Proceedings of the 36th Chinese Control Conference
July 26-28, 2017, Dalian, China

Event-Triggered Control for Multi-Agent Systems with Output Saturation

Xinlei Yi[1], Tao Yang[2], Junfeng Wu[1], Karl H. Johansson[1]

1. The ACCESS Linnaeus Centre, Electrical Engineering, KTH Royal Institute of Technology, 100 44, Stockholm, Sweden
E-mail: {xinleiy, junfengw, kallej}@kth.se

2. The Department of Electrical Engineering, University of North Texas, Denton, TX 76203 USA
E-mail: Tao.Yang@unt.edu

Abstract: We propose distributed static and dynamic event-triggered control laws to solve the consensus problem for multi-agent systems with output saturation. Under the condition that the underlying graph is undirected and connected, we show that consensus is achieved under both event-triggered control laws if and only if the average of the initial states is within the saturation level. Numerical simulations are provided to illustrate the effectiveness of the theoretical results and to show that the control laws lead to reduced need for inter-agent communications.

Key Words: Consensus, Event-triggered control, Multi-agent systems, Output saturation

1 Introduction

Due to its wide applicability, consensus in multi-agent systems has been widely investigated. The basic method is to use a distributed consensus protocol. Specifically, each agent updates its state based on its own and the states of its neighbors in such a way that the final states of all agents converge to a common value, e.g., [1–4]. However, real systems are subject to physical constraints such as input, output, digital communication channels, and sensors constraints. These constraints lead to nonlinearity in the closed-loop dynamics. Thus the behavior of each agent is affected and special attention to these constraints needs to be taken in order to understand their influence on the convergence properties. Here we list some representative examples of such constraints. For example, [5] studies global consensus for discrete-time multi-agent systems with input saturation constraint; [6] considers the leader-following consensus problem for multi-agent systems subject to input saturation; [7] and [8] investigate necessary and sufficient initial conditions for achieving consensus in the presence of output saturation.

In the aforementioned work, the agent updates its state based on the continuous communication with its neighboring agents. However, it may be impractical to require continuous communication in physical applications, as agents can be equipped with embedded microprocessors with limited resources to transmit and collect information. Event-triggered control was introduced partially to tackle this problem [9–12]. The control in event-triggered control signal is often piecewise constant between the triggering times. The triggering times are determined implicitly by the event conditions to ensure the stability of the closed-loop system. Many researchers studied event-triggered control for multi-agent systems recently, e.g., [13–22]. The key points in event-triggered control for multi-agent systems are how to design the event-triggered control law and the threshold to determine the corresponding triggering times and how to exclude Zeno behavior. For continuous-time multi-agent systems, Zeno behavior is that there are infinite number of triggers in a finite time interval [23]. Another important point is how to realize the event-triggered controller in a distributed way since on the one hand for the centralized controller, it normally requires a large amount of agents take action in a synchronous manner and it is not efficient, on the other hand for some decentralized controller, it requires a priori knowledge of some global network parameters, for example, in [15] the smallest positive eigenvalue of the Laplacian matrix needs to be known in advance.

In this paper, we first consider consensus for static event-triggered control of multi-agent systems with output saturation. Then, we study dynamic event-triggered control law, which is a multi-agent extension of an event-triggered mechanism proposed in [24]. The contributions of this paper are two-fold: 1) we show that consensus is achieved under both static and dynamic event-triggered control laws, which are distributed in the sense that they do not require any a priori knowledge of global network parameters; and 2) we show that the dynamic event-triggered control law is free from Zeno behavior.

The rest of this paper is organized as follows. Section 2 introduces the preliminaries and the problem formulation. The main results are stated in Section 3. Simulations are given in Section 4. Finally, the paper is concluded in Section 5.

Notations: $\|\cdot\|$ represents the Euclidean norm for vectors or the induced 2-norm for matrices. $\mathbf{1}_n$ denotes the column vector with each component being 1 and dimension n.

2 Preliminaries

In this section, we present some definitions from algebraic graph theory [25] and the formulation of the problem.

2.1 Algebraic Graph Theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ denote a (weighted) undirected graph with the set of agents (vertices or nodes) $\mathcal{V} = \{v_1, \dots, v_n\}$, the set of links (edges) $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and the (weighted) adjacency matrix $A = A^\top = (a_{ij})$ with nonnegative elements a_{ij}. A link $(v_i, v_j) \in \mathcal{E}$ if $a_{ij} > 0$, i.e., if agent v_i and v_j can communicate with each other. It is assumed that $a_{ii} = 0$ for all $i \in \mathcal{I}$, where $\mathcal{I} = \{1, \dots, n\}$. Let $\mathcal{N}_i = \{j \in \mathcal{I} \mid$

This work is supported by the Knut and Alice Wallenberg Foundation, the Swedish Foundation for Strategic Research, and the Swedish Research Council.

$a_{ij} > 0\}$ and $\deg_i = \sum_{j=1}^{n} a_{ij}$ denotes the neighbors' index set and (weighted) degree of agent v_i, respectively. The degree matrix of graph $\mathcal{G}$ is $Deg = \mathrm{diag}([\deg_1, \cdots, \deg_n])$. The Laplacian matrix is $L = (L_{ij}) = Deg - A$. A path of length k between agent v_i and agent v_j is a subgraph with distinct agents $v_{i_0} = v_i, \dots, v_{i_k} = v_j \in \mathcal{V}$ and edges $(v_{i_j}, v_{i_{j+1}}) \in \mathcal{E},\ j = 0, \dots, k-1$. An undirected graph is connected if there exists at least one path between any two distinct agents.

For connected graphs we have the following well known result [25].

Lemma 1 *If an undirected graph $\mathcal{G}$ is connected, then its Laplacian matrix L has a simple eigenvalue at zero with corresponding eigenvector $\mathbf{1}_n$ and all other eigenvalues are real and strictly positive.*

2.2 Problem Formulation

We consider a set of n agents that are modeled as a single integrator with output saturation:

$$\begin{cases} \dot{x}_i(t) = u_i(t) \\ y_i(t) = \mathrm{sat}_h(x_i(t)) \end{cases}, \ i \in \mathcal{I}, t \geq 0, \tag{1}$$

where $x_i(t) \in \mathbb{R}$ is the state, $u_i(t) \in \mathbb{R}$ is the control input, and $y_i(t)$ is the measured output. The saturation function $\mathrm{sat}_h : \mathbb{R} \to \mathbb{R}$ is defined as

$$\mathrm{sat}_h(s) = \begin{cases} h, & \text{if } s \geq h \\ s, & \text{if } |s| < h \\ -h, & \text{if } s \leq -h \end{cases}, \tag{2}$$

where h is a positive constant, and the interval $[-h, h]$ is referred to as the saturation level. For simplicity, for vector $z = [z_1, \dots, z_n]^\top \in \mathbb{R}^n$, we still use the notation $\mathrm{sat}_h(z)$ and define $\mathrm{sat}_h(z) = [\mathrm{sat}_h(z_1), \dots, \mathrm{sat}_h(z_n)]^\top$.

Remark 1 *For the ease of presentation, we study the case where all the agents have the same saturation level and scalar states. However, the analysis in this paper can be easily extended the cases where the agents have different saturation levels and have vector-valued states.*

In the literature, the following distributed consensus protocol is often considered, e.g., [7],

$$u_i(t) = -\sum_{j=1}^{n} L_{ij} y_j(t). \tag{3}$$

To implement (3), continuous-time outputs from all neighbours are needed. However, it is often unpractical to require continuous communication in physical applications. Moreover, if at some time t_0, $|x_i(t_0)| > h$, then due to the continuity of $x_i(t)$, there exists $t_1 > t_0$ such that $|x_i(t)| \geq h, \forall t \in [t_0, t_1]$. Then $y_i(t) = \mathrm{sat}_h(x_i(t))$ is a constant during $[t_0, t_1]$. So it is also waste to continuously transmit $y_i(t)$ during $[t_0, t_1]$ since no new information is provided.

Inspired by the idea of event-triggered control for multi-agent systems [13], instead of (3) we use the following event-triggered control

$$u_i(t) = -\sum_{j=1}^{n} L_{ij} y_j(t^j_{k_j(t)}) = -\sum_{j=1}^{n} L_{ij}\, \mathrm{sat}_h(x_j(t^j_{k_j(t)})), \tag{4}$$

where $k_j(t) = \mathrm{argmax}_k\{t^j_k \leq t\}$ with the increasing $\{t^j_k\}_{k=1}^{\infty}$, $j \in \mathcal{I}$ to be determined later. We assume $t^j_1 = 0, j \in \mathcal{I}$. Note that the control protocol (4) only updates at the triggering times and is constant between consecutive triggering times.

For simplicity, let $x(t) = [x_1(t), \dots, x_n(t)]^\top$, $\hat{x}_i(t) = x_i(t^i_{k_i(t)})$, $\hat{x}(t) = [\hat{x}_1(t), \dots, \hat{x}_n(t)]^\top$, $e_i(t) = \mathrm{sat}_h(\hat{x}_i(t)) - \mathrm{sat}_h(x_i(t))$, and $e(t) = [e_1(t), \cdots, e_n(t)]^\top = \mathrm{sat}_h(\hat{x}(t)) - \mathrm{sat}_h(x(t))$.

3 Event-Triggered Control Law

In this section, we propose the static and dynamic event-triggered control laws to determine the triggering time sequence and we show that consensus is achieved under both event-trigger control laws.

3.1 Static Event-Triggered Control Law

We first give some lemmas which will be used later.

Lemma 2 *Consider the multi-agent system (1) with the event-triggered control protocol (4). The average of all agents' states $\bar{x}(t) = \frac{1}{n}\sum_{i=1}^{n} x_i(t)$ is a constant, i.e., $\bar{x}(t) = \bar{x}(0)$ for all $t \geq 0$.*

Proof: It follows from (1) and (4) that the time derivative of the average value is given by

$$\begin{aligned} \dot{\bar{x}}(t) =& \frac{1}{n}\sum_{i=1}^{n} \dot{x}_i(t) = -\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} L_{ij}\, \mathrm{sat}_h(x_j(t^j_{k_j(t)})) \\ =& -\frac{1}{n}\sum_{j=1}^{n} \mathrm{sat}_h(x_j(t^j_{k_j(t)})) \sum_{i=1}^{n} L_{ij} = 0. \end{aligned}$$

Lemma 3 *(Lemma 3.2 in [7].) For any constants a, b with $|a| \leq h$,*

$$\int_a^b [\mathrm{sat}_h(s) - a] ds \geq 0,$$

and the equality holds if and only if $a = b$ or $b \geq a = h$ or $b \leq a = -h$.

Assume that $|\bar{x}(0)| \leq h$. Consider the Lyapunov candidate

$$V(x) = \sum_{i=1}^{n} \int_{\bar{x}(0)}^{x_i} [\mathrm{sat}_h(s) - \bar{x}(0)] ds. \tag{5}$$

From Lemma 3, we know that $V(t) \geq 0$ due that $|\bar{x}(0)| \leq h$. The derivative of $V(x)$ along the trajectories of system (1) with the event-triggered control (4) is

$$\begin{aligned}
\dot V(x) =& \sum_{i=1}^{n}[\mathrm{sat}_h(x_i(t)) - \bar x(0)]\dot x_i(t)\\
=& \sum_{i=1}^{n}\mathrm{sat}_h(x_i(t))\dot x_i(t) - \sum_{i=1}^{n}\bar x(0)\dot x_i(t)\\
=& \sum_{i=1}^{n}\mathrm{sat}_h(x_i(t))\dot x_i(t) + \sum_{i=1}^{n}\bar x(0)\sum_{j=1}^{n}L_{ij}\,\mathrm{sat}_h(\hat x_j(t))\\
=& \sum_{i=1}^{n}\mathrm{sat}_h(x_i(t))\dot x_i(t) + \sum_{j=1}^{n}\bar x(0)\,\mathrm{sat}_h(\hat x_j(t))\sum_{i=1}^{n}L_{ij}\\
=& \sum_{i=1}^{n}[\mathrm{sat}_h(\hat x_i(t)) - e_i(t)]\sum_{j=1}^{n}-L_{ij}\,\mathrm{sat}_h(\hat x_j(t))\\
=& -\sum_{i=1}^{n}\mathrm{sat}_h(\hat x_i(t))\sum_{j=1}^{n}L_{ij}\,\mathrm{sat}_h(\hat x_j(t))\\
&+\sum_{i=1}^{n}e_i(t)\sum_{j=1}^{n}L_{ij}\,\mathrm{sat}_h(\hat x_j(t))\\
=& \sum_{i=1}^{n}-q_i(t)\\
&+\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}L_{ij}e_i(t)[\mathrm{sat}_h(\hat x_j(t)) - \mathrm{sat}_h(\hat x_i(t))]\\
\le& \sum_{i=1}^{n}-q_i(t) - \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}L_{ij}e_i^2(t)\\
&-\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}L_{ij}\frac{1}{4}[\mathrm{sat}_h(\hat x_j(t)) - \mathrm{sat}_h(\hat x_i(t))]^2\\
=& \sum_{i=1}^{n}-\frac{1}{2}q_i(t) + \sum_{i=1}^{n}L_{ii}e_i^2(t),
\end{aligned} \tag{6}$$

where

$$q_i(t) = -\frac{1}{2}\sum_{j=1}^{n}L_{ij}[\mathrm{sat}_h(\hat x_j(t)) - \mathrm{sat}_h(\hat x_i(t))]^2 \ge 0, \tag{7}$$

and the last two equalities hold since

$$\begin{aligned}
\sum_{i=1}^{n}q_i(t) =& -\sum_{i=1}^{n}\frac{1}{2}\sum_{j=1}^{n}L_{ij}[\mathrm{sat}_h(\hat x_j(t)) - \mathrm{sat}_h(\hat x_i(t))]^2\\
=& \sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{sat}_h(\hat x_i(t))L_{ij}\,\mathrm{sat}_h(\hat x_j(t))\\
=& [\mathrm{sat}_h(\hat x(t))]^\top L\,\mathrm{sat}_h(\hat x(t)),
\end{aligned}$$

and the inequality holds because $ab \le a^2 + \frac{1}{4}b^2$ for all $a, b \in \mathbb{R}$.

Our first main result follows from the above discussion.

Theorem 1 *Consider the multi-agent system (1) with the even-triggered control protocol (4). Suppose that the underlying graph $\mathcal{G}$ is undirected and connected. Suppose $x(0) \neq \bar x(0)\mathbf{1}_n$. Given a constant $\sigma_i \in (0, 1)$ and the first triggering time $t_1^i = 0$, every agent v_i determines its triggering time sequence $\{t_k^i\}_{k=2}^{\infty}$ by*

$$t_{k+1}^i = \max_{r\ge t_k^i}\Big\{r : e_i^2(t) \le \frac{\sigma_i}{2L_{ii}}q_i(t), \forall t \in [t_k^i, r]\Big\}, \tag{8}$$

with $q_i(t)$ defined in (7). Then consensus is achieved if and only if

$$|\bar x(0)| \le h.$$

Proof: (Necessity) This part of the proof is a special case of the proof of Theorem 3.1 in [7] with some minor modifications. We thus omit the proof here.

(Sufficiency) If $|\bar x(0)| \le h$, then from Lemma 3 we know that $V(t) \ge 0$ and $V(t) = 0$ if and only if $x_i(t) = \bar x(0), i \in \mathcal{V}$. From (6) and (8), we have

$$\begin{aligned}
\dot V(x) \le& \sum_{i=1}^{n}-\frac{1}{2}q_i(t) + \sum_{i=1}^{n}L_{ii}e_i^2(t)\\
\le& \frac{1}{2}(1-\sigma_{\max})\sum_{i=1}^{n}-q_i(t)\\
=& -\frac{1}{2}(1-\sigma_{\max})[\mathrm{sat}_h(\hat x(t))]^\top L\,\mathrm{sat}_h(\hat x(t)) \le 0,
\end{aligned} \tag{9}$$

where $\sigma_{\max} = \max\{\sigma_1, \dots, \sigma_n\} < 1$. Note that

$$\begin{aligned}
&[\mathrm{sat}_h(x(t))]^\top L\,\mathrm{sat}_h(x(t))\\
=&[\mathrm{sat}_h(\hat x(t)) + e(t)]^\top L[\mathrm{sat}_h(\hat x(t)) + e(t)]\\
\le&2[\mathrm{sat}_h(\hat x(t))]^\top L\,\mathrm{sat}_h(\hat x(t)) + 2e^\top(t)Le(t)\\
\le&2[\mathrm{sat}_h(\hat x(t))]^\top L\,\mathrm{sat}_h(\hat x(t)) + 2\|L\|\|e(t)\|^2\\
\le&2[\mathrm{sat}_h(\hat x(t))]^\top L\,\mathrm{sat}_h(\hat x(t)) + \frac{\|L\|\sigma_{\max}}{\min_i L_{ii}}\sum_{i=1}^{n}q_i(t)\\
=&\Big(2 + \frac{\|L\|\sigma_{\max}}{\min_i L_{ii}}\Big)[\mathrm{sat}_h(\hat x(t))]^\top L\,\mathrm{sat}_h(\hat x(t)),
\end{aligned} \tag{10}$$

where the first inequality holds since L is positive semi-definite and $[a+b]^\top L[a+b] \le 2a^\top La + 2b^\top Lb, \forall a, b \in \mathbb{R}^n$, the second inequality holds because $a^\top La \le \|L\|\|a\|^2, \forall a \in \mathbb{R}^n$, and the last inequality holds due to (8). Then we have

$$\begin{aligned}
\dot V(x) \le& -\frac{(1-\sigma_{\max})\min_i L_{ii}}{4\min_i L_{ii} + 2\|L\|\sigma_{\max}}[\mathrm{sat}_h(x(t))]^\top L\,\mathrm{sat}_h(x(t))\\
\le& 0.
\end{aligned}$$

Moreover we have $\dot V(t) = 0$ if and only if $\mathrm{sat}_h(x(t)) = a\mathbf{1}_n$ for some $a \in \mathbb{R}$. This is equivalent to $x(t) = a\mathbf{1}_n$ for some $a \in \mathbb{R}$, otherwise $x(t) \ge h\mathbf{1}_n$ and $x(t) \neq h\mathbf{1}_n$ or $x(t) \le -h\mathbf{1}_n$ and $x(t) \neq -h\mathbf{1}_n$, however, both cases contradict $|\bar x(t)| \le h$. Then by LaSalle Invariance Principle [26] $\lim_{t\to+\infty} x_i(t) = \bar x(0)$, for all $i \in \mathcal{I}$.

Remark 2 *The control law (8) is referred to as a static static triggering law since it depends only on the current state of the considered agent. The event-triggered control law is distributed since each agent's control action only depends on its neighbours' state information, without any prior knowledge of global parameters, such as the eigenvalue of the Laplacian matrix.*

The event-triggered control law reduces the overall communication among agents. It is essential to exclude Zeno behavior [23]. However, we do not know whether Zeno behavior can be excluded or not in the above event-triggered control law. In order to exclude Zeno behavior, in the next section, we propose a dynamic event-triggered control law.

3.2 Dynamic Event-triggered Control Law

Inspired by [24], we introduce an internal dynamic variable η_i to agent v_i:

$$\dot{\eta}_i(t) = -\beta_i \eta_i(t) + \frac{\sigma_i}{2} q_i(t) - L_{ii} e_i^2(t), \tag{11}$$

with $\eta_i(0) > 0$, $\beta_i > 0$ and $\sigma_i \in (0, 1)$. This dynamic variable is correlated in the event-triggered rule, as defined in our second main result.

Theorem 2 *Consider the multi-agent system (1) with the even-triggered control protocol (4). Suppose that the underlying graph $\mathcal{G}$ is undirected and connected. Suppose $x(0) \neq \bar{x}(0)\mathbf{1}_n$. Given a constant $\theta_i > 0$ and the first triggering time $t_1^i = 0$, agent v_i determines its triggering time sequence $\{t_k^i\}_{k=2}^{\infty}$ by*

$$t_{k+1}^i = \max_{r \geq t_k^i} \Big\{ r : \theta_i \Big(L_{ii} e_i^2(t) - \frac{\sigma_i}{2} q_i(t) \Big) \leq \eta_i(t), \quad \forall t \in [t_k^i, r] \Big\}, \tag{12}$$

with $q_i(t)$ defined in (7) and $\eta_i(t)$ defined in (11). Then (i) there is no Zeno behavior; (ii) consensus is achieved if and only if

$$|\bar{x}(0)| \leq h.$$

Proof: (i) From equation (11) and condition (12), we have

$$\dot{\eta}_i(t) \geq -\beta_i \eta_i(t) - \frac{1}{\theta_i} \eta_i(t).$$

Thus

$$\eta_i(t) \geq \eta_i(0) e^{-(\beta_i + \frac{1}{\theta_i})t} > 0.$$

Next, we show that there is no Zeno behavior by contradiction. Suppose that there exists Zeno behavior, then there exists an agent v_i, such that $\lim_{k\to\infty} t_k^i = T_0$, where T_0 is a positive constant. Let $\varepsilon_0 = \frac{1}{4\sqrt{\theta_i L_{ii}^3} h} e^{-\frac{1}{2}(\beta_i + \frac{1}{\theta_i})T_0} > 0$. Then, there exists a positive integer $N(\varepsilon_0)$ such that

$$t_k^i \in [T_0 - \varepsilon_0, T_0], \ \forall k \geq N(\varepsilon_0). \tag{13}$$

Since $|\operatorname{sat}_h(s)| \leq h$, we have $|u_i(t)| \leq 2hL_{ii}$. From $q_i(t) \geq 0$ and $|\operatorname{sat}_h(s_1) - \operatorname{sat}_h(s_2)| \leq |s_1 - s_2|$, we conclude that one sufficient condition to guarantee the inequality in condition (12) is

$$|\hat{x}_i(t) - x_i(t)| \leq \frac{1}{\sqrt{\theta_i L_{ii}}} e^{-\frac{1}{2}(\beta_i + \frac{1}{\theta_i})t}. \tag{14}$$

Again from $|\dot{x}_i(t)| = |u_i(t)| \leq 2hL_{ii}$ and $|\hat{x}_i(t_k^i) - x_i(t_k^i)| = 0$ for any triggering time t_k^i, we conclude that one sufficient condition to inequality (14) is

$$(t - t_k^i) 2hL_{ii} \leq \frac{1}{\sqrt{\theta_i L_{ii}}} e^{-\frac{1}{2}(\beta_i + \frac{1}{\theta_i})t}. \tag{15}$$

Then

$$\begin{aligned} t_{N(\varepsilon_0)+1}^i - t_{N(\varepsilon_0)}^i &\geq \frac{1}{2\sqrt{\theta_i L_{ii}^3} h} e^{-\frac{1}{2}(\beta_i + \frac{1}{\theta_i}) t_{N(\varepsilon_0)+1}^i} \\ &\geq \frac{1}{2\sqrt{\theta_i L_{ii}^3} h} e^{-\frac{1}{2}(\beta_i + \frac{1}{\theta_i})T_0} = 2\varepsilon_0, \end{aligned} \tag{16}$$

which contradicts (13). Therefore, there is no Zeno behavior.

(ii) (Necessity) This part of the proof is a special case of the proof of Theorem 3.1 in [7] with some minor modifications. We thus omit the proof here.

(Sufficiency) Let $\eta(t) = [\eta_1(t), \ldots, \eta_n(t)]^\top$. Consider the Lyapunov candidate

$$W(x, \eta) = V(x) + \sum_{i=1}^{n} \eta_i, \tag{17}$$

where $V(t)$ is given (5). If $|\bar{x}(0)| \leq h$, then it follows from Lemma 3 that $V(x) \geq 0$ and $V(x) = 0$ if and only if $x_i = \bar{x}(0), i \in \mathcal{I}$. The derivative of $W(x, \eta)$ along the trajectories of system (1) and system (11) with the event-triggered control protocol (4) is

$$\begin{aligned} \dot{W}(x, \eta) &= \dot{V}(x) + \sum_{i=1}^{n} \dot{\eta}_i(t) \\ &\leq \frac{1}{2}(1 - \sigma_{max}) \sum_{i=1}^{n} -q_i(t) - \sum_{i=1}^{n} \beta_i \eta_i(t) \leq 0. \end{aligned} \tag{18}$$

Then by LaSalle Invariance Principle [26], we have

$$\lim_{t\to\infty} x_i(t) = \bar{x}(0), i \in \mathcal{I}.$$

Remark 3 *The static triggering law (8) can be seen as a limit case of the dynamic triggering law (12) when θ_i grows large.*

4 Simulations

In this section, a numerical example is provided to demonstrate the theoretical results. We choose $h = 1$ in the saturation function. Consider a connected network of four agents with the Laplacian matrix

$$L = \begin{bmatrix} 5.7 & -2.2 & 0 & -3.5 \\ -2.2 & 7.9 & -5.7 & 0 \\ 0 & -5.7 & 6.7 & -1 \\ -3.5 & 0 & -1 & 4.5 \end{bmatrix},$$

whose topology is shown in Fig. 1. The initial value of each agent is randomly selected within the interval $[-5, 5]$. We first choose $x(0) = [2.513, -2.449, 0.060, 1.991]^\top$, the average initial state is $\bar{x}(0) = 0.5288$ and the condition $|\bar{x}(0)| \leq h$ is thus satisfied. Therefore, according to Theorems 1 and 2, consensus is achieved. Fig. 2 (a) shows the state evolution under the static triggering law (8) with $\sigma_i = 0.9$. Fig. 2 (b) shows the corresponding triggering times for each agent. Fig. 3 (a) shows the state evolution under the dynamic triggering law (12) with $\sigma_i = 0.9$, $\eta_i(0) = 10$, $\beta_i = 1$ and $\theta_i = 1$. Fig. 3 (b) shows the corresponding triggering times for each agent. It can be seen that 1) under both event-triggered laws consensus is achieved; 2) comparing with the static triggering law (8), the dynamic triggering law (12) determines less triggering times for each agent; 3) the trajectories under the static triggering law (8) are more smooth than the trajectories under the dynamic triggering law (12).

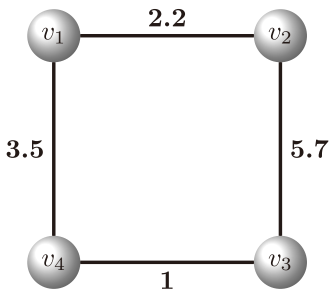

Fig. 1: The communication topology.

We next choose the initial value as $x(0) = [2.513, 0.551, 0.060, 1.991]^\top$. The average initial state is $\bar{x}(0) = 1.2788$, so the condition $|\bar{x}(0)| \leq h$ is not satisfied in this case. Therefore, according to Theorems 1 and 2, consensus is not achieved. Fig. 4 shows the state evolution under the static triggering law (8) with $\sigma_i = 0.9$. Fig. 5 shows the state evolution under the dynamic triggering law (12) with $\sigma_i = 0.9$, $\eta_i(0) = 10$, $\beta_i = 1$ and $\theta_i = 1$. It can be seen that under both event-triggered laws consensus is not achieved in this case as predicted since the condition of the saturation level is not fulfilled.

5 Conclusions

In this paper, we proposed a static and a dynamic event-triggered control law for multi-agent systems subject to output saturation. We showed that, if the communication graph is undirected and connected, both event-triggered control laws solve the consensus problem if and only if the average of the initial states is within the saturation level. In addition, the dynamic event-triggered control law was shown to be free of Zeno behavior. Future research directions include considering directed communication graphs, input saturations and interconnection saturations.

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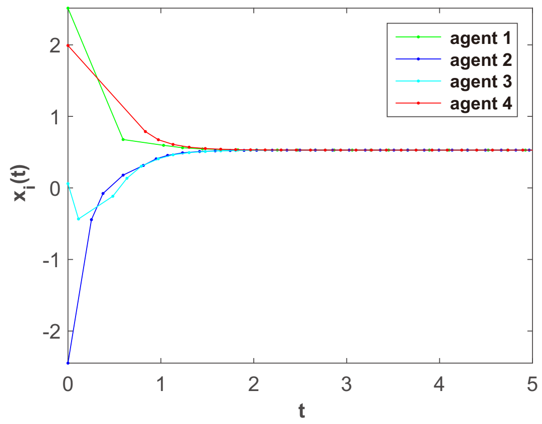
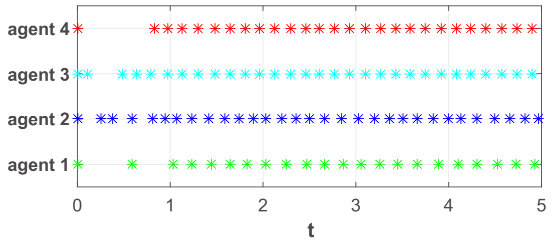
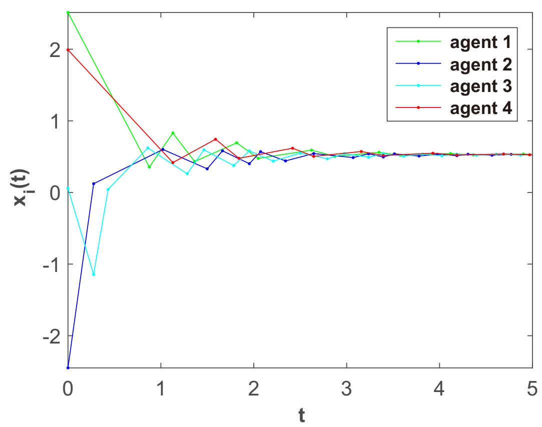


(a)

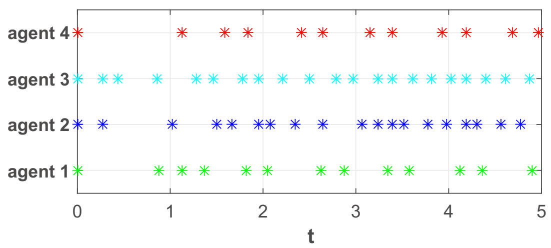


(b)

Fig. 2: (a) The state evolution under the static triggering law (8). (b) The triggering times for each agent under the static triggering law (8).



(a)



(b)

Fig. 3: (a) The state evolution under the dynamic triggering law (12). (b) The triggering times for each agent under the dynamic triggering law (12).

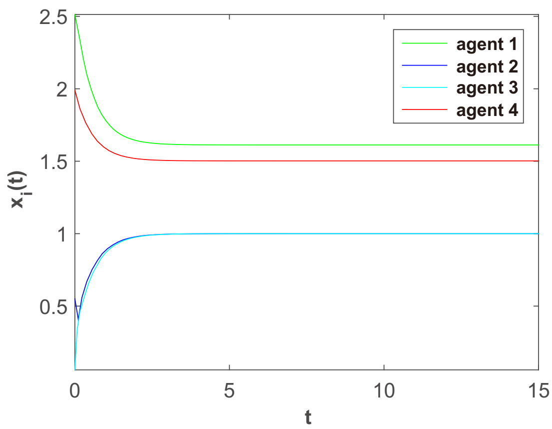


Fig. 4: The state evolution under static triggering law (8).

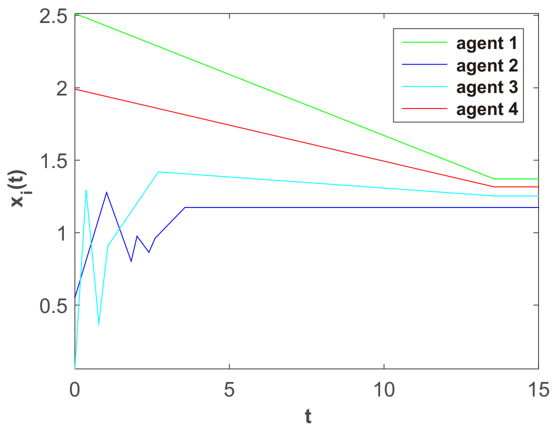


Fig. 5: The state evolution under dynamic triggering law (12).

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