*T}\phi_i(t) + \epsilon_i(x_i(t)) - \delta_i(t)
\end{aligned}$$

$$
\begin{aligned}
&- W_i(t)^T\phi_i(t)\Big) + \frac{1}{2\nu_1}\sum_{i=1}^N L_{ii}\delta_i(t)^T\delta_i(t) \\
&+ \frac{p}{2\nu_2}\sum_{i=1}^N \mathrm{Tr}\left[\tilde{W}_i(t)^T\tilde{W}_i(t)\right] + \frac{\epsilon_f^2\Delta}{2\nu_3} \\
&+ \frac{\nu_1+\nu_2+\nu_3}{2}\sum_{i=1}^N (\alpha_i\exp(-\beta_i t)+\epsilon_0) \\
\le &\sum_{i=1}^N \left(\frac{\Delta}{2\nu_1}+\frac{\nu_4}{2}-1\right)\delta_i(t)^T\delta_i(t) + \frac{p\eta\Delta}{\nu_2}V_2(t) \\
&- \sum_{i=1}^N \delta_i(t)^T\tilde{W}_i(t)^T\phi_i(t) + \left(\frac{\Delta}{2\nu_3}+\frac{1}{2\nu_4}\right)\bar{\epsilon}_f^2 \\
&+ \frac{\nu_1+\nu_2+\nu_3}{2}\sum_{i=1}^N (\alpha_i\exp(-\beta_i t)+\epsilon_0).
\end{aligned}
$$

In addition,

$$
\begin{aligned}
\dot{V}_2(t) =& \frac{1}{\eta}\sum_{i=1}^N \mathrm{Tr}\left[\tilde{W}_i(t)^T\dot{W}_i(t)\right] \\
=& \sum_{i=1}^N \mathrm{Tr}\left[\tilde{W}_i(t)^T\left(\phi_i(t)\delta_i(t)^T - \sigma_i W_i(t)\right)\right] \\
=& \sum_{i=1}^N \left(\delta_i(t)^T\tilde{W}_i(t)^T\phi_i(t) - \sigma_i\mathrm{Tr}\left[\tilde{W}_i(t)^T W_i(t)\right]\right) \\
\le& \sum_{i=1}^N \Big(\delta_i(t)^T\tilde{W}_i(t)^T\phi_i(t) - \frac{\sigma_i}{2}\mathrm{Tr}\left[\tilde{W}_i(t)^T\tilde{W}_i(t)\right] \\
&+ \frac{\sigma_i}{2}\mathrm{Tr}\left[W_i^{*T}W_i^* - W_i(t)^T W_i(t)\right]\Big) \\
\le& \sum_{i=1}^N \Big(\delta_i(t)^T\tilde{W}_i(t)^T\phi_i(t) \\
&+ \frac{\sigma_i}{2}\mathrm{Tr}\left[W_i^{*T}W_i^* - W_i(t)^T W_i(t)\right]\Big) - \eta\sigma_m V_2(t),
\end{aligned}
$$

where $\sigma_m = \min_i \sigma_i$. Combining the analysis above,

$$
\begin{aligned}
\dot{V}(t) \le& -\left(1-\frac{\Delta}{2\nu_1}-\frac{\nu_4}{2}\right)\sum_{i=1}^N \delta_i(t)^T\delta_i(t) \\
&+ \frac{\nu_1+\nu_2+\nu_3}{2}\sum_{i=1}^N (\alpha_i\exp(-\beta_i t)+\epsilon_0) \\
&- \eta\left(\sigma_m - \frac{p\Delta}{\nu_2}\right)V_2(t) + \left(\frac{\Delta}{2\nu_3}+\frac{1}{2\nu_4}\right)\bar{\epsilon}_f^2 \\
&+ \sum_{i=1}^N \frac{\sigma_i}{2}\mathrm{Tr}\left[W_i^{*T}W_i^* - W_i(t)^T W_i(t)\right].
\end{aligned}
$$

There exists a matrix $M \in \mathbb{R}^{N\times N}$ such that $L = M^T\Lambda M$ and $\Lambda = \mathrm{diag}\{\lambda_1,\lambda_2,\dots,\lambda_N\}$. From [12], we have $\sum_{i=1}^N \delta_i(t)^T\delta_i(t) \ge 2V_1(t)/\lambda_N(M^T\bar{\Lambda}^{-1}M)$ where $\bar{\Lambda} = \mathrm{diag}\{\lambda_2,\lambda_2,\lambda_3,\dots,\lambda_N\}$. In addition, $\nu_1,\nu_2,\nu_4$ are chosen such that

$$
\begin{cases}
\dfrac{\Delta}{2\nu_1}+\dfrac{\nu_4}{2} < 1, \\
\nu_2 > \dfrac{p\Delta}{\sigma_m}.
\end{cases}
$$

Then, we can write

$$
\dot{V}(t) \le -\varphi V(t) + \omega(t) \tag{12}
$$

where $\varphi = \min\{\varphi_1,\varphi_2\} > 0$,

$$
\begin{aligned}
\varphi_1 &= \left(1-\frac{\Delta}{2\nu_1}-\frac{\nu_4}{2}\right)\frac{2}{\lambda_N(M^T\bar{\Lambda}^{-1}M)}, \\
\varphi_2 &= \eta\left(\sigma_m - \frac{p\Delta}{\nu_2}\right), \\
\omega(t) &= \frac{\nu_1+\nu_2+\nu_3}{2}\sum_{i=1}^N (\alpha_i\exp(-\beta_i t)+\epsilon_0) \\
&\quad + \sum_{i=1}^N \frac{\sigma_i}{2}\mathrm{Tr}\left[W_i^{*T}W_i^* - W_i(t)^T W_i(t)\right], \\
&\quad + \left(\frac{\Delta}{2\nu_3}+\frac{1}{2\nu_4}\right)\bar{\epsilon}_f^2.
\end{aligned}
$$

Solving the differential inequality (12) yields

$$
V(t) \le V(0)\exp(-\varphi t) + \exp(-\varphi t)\int_0^t \exp(\varphi\tau)\omega(\tau)\,d\tau. \tag{13}
$$

Since we have shown the boundedness of $V(t)$, and thus of $x_i$ and $W_i$, there exists a constant $U > 0$ such that $||u_i(t)|| \le U$. Let $t_k^i$ be the $k$-th triggering time for agent $i$. Consider $t \in [t_k^i, t_{k+1}^i)$ and some $\varsigma > 0$. Then,

$$
\begin{aligned}
\frac{d}{dt}||e_i(t)||^2 &= -2e_i(t)^T u_i(t) \\
&\le \varsigma||e_i(t)||^2 + \varsigma^{-1}||u_i(t)||^2 \\
&\le \varsigma||e_i(t)||^2 + \varsigma^{-1}U^2,
\end{aligned}
$$

so that

$$
\begin{aligned}
||e_i(t)||^2 &\le ||e_i(t_k^i)||^2 e^{\varsigma(t-t_k^i)} + \varsigma^{-1}U^2\int_{t_k^i}^t e^{\varsigma(t-\tau)}\,d\tau \\
&= (\varsigma^{-1}U)^2\left(e^{\varsigma(t-t_k^i)} - 1\right),
\end{aligned}
$$

because $e_i(t_k^i) = 0$ for any $k$. In order for agent $i$ to trigger at $t_{k+1}^i$, a necessary condition is

$$
||e_i(t_{k+1}^{i-})||^2 > \frac{\alpha_i}{L_{ii}}(e^{-\beta_i t_{k+1}^i} + \epsilon_0).
$$

By setting $\varsigma = 1$,

$$
t_{k+1}^i - t_k^i > \ln\left[1 + \frac{\alpha_i}{L_{ii}U^2}\left(e^{-\beta_i t_{k+1}^i} + \epsilon_0\right)\right] > 0,
$$

which means there is a strictly positive lower bound on the inter-communication interval. Therefore, Zeno behaviour is excluded. Following from (13), since $\epsilon(t) \le \frac{2}{N\lambda_2}V_1(t) \le \frac{2}{N\lambda_2}V(t)$,

$$
\begin{aligned}
\lim_{t\to\infty}\varepsilon(t) &\le \limsup_{t\to\infty}\frac{2\omega(t)}{N\lambda_2\varphi} \\
&= \sum_{i=1}^N \frac{\sigma_i}{N\lambda_2\varphi}\mathrm{Tr}\left[W_i^{*T}W_i^* - \bar{W}_i^T\bar{W}_i\right] \\
&\quad + \left(\frac{\nu_4\Delta+\nu_3}{N\lambda_2\nu_3\nu_4\varphi}\right)\bar{\epsilon}_f^2 + \frac{\epsilon_0(\nu_1+\nu_2+\nu_3)}{\lambda_2\varphi},
\end{aligned}
$$

which completes the proof.

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