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Reducing communication and actuation in distributed control systems

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Abstract— **This paper addresses the problem of reducing the number of transmissions and control updates in a distributed control network of interconnected linear systems. Each node in the network decides when to transmit its state through the network and when to update the control law. Both decisions are event-driven and based on local information. It is shown that the stability of the system is preserved and the state of the system converges to a small region around the origin, whose size depends on the parameters of the transmission and control update trigger functions. A strictly positive lower bound for the inter-event times is derived. The results are illustrated through simulations showing the effectiveness of the proposed approach.**

I. Introduction

A distributed Networked Control System (NCS) consists of numerous coupled subsystems (also called agents or nodes), which are geographically distributed and exchange information over a communication network.

Event-triggered policies have been proposed to reduce the number of transmissions [1]-[4] and the need of feedback [5] in control networks. Hence, there is a natural interest in applying these techniques to decentralized NCS since the design of a centralized controller is inappropriate for a large number of subsystems as it requires a very powerful communication network and extremely detailed models of subsystem interconnections to compute the control action.

There are recent contributions on distributed event-triggered control, which basically follow two directions. The first approach assumes sophisticated measurement devices in order to get relative measurements of neighboring nodes and focuses on the design of triggering rules to reduce the number of the actuator updates for a more efficient usage of the limited resources of embedded processors, in which the control task has to share computational and communication resources with other tasks [6], [7]. The second approach tries to reduce the communication between the subsystems [8]-[13]. A node broadcasts the state to its neighbors when the error reaches a certain threshold. How this error and this bound are defined separates the different approaches in the literature, e. g. deadband control [2], Lyapunov approaches to event-based control [5],[14] or self-triggering [15], [16].

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On the one hand, the drawback of the first direction is obvious and lies in the requirement of involved measurement devices to provide the relative information. On the other hand, the second approach might lead to a frequent adaption of the control input specially if the number of neighbors is large. In fact, the control signal is updated whenever a new measurement is received from a neighboring agent.

The importance of reducing the number of control actions in order to save energy has been showed up in recent publications such as [17]-[19]. In [17] a first-order linear stochastic process is sampled periodically and a sporadic controller decides whether to apply a new control action or not based on the cost of control actions. Whereas in [18] and [19] optimization problems are solved in order to not exceed certain limits on the switching rate, and to maximize the time elapsed between two consecutive executions of the control task, respectively. Furthermore, reducing actuation is also important because some actuators are subject to wear. After some time in operation, this wear may result in phenomena that deteriorate the control performance, such as friction or hysteresis in mechanical actuators [20].

In a single control loop the reduction of communication usually imples the reduction of actuator updates [5], [21]. However, this does not necessary hold in distributed systems. In [22] decentralized event-triggering is proposed though the controller design is centralized. To the best of the author's knowledge, both aspects, i.e., reduction of actuation and communication, have not been considered simultaneously in the context of distributed control systems. This is addressed in this paper.

This paper presents a distributed control approach for interconnected linear systems in which the decision of when to transmit the state through the network and when to update the control law are event-driven. Specifically, we propose two sets of trigger functions. The first set detects when the error between the current and the last broadcasted state reaches a certain time-varying threshold, and the second set of trigger functions checks a defined error for the control inputs at broadcasting events. The control law is updated when this error exceeds a given threshold. The proposed design guarantees the convergence of the system to an arbitrary small region around the equilibrium, while reducing the number of control updates in each node. Moreover, the existence of a state independent strictly positive lower bound for the inter-execution time is guaranteed. It is also shown that there exists a trade-off between the design of communication and actuation triggering rules. These results are discussed and illustrated through simulations.

The remainder of the paper is organized as follows:

Section II contains the problem statement of this work. The design of the triggering mechanisms for the communication and the control update is presented in Section III. The performance analysis and a discussion of the results are given in Section IV. Finally, some examples are given in Section V to illustrate the proposed implementation. The paper is concluded with a summary of the results of this paper and potential future works.

II. Problem statement

Consider a system of N linear time-invariant subsystems. The dynamics of each subsystem are given by

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j \in N_i} H_{ij} x_j(t), \qquad (1)$$

$\forall i = 1, ..., N$, where N_i is the set of neighbors of subsystem i, i.e., subsystems which affect its dynamics, and H_{ij} is the interaction term between agents i and j. The state $x_i(t)$ of the ith agent has dimension n_i, $u_i(t)$ is the m_i-dimensional local control signal of agent i, and A_i, B_i and H_{ij} are matrices of appropriate dimensions.

Let us assume that the state $x_i(t)$ is measurable. Each agent i sends its state through the network to its neighbors when an event is triggered. The time instances at which agent i broadcasts its state are denoted by $\{t^i_k\}^{\infty}_{k=0}$, where $t^i_k < t^i_{k+1}$ for all k, and $\hat{x}_i(t)$ is the broadcasted state.

In [11], the control signal was computed based on the broadcasted states as

$$u_i(t) = K_i \hat{x}_i(t^i_k) + \sum_{j \in N_i} L_{ij} \hat{x}_j(t), t \in [t^i_k, t^i_{k+1}), \qquad (2)$$

$\forall i = 1, ..., N$, where K_i is the feedback gain for the nominal subsystem i and L_{ij} is a set of decoupling gains.

Thus, $u(t)$ is a piecewise constant function. Accordingly, the control law of agent i is updated when an event is triggered by itself or by any of its neighbors. This might lead to very frequent control updates if the number of neighbors is large. However, the change of the control signal $u_i(t)$ might be small due to, e. g., a weak coupling. In this situation an update of the control signal is generally not needed.

We propose a new control law in which $u_i(t)$ is not updated at each broadcasting event, but when an additional condition is fulfilled. We consider two mechanisms controlled by events. The first one is the transmission of information between nodes (*transmission events*), and the second one is the update of the control law (*control update events*). The description of the trigger-functions which handle the occurrance of these events is given next.

III. Trigger functions

A. Transmission events

The occurrence of a transmission event is defined by trigger functions $f_{x,i}$ which only depend on local information of agent i and take values in $\mathbb{R}$.

The sequence of broadcasting times t^i_k are determined recursively by the event trigger function as $t^i_{k+1} = \inf\{t : t > t^i_k, f_{x,i}(t, e_{x,i}(t)) > 0\}$.

We define the error between the current state x_i and the latest broadcasted state $\hat{x}_i$ as

$$e_{x,i}(t) = \hat{x}_i(t) - x_i(t), \qquad (3)$$

and we consider time-dependent trigger functions defined by

$$f_{x,i}(t, e_{x,i}(t)) = \|e_{x,i}(t)\| - c_{x,0} - c_{x,1} e^{-\alpha t}, \qquad (4)$$

with $c_{x,0} > 0, c_{x,1} \geq 0$, and $\alpha > 0$. An event is detected when $f_{x,i}(t, e_{x,i}(t)) > 0$, and the error $e_{x,i}$ is reset to zero. Note that the error remains bounded by

$$\|e_{x,i}(t)\| \leq c_{x,0} + c_{x,1} e^{-\alpha t}, \qquad (5)$$

which is a decaying bound with t.

This type of trigger functions has been shown to decrease the number of events while maintaining a good performance of the system [11]. The case $c_{x,0} = 0$ is excluded in this paper. The reason is discussed later. However, the case $c_{x,1} = 0$ is admitted leading to static trigger functions, widely studied in the literature [2], [4].

B. Control update events

Let us denote the time instances at which the control update of the agent i occurs as $\{t^i_l\}^{\infty}_{l=0}, \forall i = 1, \ldots, N$.

The control law is defined for the inter-event time period as

$$\hat{u}_i(t) = K_i \hat{x}_i(t^i_l) + \sum_{j \in N_i} L_{ij} \hat{x}_j(t^i_l), t \in [t^i_l, t^i_{l+1}). \qquad (6)$$

In order to determine the occurrence of an event, we define

$$e_{u,i}(t) = \hat{u}_i(t) - u_i(t), \qquad (7)$$

where $u_i(t)$ is given by (2). The set of trigger functions is given by

$$f_{u,i}(e_{u,i}(t)) = \|e_{u,i}(t)\| - c_u, \quad c_u > 0. \qquad (8)$$

The sequence of control updates is determined recursively. However, whereas the transmission events can occur at any time t because $x_i(t)$ is a continuous function, $u_i(t)$ in (2) is not continuous but piecewise constant and only changes its value at transmission events, that is, the events on the control update are a subsequence of the transmission events.

Denote $\bar{N}_i = i \cup N_i$ and the set of all broadcasting times for the agent i in $\bar{N}_i$ as

$$\{t^{\bar{N}_i}_k\} = \{t^i_k\} \cup \{\{t^j_k\} : j \in N_i\}.$$

Thus, $t^i_{l+1} = \inf\{t^{\bar{N}_i}_k : t^{\bar{N}_i}_k > t^i_l, f_{u,i}(t^{\bar{N}_i}_k) > 0\}$. Hence, it holds $\{t^i_l\} \subset \{t^{\bar{N}_i}_k\}$.

An example of the proposed design is given in Fig. 1. Assume that Agent 1 sends and receives information to/from its neighborhood through a network. At $t = t^2_k$ it receives a broadcasted state $\hat{x}_2$ from Agent 2. Agent 1 computes u_1 according to the new value received. For example, if Agent 2 is its only neighbor, $u_1(t^2_k) = K_1 \hat{x}_1(t^2_k) + L_{12} \hat{x}_2(t^2_k) = K_1 \hat{x}_1(t^1_{k-1}) + L_{12} \hat{x}_2(t^2_k)$, where t^1_{k-1} is assumed to be the last broadcasting event time for Agent 1. After computing u_1, it checks if the difference between this value and the

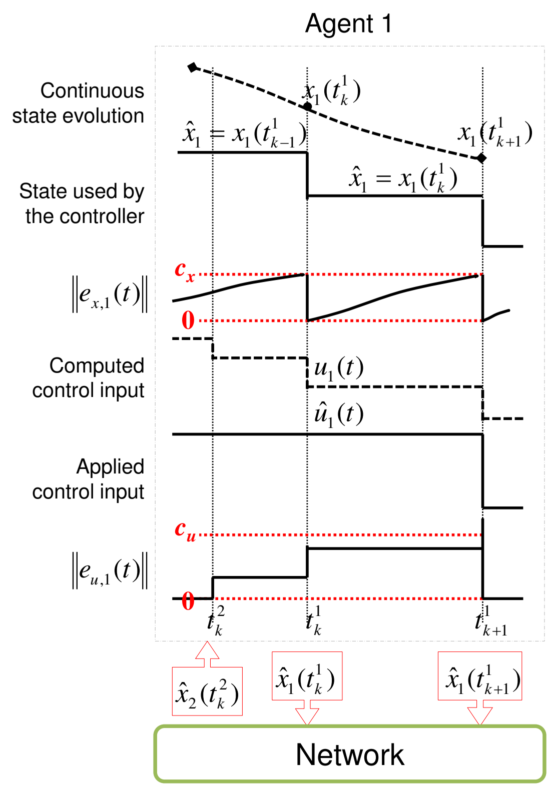

Fig. 1. Illustrative example of transmission and control update events.

current control signal applied exceeds the threshold c_u. Since this threshold is not exceeded, it does not update $\hat{u}_1$. At $t = t_k^1$, Agent 1 detects an event because $e_{x,1}$ reaches the threshold c_x. $x_1(t_k^1)$ is broadcasted through the network and u_1 is computed again. Because $\|e_{u,1}\| < c_u$, $\hat{u}_1$ is not modified. Finally, a new event occurs at $t = t_{k+1}^1$ resulting in a broadcast and a control update since $\|e_{u,1}\| \geq c_u$.

IV. Performance analysis

A. Preliminaries

The dynamics of the subsystems (1) with control law (6) and trigger functions (4) and (8) can be rewritten in terms of $e_{x,i}(t)$ and $e_{u,i}(t)$ as follows

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_i(u_i(t) + e_{u,i}(t)) + \sum_{j\in N_i} H_{ij}x_j(t) \\ &= A_{K,i}x_i(t) + \sum_{j\in N_i} \Delta_{ij}x_j(t) + B_i K_i e_{x,i}(t) \\ &+ B_i \sum_{j\in N_i} L_{ij}e_{x,j}(t) + B_i e_{u,i}(t),\end{aligned}$$

where $A_{K,i} = A_i + B_i K_i$ and $\Delta_{ij} = H_{ij} + B_i L_{ij}$. Note that $A_{K,i}$ is the closed loop matrix of subsystem i, assumed to be Hurwitz, and Δ_{ij} reflects the effect of the uncertainties on the interconnections model.

Let us define the stack vectors

$$x^T(t) = \left(x_1^T(t) \dots x_N^T(t)\right),$$
$$e_x^T(t) = \left(e_{x,1}^T(t) \dots e_{x,N}^T(t)\right)$$
$$e_u(t) = \left(e_{u,1}(t) \dots e_{u,N}(t)\right)^T, \quad (9)$$

whose dimension is $n = \sum_{i=1}^N n_i$, and the matrices

$$A_K = \text{diag}(A_{K,1}, ..., A_{K,N})$$
$$B = \text{diag}(B_1, ..., B_N)$$
$$K = \begin{pmatrix} K_1 & L_{12} & \cdots & L_{1N} \\ L_{21} & K_2 & \cdots & L_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ L_{N1} & L_{N2} & \cdots & K_N \end{pmatrix}$$
$$\Delta = \begin{pmatrix} 0 & \Delta_{12} & \cdots & \Delta_{1N} \\ \Delta_{21} & 0 & \cdots & \Delta_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{N1} & \Delta_{N2} & \cdots & 0 \end{pmatrix}. \quad (10)$$

Note that $H_{ij}, L_{ij}, \Delta_{ij} := 0$ if $j \notin N_i$. Accordingly, the overall system dynamics are given by

$$\dot{x}(t) = (A_K + \Delta)x(t) + BKe_x(t) + Be_u(t). \quad (11)$$

As the broadcasted states $\hat{x}_i$ remain constant between consecutive events, the dynamics of the state error in each interval are given by

$$\dot{e}_x(t) = -(A_K + \Delta)x(t) - BKe_x(t) - Be_u(t). \quad (12)$$

The state error of the overall system is bounded by $\|e_x(t)\| \leq \sqrt{N}(c_{x,0} + c_{x,1}e^{-\alpha t})$, according to (5). However, $e_u(t)$ is not strictly bounded by c_u because $u_i(t)$ is not a continuous function but piecewise constant.

Assumption 1: Simultaneous broadcasting events in any neighborhood $\bar{N}_i$ is not allowed, i.e., two neighboring nodes cannot transmit at the same instance of time.

The previous assumption will serve to establish a bound on the control error, otherwise a more conservative bound would be obtained. Moreover, this assumption seems reasonable from the network protocol perspective. Assumption 1 might induce delays in the case where two nodes want to transmit at the same time. However, we assume that this delay is negligible in this paper. The effect of delays and packet losses on event-triggered control of distributed control systems has been already studied in [23]. Hence, similar results could be inferred assuming that the induced delay is at most the bound derived for the transmission delay in the cited paper.

Moreover, in case that two broadcasted states would be received by one agent, it could enqueue the data and do the computation of the control law one by one.

Lemma 2: If Assumption 1 holds, the control error of the subsystem i is bounded by

$$\|e_{u,i}(t)\| \leq \bar{c}_{u,i}(t), \quad (13)$$

with $\bar{c}_{u,i}(t) = c_u + (c_{x,0} + c_{1,x}e^{-\alpha t}) \cdot \max\{\|K_i\|, \|L_{ij}\| : j \in N_i\}$.

Proof: Assume that the last broadcasting event on the subsystem i occurred at $t = t_k^{\bar{N}_i}$, meaning that its own events and the neighbors' are included. If this last event did not yield a control update it means that $\|e_{u,i}(t_k^{\bar{N}_i})\| < c_u$. Assume that at $t = t_{k+1}^{\bar{N}_i}$ there is a new broadcast in $\bar{N}_i$. There are two possibilities:

- It is the subsystem i which triggers the event. Thus,

$$\begin{aligned}\|e_{u,i}(t_{k+1}^{\bar{N}_i})\| &= \|e_{u,i}(t_k^{\bar{N}_i}) + u_i(t_k^{\bar{N}_i}) - u_i(t_{k+1}^{\bar{N}_i})\| \\ &= \|e_{u,i}(t_k^{\bar{N}_i}) + K_i(\hat{x}_i(t_k^{\bar{N}_i}) - \hat{x}_i(t_{k+1}^{\bar{N}_i}))\| \\ &\leq \|e_{u,i}(t_k^{\bar{N}_i})\| + \|K_i\|\|\hat{x}_i(t_k^{\bar{N}_i}) - \hat{x}_i(t_{k+1}^{\bar{N}_i})\| \\ &\leq c_u + \|K_i\|(c_{x,0} + c_{1,x}e^{-\alpha t_{k+1}^{\bar{N}_i}}).\end{aligned}$$

- If the event has been triggered for any neighbor $j \in N_i$, analogously it yields

$$\begin{aligned}\|e_{u,i}(t_{k+1}^{\bar{N}_i})\| &= \|e_{u,i}(t_k^{\bar{N}_i}) + L_{ij}(\hat{x}_j(t_k^{\bar{N}_i}) - \hat{x}_j(t_{k+1}^{\bar{N}_i}))\| \\ &\leq c_u + \|L_{ij}\|(c_{x,0} + c_{1,x}e^{-\alpha t_{k+1}^{\bar{N}_i}}).\end{aligned}$$

Since this holds for all t and considering the worst case, it yields (13). ■

Lemma 3: If Assumption 1 holds, the control error of the overall system is bounded by

$$\|e_u(t)\| \leq \sqrt{N}(c_u + \|\mu(K)\|_{\max}(c_{x,0} + c_{1,x}e^{-\alpha t})) = \bar{c}_u(t), \tag{14}$$

where

$$\mu(K) = \begin{pmatrix} \|K_1\| & \|L_{12}\| & \cdots & \|L_{1N}\| \\ \|L_{21}\| & \|K_2\| & \cdots & \|L_{2N}\| \\ \vdots & \vdots & \ddots & \vdots \\ \|L_{N1}\| & \|L_{N2} & \cdots & \|K_N\| \end{pmatrix}, \tag{15}$$

and $\|\cdot\|_{\max}$ denotes the entry-wise max norm of a matrix.

Proof: From (9) and (13) it follows that

$$\|e_u(t)\| \leq \sqrt{\sum_{i=1}^{N} \bar{c}_{u,i}^2(t)} \leq \sqrt{N(\max\{\bar{c}_{u,i}(t)\})^2},$$

which is equivalent to (14). ■

Remark 4: Note that even though constant trigger functions are defined to the control update, the effective bound on the control input is time variant due to the trigger mechanism on the state error.

B. Main result

Assumption 5: We assume that $A_{K,i}$, $i = 1, \ldots, N$ is diagonalizable so that the Jordan form of $A_{K,i}$ is diagonal and its elements are the eigenvalues of $A_{K,i}$, $\lambda(A_{K,i})$. This assumption facilitates the calculations, but the extension to general Jordan blocks is straightforward.

Assumption 6: The coupling terms Δ_{ij} are such that $\kappa(V)\|\Delta\| < |\lambda_{max}(A_K)|$, where $\|\cdot\|$ is the induced 2-norm, $\lambda_{max}(A_K) = \max\{\Re e(\lambda) : \lambda \in \lambda(A_K)\}$, and $\kappa(V) = \|V\|\|V^{-1}\|$, being V the matrix of the eigenvectors of A_K.

Theorem 7: Consider the interconnected linear system (11). If trigger functions (4) are defined for the broadcasting with $0 < \alpha < |\lambda_{max}(A_K)| - \kappa(V)\|\Delta\|$, and trigger functions (8) for the control update, then, for all initial conditions $x(0)$ and $t \geq 0$, it follows that

$$\begin{aligned}\|x(t)\| \leq &(\kappa(V)\|x(0)\| - k_1 - k_2)e^{-(|\lambda_{max}(A_K)| - \kappa(V)\|\Delta\|)t} \\ &+ k_1 + k_2 e^{-\alpha t},\end{aligned} \tag{16}$$

where $k_1 = \kappa(V)\sqrt{N}\frac{(\|BK\| + \|B\|\|\mu(K)\|_{\max})c_{x,0} + \|B\|c_u}{|\lambda_{max}(A_K)| - \kappa(V)\|\Delta\|}$, $k_2 = \kappa(V)\sqrt{N}\frac{(\|BK\| + \|B\|\|\mu(K)\|_{\max})c_{x,1}}{|\lambda_{max}(A_K)| - \kappa(V)\|\Delta\| - \alpha}$. Furthermore, the system does not exhibit Zeno behavior.

Proof: The state of the system at any time is given by $x(t) = e^{(A_K + \Delta)t}x(0)$ $+ \int_0^t e^{(A_K + \Delta)(t-s)}(BKe_x(s) + Be_u(s))ds$. The error e_x is bounded by $\sqrt{N}(c_{x,0} + c_{x,1}e^{-\alpha t})$ and the bound on e_u is derived in Lemma 3. Moreover, according to [25], if $\|e^{At}\| \leq ce^{\beta t}$, then $\|e^{(A+E)t}\| \leq ce^{(\beta + c\|E\|)t}$ for given matrices A, E. Thus, $\|e^{(A_K + \Delta)t}\| \leq \kappa(V)e^{(\lambda_{max}(A_K) + \kappa(V)\|\Delta\|)t}$. Because $\lambda_{max}(A_K) < 0$ and from Assumption 6 $|\lambda_{max}(A_K)| - \kappa(V)\|\Delta\| > 0$, it follows $\|e^{(A_K + \kappa(V)\Delta)t}\| \leq \kappa(V)e^{-(|\lambda_{max}(A_K)| - \kappa(V)\|\Delta\|)t}$.

With these considerations, the bound on $x(t)$ can be calculated following the methodology of [11] to derive (16), showing that the system is globally ultimately bounded.

The Zeno behavior exclusion in the broadcasting and, as a consequence, in the control update, can also be proved similar to in [11], resulting in the following lower bound for the inter-event time

$$\tau_x = \frac{c_{x,0}}{\gamma_1 + \sqrt{N}(\gamma_2 + \gamma_3 + \gamma_4)}, \tag{17}$$

where $\gamma_1 = \kappa(V)\|x(0)\|\|A_K + \Delta\|$, $\gamma_2 = \mu_x c_{x,0}\big(1 + \frac{\kappa(V)\|A_K + \Delta\|}{|\lambda_{max}(A_K)| - \kappa(V)\|\Delta\|}\big)$, $\gamma_3 = \mu_x c_{x,1}\big(1 + \frac{\kappa(V)\|A_K + \Delta\|}{|\lambda_{max}(A_K)| - \kappa(V)\|\Delta\| - \alpha}\big)$, $\gamma_4 = \|B\|c_u\big(1 + \frac{\kappa(V)\|A_K + \Delta\|}{|\lambda_{max}(A_K)| - \kappa(V)\|\Delta\|}\big)$, and $\mu_x = \|BK\| + \|B\|\|\mu(K)\|_{\max}$. ■

Note that (17) is strictly positive since $c_{x,0} > 0$.

C. Discussion

The previous analysis is based on two sets of trigger functions to detect transmission and control updates events. One concern that can be raised is how the values of the parameters of these trigger functions can be selected or if there is any relation between them.

Let us first assume the case $c_{1,x} = 0$ yielding to static trigger functions. It follows that $\|e_{x,i}\| \leq c_{x,0}$ and $\|e_{u,i}\| \leq c_u + c_{x,0} \cdot \max\{\|K_i\|, \|L_{ij}\| : j \in N_i\}$, according to (5) and (13), respectively. Assume that the last control update event occurred at $t = t^*$ and we denote the number of transmission events between t^* and the next control execution for the agent i as n_e^i. A lower bound for n_e^i can be derived following the ideas of Lemma 2. Because $\|e_{u,i}(t) - e_{u,i}(t^*)\| = \|e_{u,i}(t)\| \leq \sum_{k=1}^{n_e} \max\{\|K_i\|, \|L_{ij}\| : j \in N_i\} \cdot c_{x,0} = \max\{\|K_i\|, \|L_{ij}\| : j \in N_i\} \cdot n_e c_{x,0}$ and the next control update event will not be triggered before $\|e_{u,i}\| = c_u \leq c_u + c_{x,0} \cdot \max\{\|K_i\|, \|L_{ij}\| : j \in N_i\}$. Thus,

$$n_e^i \geq \frac{c_u}{\max\{\|K_i\|, \|L_{ij}\| : j \in N_i\} \cdot c_{x,0}}. \tag{18}$$

Equation (18) shows the trade-off between c_u and $c_{x,0}$ and gives an idea on how one of this parameters should be chosen according to the other one. Moreover, (18) can be translated into a relationship between the inter-execution times of the control law (6), denoted τ_u^i, and the minimum broadcasting period (17). It holds that $\tau_u^i \geq n_e^i \tau_x \geq \frac{c_u}{\max\{\|K_i\|,\|L_{ij}\|:j\in N_i\}\cdot(\gamma_1+\sqrt{N}(\gamma_2+\gamma_4))}$.

Note that $\gamma_3 = 0$ because we are analyzing the case $c_{x,1} = 0$. Let τ_u be $\tau_u = \min\{\tau_u^i\}$. It yileds $\tau_u \geq \frac{c_u}{\|\mu(K)\|_{\max}(\gamma_1+\sqrt{N}(\gamma_2+\gamma_4))}$. Hence, $c_{x,0}$ and c_u can be chosen to meet some constraints on τ_x and τ_u.

In the design of Section III the case $c_{x,0} = 0$ was excluded and the reason is given next. Assume that $c_{x,0} = 0$. Thus, following a similar procedure as in the previous case, $\|e_{u,i}(t)\| \leq \max\{\|K_i\|,\|L_{ij}\| : j \in N_i\}\cdot n_e c_{x,1} e^{-\alpha t^*}$, where n_e is the number of broadcasting events and t^* the time of the last control update event. Moreover, the next event is not triggered before $\|e_{u,i}\|$ reaches the threshold c_u. In this case, it holds that

$$n_e \geq \frac{c_u}{\max\{\|K_i\|,\|L_{ij}\|:j\in N_i\}\cdot c_{x,1} e^{-\alpha t^*}}. \quad (19)$$

Note that the lower bound for n_e in (19) goes to infinity when $t^* \to \infty$, which means that when the values of times are large, many transmission events are required to trigger a new control update and may lead to small inter-event times. One possible solution is to accomodate the threshold c_u to the decreasing bound on the state $c_{x,1}e^{-\alpha t}$.

V. Simulation results

A. System description

The system considered is a collection of $N \times N$ inverted pendulums of mass m and length l coupled by springs with rate k. The topology of the system is depicted in Fig. 2. Each

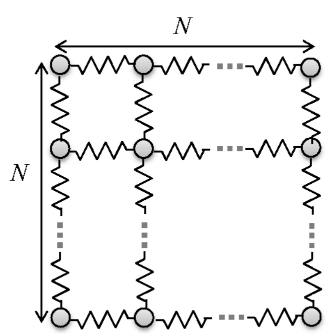

Fig. 2. Scheme of the coupled pendulums mesh.

subsystem can be described as

$$\dot{x}_i = \begin{pmatrix} A_i & \mathbf{0} \\ \mathbf{0} & A_i \end{pmatrix} x_i + \begin{pmatrix} B_i & \mathbf{0} \\ \mathbf{0} & B_i \end{pmatrix} u_i + \sum_{j\in N_i} \begin{pmatrix} H_{ij} & \mathbf{0} \\ \mathbf{0} & H_{ij} \end{pmatrix} x_j,$$

where $A_i = \begin{pmatrix} 0 & 1; & \frac{g}{l} - \frac{|N_i|k}{ml^2} & 0 \end{pmatrix}$, $B_i = \begin{pmatrix} 0; & \frac{1}{ml^2} \end{pmatrix}$, $H_{ij} = \begin{pmatrix} 0 & 0; & \frac{k}{ml^2} & 0 \end{pmatrix}$, $x_i = \begin{pmatrix} x_{i_1} & x_{i_2} & x_{i_3} & x_{i_4} \end{pmatrix}^T$ and $u_i = \begin{pmatrix} u_{i_1} & u_{i_2} \end{pmatrix}^T$.

The feedback gains K_i are designed to place the poles at $\{-2,-2,-1,-1\}$. The decoupling gains are designed to decouple the system with uncertainties $\|\Delta_{ij}\| < 0.35\frac{|\lambda_{max}(A_{K,i})|}{\kappa(V)}$.

B. Performance

Fig. 3 shows the output of the system in a 3D space for a mesh of 6×6 pendulums. The coordinates in the XY plane over time are plotted. Trigger functions with $c_{x,0} = 0.02$, $c_{x,1} = 0.5$, $\alpha = 0.6$ and $c_u = 0.1$ are considered.

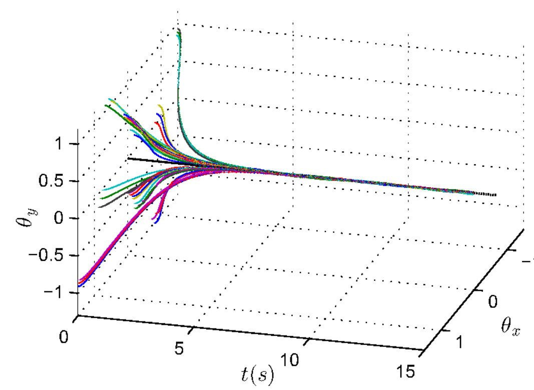

Fig. 3. x_{i_1} for a 6×6 mesh.

Let us focus on one particular subsystem, for example the agent (2,2) (second row, second column). The state and the control signals are illustrated in Fig. 4. The number of broadcasting events in all the neighborhood of this particular agent, which has four neighbors, is 170, while the number of control updates in the agent (2,2) is 90, so that 47% of the transmissions do not end into a control update because the threshold c_u is not reached.

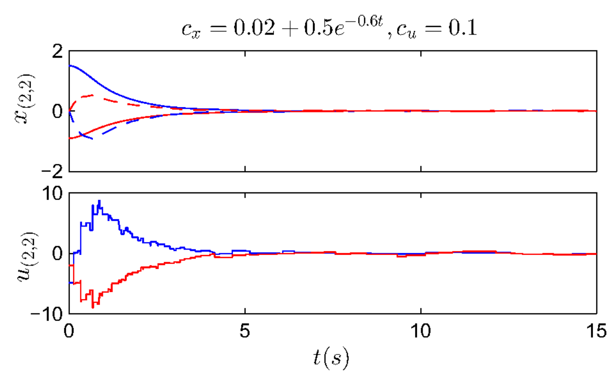

Fig. 4. State and control signals for agent (2,2).

If this experiment is repeated for the case in which trigger functions (8) are not considered, the number of broadcasting events in the neighborhood of (2,2) is 140, which is equal to the number of control updates. Thus, the proposed design with trigger functions (8) as expected might cause an increase of network transmissions, in this case 21% while saving almost half of the changes on the control signal. Moreover, if we compute the average broadcasting period for the entire

TABLE I

AVERAGE BROADCASTING PERIOD VARIATIONS WITH N.

$N \times N$	16	36	64	81	100
$\bar{\tau}_{x,k}$	0.5422	0.5202	0.4813	0.4676	0.4765

TABLE II

AVERAGE TRANSMISSION AND CONTROL UPDATE EVENTS WITH c_u.

c_u	0.02	0.05	0.1	0.2
$\bar{n}_x$	86.20	83.98	95.46	181.48
$\bar{n}_u$	93.11	75.00	67.28	57.58

network as $\frac{N^2 t_{sim}}{\text{No. events}}$ it yields 0.5202 s for the first case and 0.5954 s for the case without using the event-triggered control update. Hence, for the overall network the difference is not relevant. These results are extended for different values of N in Table I. Note that the variations of the average period with the number of agents are not significant.

The influence of the paramter c_u for given parameters $c_{x,0} = 0.02, c_{x,1} = 0.5$ and $\alpha = 0.6$ can be analyzed and the results are illustrated in Table II. For a mesh of 6×6 subsystems the following values are computed for each value of c_u and simulation time $t = 15$ s:

- Average number of transmissions through the network defined as $\bar{n}_x = \frac{\sum_{i=1}^{N^2} |\{t_k^i\}|}{N^2} |\overline{N_i}|$, where $|\{t_k^i\}|$ is the cardinality of the set $\{t_k^i\}$ and $|\overline{N_i}|$ is the average for the number of neighboring agents.
- Average number of control updates defined as $\bar{n}_u = \frac{\sum_{i=1}^{N^2} |\{t_l^i\}|}{N^2}$.

Note that the best choice of the values of $c_u, c_{x,0}$ and $c_{x,1}$ depends on the communication and actuation costs of the implementation, and the lower bounds on the inter-event times that should be guaranteed in the system. We can say that a value $c_u \in [0.05, 0.1]$ would be a good option because the decrease of the control events is notable while the increase in communication events is assumable. If $c_u = 0.02$ all broadcasting events lead into a control update ($\bar{n}_u$ is actually larger than $\bar{n}_x$, but this is due to the error induced by the statistical treatment of the data).

VI. CONCLUSIONS AND FUTURE WORK

This paper has presented a framework to reduce broadcasts and control updates in a distributed control network. Two sets of trigger functions have been proposed. The system is shown to be globally ultimately bounded and converges to a region which depends on the parameters of the transmission and control update trigger functions. A lower bound for the inter-event time has been derived excluding Zeno behavior. The theoretical results have been illustrated through simulations showing the trade-off between the design parameters.

The proposed design of constant thresholds in the control update mechanisms miss the property of asymptotic stability of the overall system that can be achieved with pure exponential trigger functions as described in [11]. One interesting direction for future work is the design of control update trigger functions which preserve this property.

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