Automatica 92 (2018) 9–17

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stochastic self-triggered model predictive control for linear systems with probabilistic constraints☆

Li Dai [a,*,1], Yulong Gao [b,1], Lihua Xie [c], Karl Henrik Johansson [b], Yuanqing Xia [a]

[a] School of Automation, Beijing Institute of Technology, Beijing 100081, PR China
[b] ACCESS Linnaeus Center, KTH Royal Institute of Technology, SE-10044 Stockholm, Sweden
[c] School of Electrical and Electronic Engineering, Nanyang Technological University, 639798, Singapore

ARTICLE INFO

Article history:
Received 22 March 2017
Received in revised form 3 October 2017
Accepted 14 January 2018
Available online 14 March 2018

Keywords:
Stochastic systems
Probabilistic constraints
Model predictive control (MPC)
Self-triggered control

ABSTRACT

A stochastic self-triggered model predictive control (SSMPC) algorithm is proposed for linear systems subject to exogenous disturbances and probabilistic constraints. The main idea behind the self-triggered framework is that at each sampling instant, an optimization problem is solved to determine both the next sampling instant and the control inputs to be applied between the two sampling instants. Although the self-triggered implementation achieves communication reduction, the control commands are necessarily applied in open-loop between sampling instants. To guarantee probabilistic constraint satisfaction, necessary and sufficient conditions are derived on the nominal systems by using the information on the distribution of the disturbances explicitly. Moreover, based on a tailored terminal set, a multi-step open-loop MPC optimization problem with infinite prediction horizon is transformed into a tractable quadratic programming problem with guaranteed recursive feasibility. The closed-loop system is shown to be stable. Numerical examples illustrate the efficacy of the proposed scheme in terms of performance, constraint satisfaction, and reduction of both control updates and communications with a conventional time-triggered scheme.

1. Introduction

Networked control systems are usually subject to constraints and uncertainties. The constraints include not only the traditional system constraints, such as state constraints, but also communication constraints, such as a limited bandwidth in wireless communication networks. For such systems, an integrative model predictive control (MPC) and event-based control approach is a natural idea which could ensure the system constraint satisfaction and trade off the performance of control systems and the usage of communication resources. Thus, the research of event-based MPC is of great interest.

Two specific types of event-based control are event-triggered and self-triggered control. Different from event-triggered control which requires the continuous monitoring of system states, self-triggered control determines the next update time in advance based on the information at the current sampling instant. Also, self-triggered control allows the shut-down of the sensors between two updates, resulting in a lower sampling frequency to prolong the lifespan of sensors powered by batteries. Please refer to Heemels, Johansson, and Tabuada (2012) and Hetel et al. (2017) for an overview of event-based control.

This paper considers a self-triggered implementation of stochastic MPC (SMPC) for linear systems with stochastic disturbances. One main feature of SMPC is the presence of probabilistic constraints, which require the constraints to be satisfied with given probability thresholds. Such constraints can mitigate the conservativeness introduced by hard constraints of robust MPC (RMPC). SMPC has found applications in diverse fields, e.g., building climate control (Long, Liu, Xie, & Johansson, 2014) or chemical processes (Qin & Badgwell, 2003). To the best of our knowledge, stochastic self-triggered MPC (SSMPC) has not been explored up to now. One remarkable challenge is how to characterize the 'propagation' of uncertainties during two sampling instants and formulate a computationally tractable optimization problem for determining sampling instants and control design.

Some developments of self-triggered MPC are available. Many of these results are proposed for systems without uncertainties (Barradas Berglind, Gommans, & Heemels, 2012; Hashimoto, Adachi, & Dimarogonas, 2017; Henriksson, Quevedo, Sandberg,

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Riccardo Scattolini under the direction of Editor Ian R. Petersen.

* Corresponding author.
E-mail addresses: li.dai@bit.edu.cn (L. Dai), yulongg@kth.se (Y. Gao), elhxie@ntu.edu.sg (L. Xie), kallej@kth.se (K.H. Johansson), xia_yuanqing@bit.edu.cn (Y. Xia).
1 Both authors contributed equally to this work.

https://doi.org/10.1016/j.automatica.2018.02.017
0005-1098/

& Johansson, 2012). For systems with uncertainties, most results account for the synthesis of self-triggered control and RMPC which aims to guarantee robust constraint satisfaction. The interested reader can refer to Aydiner, Brunner, and Heemels (2015) and Brunner, Heemels, and Allgöwer (2014, 2016). By maximizing the inter-sampling time subject to constraints on the cost function, a robust self-triggered MPC (RSMPC) algorithm is presented for constrained linear systems with bounded additive disturbances in Brunner et al. (2014), which employs the robust Tube MPC method in Mayne, Seron, and Raković (2005) to guarantee constraint satisfaction. In Brunner et al. (2014), all constraint parameters are determined by fixing the maximal inter-sampling time, which has the drawback of leading to a conservative region of attraction. To alleviate the conservatism, a RSMPC algorithm based on a more advanced Tube method (Raković, Kouvaritakis, Findeisen, & Cannon, 2012) is proposed in Aydiner et al. (2015), where the cost function is defined depending on the length of the inter-sampling time such that the constraint parameters are not affected by the maximal sampling interval. By combining with the self-triggering mechanism in Aydiner et al. (2015), a recent RSMPC method is presented in Brunner et al. (2016) with the focus of extending the Tube method in Chisci, Rossiter, and Zappa (2001) to evaluate the effect of the uncertainty on the prediction of the self-triggered setup.

Inspired by Aydiner et al. (2015) and Brunner et al. (2016), we design a self-triggered strategy for SMPC. Notice that inherent differences between SMPC and RMPC make our SSMPC algorithm largely different from the ones presented in Aydiner et al. (2015) and Brunner et al. (2016). Following the ideas of Tube MPC (Kouvaritakis, Cannon, Raković, & Cheng, 2010), we construct stochastic tubes as tight as possible by explicitly using the distributions of the disturbances. Since a crucial assumption of feedback at every time step in Kouvaritakis et al. (2010) is not satisfied in the self-triggered setting (which allows open-loop operations between sampling instants), some appropriate and non-trivial modifications are needed: (i) by considering the multi-step open-loop operation between control updates, three predicted controllers are defined for different phases of the prediction horizon, making it more complex than (Kouvaritakis et al., 2010) to evaluate the effect of the uncertainty on predictions and construct equivalent deterministic constraints; (ii) the inter-sampling time as an optimizing variable is included in the cost function and a tuning parameter is introduced to provide a trade-off between performance and communication; (iii) an improved terminal set, which is adapted to different inter-sampling times, is designed to make the constraints recursively feasible.

The present paper is the first work on SSMPC, which extends the existing literatures on MPC considerably. The main contributions are summarized in the following. (i) Our joint design of the self-triggering mechanism and the SMPC controller effectively reduces the amount of communication, while guaranteeing control performance with specific level of trade-off. (ii) The MPC optimization problem is transformed into a tractable quadratic programming problem by using information on the disturbance distribution. (iii) For the self-triggering mechanism, the probability of constraint violation can be tight to the specified limit. (iv) Both recursive feasibility and closed-loop stability are guaranteed. To illustrate the effectiveness of the algorithm, numerical experiments are carried out to compare the proposed SSMPC with a periodically-triggered SMPC (PSMPC), RSMPC, and unconstrained MPC (LQR).

The remainder of this paper is structured as follows. Problem formulation is set up in Section 2. In Section 3, a multi-step open-loop MPC optimization problem is formulated incorporating probabilistic constraints and specific terminal sets. In Section 4, a SSMPC algorithm is developed and main results are established. Section 5 presents numerical simulations and Section 6 concludes.

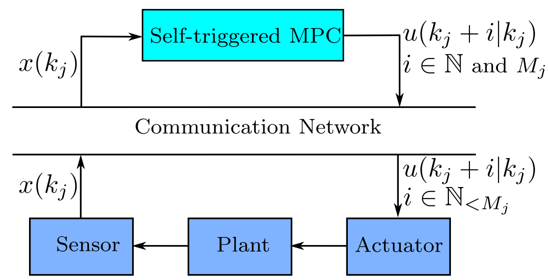


Fig. 1. The self-triggered MPC framework.

Notation 1.1. *Let* $\mathbb{N} \triangleq \{0, 1, \ldots\}$. *For some* $q, s \in \mathbb{N}$ *and* $q < s$, *let* $\mathbb{N}_{\geq q}$, $\mathbb{N}_{>q}$, $\mathbb{N}_{\leq q}$, $\mathbb{N}_{<q}$, *and* $\mathbb{N}_{[q,s]}$ *denote the sets* $\{r \in \mathbb{N} \mid r \geq q\}$, $\{r \in \mathbb{N} \mid r > q\}$, $\{r \in \mathbb{N} \mid r \leq q\}$, $\{r \in \mathbb{N} \mid r < q\}$, *and* $\{r \in \mathbb{N} \mid q \leq r \leq s\}$, *respectively. Let* I *and* $\mathbf{0}$ *denote an identity matrix and a zero matrix or zero vector of appropriate dimension. When* $\leq, \geq, <, >$, *and* $|\cdot|$ *are applied to vectors, they are interpreted element-wise. For* $W \in \mathbb{R}^{n\times n}$, $W \succ 0$ *means that* W *is symmetric and positive definite. For* $x \in \mathbb{R}^n$ *and* $W \succ 0$, $\|x\|_W^2 \triangleq x^T W x$. *For* $x_i \in \mathbb{R}^n$, $i \in \mathbb{N}$, *define* $\sum_{i=a}^{b} x_i = \mathbf{0}$ *if* $a > b$. *Pr denotes the probability,* $\mathbb{E}$ *the expectation,* $\mathbb{E}_k$ *the conditional expectation of a random variable given the state at time k, and* $(k+i|k)$ *a prediction of a variable i steps ahead from time k.*

2. Problem formulation

The self-triggered MPC framework of this paper is shown in Fig. 1, in which the notations are introduced below. Consider a linear time-invariant system

$$x(k+1) = Ax(k) + Bu(k) + w(k), \ k \in \mathbb{N}, \tag{1}$$

where $x(k) \in \mathbb{R}^{N_x}$ is the state, $u(k) \in \mathbb{R}^{N_u}$ the control input, $w(k) \in \mathbb{R}^{N_w}$ the stochastic disturbance, and (A, B) a stabilizable pair. Notice that $N_x = N_w$. Assume that $w(k)$, $k \in \mathbb{N}$, are independent and identically distributed (i.i.d.) and the elements of $w(k)$ have zero mean. The distribution F_i of the ith element of $w(k)$ is assumed to be known and continuous with a bounded support $[-\sigma_i, \sigma_i]$, $\sigma_i > 0$, and correspondingly we have $w(k) \in \mathcal{W} \triangleq \{w \mid |w| \leq \sigma\}$, $\sigma = [\sigma_1\, \sigma_2 \ldots \sigma_{N_w}]^T$. Moreover, system (1) is subject to n_c probabilistic constraints $Pr\{g_\ell^T x(k) \leq h_\ell\} \geq p_\ell$, $\ell \in \mathbb{N}_{[1,n_c]}$, $k \in \mathbb{N}$, where $g_\ell \in \mathbb{R}^{N_x}$, $h_\ell \in \mathbb{R}$, and $p_\ell \in [0, 1]$. In the sequel, we will focus on one probabilistic constraint

$$Pr\{g^T x(k) \leq h\} \geq p, \ k \in \mathbb{N}, \tag{2}$$

as the other constraints can be treated in a similar way.

In a periodically-triggered MPC scheme, the predictive control input at time k can be designed as

$$u(k+i|k) = Kx(k+i|k) + c(k+i|k), \ i \in \mathbb{N}, \tag{3}$$

where $K \in \mathbb{R}^{N_u\times N_x}$ is chosen offline such that the matrix $\Phi \triangleq A + BK$ is Schur stable and for a prediction horizon $N \in \mathbb{N}_{\geq 1}$, $c(k+i|k)$ for $i \in \mathbb{N}_{\leq N-1}$ are optimization variables and $c(k+i|k) = \mathbf{0}$ for $i \in \mathbb{N}_{\geq N}$. At each time instant k, $u(k) = Kx(k) + c(k|k)$ is applied to the system.

To reduce the amount of communication, in the self-triggered scheme, the states $x(k)$ are only measured and transmitted to the controller at sampling instants $k_j \in \mathbb{N}$, $j \in \mathbb{N}$, which evolve as $k_{j+1} = k_j + M_j$ with $k_0 = 0$. The inter-sampling time $M_j \in \mathbb{N}_{[1,N-1]}$ is determined by a self-triggering mechanism based on the state at sampling instant k_j. Since the values of $x(k_j + i|k_j)$, $i \in \mathbb{N}_{[1,M_j-1]}$, cannot be determined at time k_j in the presence of stochastic disturbances, the predictive control sequence in (3) is not applicable to control the system at time instants between k_j and k_{j+1} in an open-loop fashion and we redefine it in the self-triggered setup as

$$u(k_j+i|k_j) = Kz(k_j+i|k_j) + c(k_j+i|k_j), \quad i \in \mathbb{N}_{\leq M_j-1}, \tag{4}$$

$$u(k_j+i|k_j) = Kx(k_j+i|k_j) + c(k_j+i|k_j), \quad i \in \mathbb{N}_{[M_j,N-1]}, \tag{5}$$

$$u(k_j+i|k_j) = Kx(k_j+i|k_j), \ i \in \mathbb{N}_{\geq N}, \tag{6}$$

where the nominal state $z(k_j+i|k_j) \triangleq \mathbb{E}[x(k_j+i|k_j)]$ evolves as

$$z(k_j+i+1|k_j) = \Phi z(k_j+i|k_j) + Bc(k_j+i|k_j), \ i \in \mathbb{N},$$

with $z(k_j|k_j) = x(k_j)$. The predictive controller (4) is designed with respect to nominal state predictions, which are deterministic and only depend on $x(k_j)$. And (5) is designed with respect to disturbed state predictions, which are introduced aiming to reduce the effect of the uncertainty in the prediction. After the Nth prediction time, the predictive controller (6) is given by the state feedback law. Notice that the number of decision variables is a finite N. As shown in Fig. 1, after solving an MPC optimization problem at sampling instant k_j, the first M_j predictive control inputs, i.e., $u(k_j|k_j)$, $u(k_j+1|k_j), \ldots, u(k_j+M_j-1|k_j)$, are transmitted to the actuator and are applied to the system until the next sampling instant k_{j+1}.

The goal is to design $\mathbf{c}(k_j) = [c^T(k_j|k_j)\ c^T(k_j+1|k_j) \ldots c^T(k_j+N-1|k_j)]^T \in \mathbb{R}^{N_u \times N}$ and to maximize M_j at each sampling instant k_j, such that a low frequency of control updates and communication is achieved, while stabilizing a neighborhood of the origin and guaranteeing probabilistic constraint satisfaction.

3. Optimization problem formulation

In this section, the problem described in Section 2 is formulated to a computationally tractable MPC optimization problem with a fixed inter-sampling time $M \in \mathbb{N}_{[1,N-1]}$. Define the prototype optimization problem $\mathcal{P}_o^M(\mathbf{c}(k_j))$ as follows.

$$\min_{\mathbf{c}(k_j)} J^M(\mathbf{c}(k_j)) \triangleq \frac{1}{\alpha}\sum_{i=0}^{M-1}\mathbb{E}_{k_j}[\|x(k_j+i|k_j)\|_Q^2 + \|u(k_j+i|k_j)\|_R^2 - \ell_{ss}] + \sum_{i=M}^{\infty}\mathbb{E}_{k_j}[\|x(k_j+i|k_j)\|_Q^2 + \|u(k_j+i|k_j)\|_R^2 - \ell_{ss}] \tag{7}$$

subject to (4)–(6) with $M_j = M$, $z(k_j|k_j) = x(k_j)$, and

$$\forall i \in \mathbb{N}_{\leq M-2}: \ z(k_j+i+1|k_j) = Az(k_j+i|k_j) + Bu(k_j+i|k_j) \tag{8a}$$

$$\forall i \in \mathbb{N}: \ x(k_j+i+1|k_j) = Ax(k_j+i|k_j) + Bu(k_j+i|k_j) + w(k_j+i) \tag{8b}$$

$$\forall i \in \mathbb{N}_{\geq 1}: \ Pr\{g^T x(k_j+i|k_j) \leq h\} \geq p. \tag{8c}$$

Therein, $Q \succ 0$ and $R \succ 0$ are weighting matrices and scalar $\alpha \geq 1$ is a tuning parameter. The constant $\ell_{ss} \triangleq \lim_{i\to\infty}\mathbb{E}_{k_j}[\|x(k_j+i|k_j)\|_Q^2 + \|u(k_j+i|k_j)\|_R^2]$ can be computed offline by Theorem 2 of Cannon, Kouvaritakis, and Wu (2009). Although the infinite-horizon cost in (7) can be expressed as a quadratic function (see Remark 3.2), solving $\mathcal{P}_o^M(\mathbf{c}(k_j))$ online is still unrealistic due to the presence of an infinite number of probabilistic constraints.

Remark 3.1. Based on the sampling interval, cost function (7) is divided into two parts similar to Aydiner et al. (2015) and Brunner et al. (2016). In Aydiner et al. (2015) and Brunner et al. (2016), the cost function consists of finite-horizon costs and a terminal cost which are defined by nominal systems. Considering the presence of stochastic uncertainties, we define (7) in expectation.

Remark 3.2. Using the probabilistic distribution of w and extending the proof of Theorem 2 in Cannon et al. (2009), cost function (7) can be written as a quadratic form of $\mathbf{c}(k_j)$ given by $J^M(\mathbf{c}(k_j)) = \mathbf{c}^T(k_j)P_{cc}\mathbf{c}(k_j) + \mathbf{c}^T(k_j)P_{cx}x(k_j) + f_x(x(k_j)) + f_w$, where $P_{cc} \succ 0$, P_{cc} and P_{cx} are constant matrices, and function $f_x(x(k_j))$ and constant f_w are determined, respectively, by the state $x(k_j)$ and the distribution of w regardless of the choice of $\mathbf{c}(k_j)$.

3.1. Probabilistic constraint handling strategy

To render $\mathcal{P}_o^M(\mathbf{c}(k_j))$ computationally tractable, we will convert probabilistic constraints (8c) to deterministic ones, such that the observed probability of constraint satisfaction is tight to the specified value and the derived constraints are recursively feasible for the closed-loop system. Under the assumption that the first M inputs in the sequence are applied in an open-loop fashion, Lemma 3.1 gives an equivalent form of (8c).

Lemma 3.1. *For any $M \in \mathbb{N}_{[1,N-1]}$ and any sampling instant $k_j, j \in \mathbb{N}$, probabilistic constraints $Pr\{g^T x(k_j+i|k_j) \leq h\} \geq p$ for $i \in \mathbb{N}_{\geq 1}$ are satisfied if and only if $\mathbf{c}(k_j)$ satisfies*

$$g^T\Phi^i x(k_j) + g^T H_i \mathbf{c}(k_j) \leq h - \gamma_i^M, \ i \in \mathbb{N}_{\geq 1}, \tag{9}$$

where $H_i \triangleq [\Phi^{i-1}B\ \Phi^{i-2}B \ldots B\ \mathbf{0} \ldots \mathbf{0}]$ and γ_i^M is defined as the minimum value such that

$$\begin{cases} Pr\{g^T A^{i-1} w(k_j|k_j) + \cdots + g^T w(k_j+i-1|k_j) \\ \qquad \leq \gamma_i^M\} = p, \ i \in \mathbb{N}_{[1,M]}, \\ Pr\Big\{g^T \Phi^{i-M}\sum_{\ell=0}^{M-1} A^\ell w(k_j+\ell|k_j) + \sum_{\ell=0}^{i-M-1} g^T\Phi^\ell \\ w(k_j+i-1-\ell|k_j) \leq \gamma_i^M\Big\} = p, \ i \in \mathbb{N}_{\geq M+1}. \end{cases} \tag{10}$$

Proof. For $i \in \mathbb{N}_{[1,M]}$, it holds by (4) that $x(k_j+i|k_j) = \Phi^i x(k_j) + H_i\mathbf{c}(k_j) + A^{i-1}w(k_j|k_j) + \cdots + w(k_j+i-1|k_j)$. Further, for $i \in \mathbb{N}_{\geq M+1}$, it follows from (5) that $x(k_j+i|k_j) = \Phi^i x(k_j) + H_i\mathbf{c}(k_j) + \Phi^{i-M}\sum_{\ell=0}^{M-1}A^\ell w(k_j+\ell|k_j) + \sum_{\ell=0}^{i-M-1}\Phi^\ell w(k_j+i-1-\ell|k_j)$. Hence, it follows directly by (10) that (8c) are equivalent to deterministic constraints (9).

In practice, the values of γ_i^M can be approximated using univariate convolutions with arbitrarily small approximation error (see Dai, Xia, Gao, & Cannon, 2017, Remark 3.2). The approximation can be performed offline in polynomial time. By modifying Lemma 3.1, recursively feasible constraints are derived in Theorem 3.1 ensuring the satisfaction of (2).

Theorem 3.1. *Given any $M \in \mathbb{N}_{[1,N-1]}$ and any sampling instant k_j, $j \in \mathbb{N}$, let for all $k \in \mathbb{N}_{[k_j,k_{j+1}-1]}$*

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + w(k), \\ u(k) = Kz(k|k_j) + c(k|k_j). \end{cases} \tag{11}$$

Suppose $\mathbf{c}(k_j)$ satisfies

$$g^T\Phi^i x(k_j) + g^T H_i\mathbf{c}(k_j) \leq h - \beta_i^M, \ i \in \mathbb{N}_{\geq 1}, \tag{12}$$

where β_i^M is defined as the maximum element of the ith column of Γ in (13) with $d_i^M \triangleq \max_{w\in\mathcal{W}} g^T\Phi^{i-M-1}w$, $b_i^M \triangleq \max_{w\in\mathcal{W}} g^T\Phi^{i-M}$

$$\Gamma=\begin{bmatrix}\gamma_1^M & \cdots & \gamma_M^M & \gamma_{M+1}^M & \gamma_{M+2}^M & \gamma_{M+3}^M & \cdots\\ 0 & \cdots & 0 & b_{M+1}^M+\xi_{M+1}^M & b_{M+2}^M+\xi_{M+2}^M & b_{M+3}^M+\xi_{M+3}^M & \cdots\\ 0 & \cdots & 0 & 0 & b_{M+2}^M+d_{M+2}^M+\xi_{M+1}^M & b_{M+3}^M+d_{M+3}^M+\xi_{M+2}^M & \cdots\\ 0 & \cdots & 0 & 0 & 0 & b_{M+3}^M+d_{M+3}^M+d_{M+2}^M+\xi_{M+1}^M & \cdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots\end{bmatrix}. \quad (13)$$

Box I.

$\sum_{\ell=0}^{M-1}A^\ell w$, *and* ξ_i^M *the minimum value such that* $Pr\{\sum_{\ell=0}^{i-M-1}g^T\Phi^\ell w \le \xi_i^M\}=p$. *Then, for the closed-loop system (11), (i) there exists at least one solution* $\mathbf{c}(k_{j+\ell})$ *satisfying (12) for* $M=1$ *at all future sampling instants* $k_{j+\ell},\ \ell\in\mathbb{N}_{\ge 1}$*; (ii) probabilistic constraints (2) are satisfied for all* $k\in\mathbb{N}$.

Proof. For (i), if β_i^M is defined as the first row of (13) given in Box I, then (12) is equivalent to (9). Next, consider the feasibility of (12) for $M=1$ at time k_{j+1}. Define T as the shift matrix with ones on the superdiagonal and zeros elsewhere. Based on the information available at current sampling instant k_j and constructing a candidate solution $\tilde{\mathbf{c}}(k_{j+1})\triangleq T^M\mathbf{c}(k_j)=[c^T(k_j+M|k_j)\dots c^T(k_j+N-1|k_j)\ \mathbf{0}\dots\mathbf{0}]^T$, it yields that $x(k_{j+1}+i|k_{j+1})=\Phi^{i+M}x(k_j)+H_{i+M}\mathbf{c}(k_j)+\Phi^ie(k_{j+1}|k_{j+1})+\Phi^{i-1}w(k_{j+1}|k_{j+1})+\cdots+w(k_{j+1}+i-1|k_{j+1}),\ i\in\mathbb{N}_{\ge 1}$, with $e(k_{j+1}|k_{j+1})\triangleq A^{M-1}w(k_j|k_j)+\cdots+w(k_j+M-1|k_j)$. Since $e(k_{j+1}|k_{j+1})$ depends on $w(k_j|k_j),\dots,w(k_j+M-1|k_j)$ which have already been realized at time k_{j+1}, the worst-case bound on $e(k_{j+1}|k_{j+1})$ needs to be considered explicitly at time k_j. Hence, to ensure $\tilde{\mathbf{c}}(k_{j+1})$ is a feasible solution at time k_{j+1}, we must require at time k_j

$$g^T\Phi^ix(k_j)+g^TH_i\mathbf{c}(k_j)\le h-(b_i^M+\xi_i^M),\ i\in\mathbb{N}_{\ge M+1}, \quad (14)$$

which corresponds to defining β_i^M in (12) as the second row of (13). By the same arguments, the feasibility of (12) for $M=1$ at time $k_{j+\ell},\ell\in\mathbb{N}_{\ge 2}$, can be ensured, if β_i^M is defined as the $(\ell+1)$th row of (13) and it holds at time k_j that

$$g^T\Phi^ix(k_j)+g^TH_i\mathbf{c}(k_j)\le h-(b_i^M+d_i^M+d_{i-1}^M+\cdots +d_{i-\ell+2}^M+\xi_{i-\ell+1}^M),\ i\in\mathbb{N}_{\ge M+\ell}. \quad (15)$$

Taking the intersection of (9), (14), and (15) for $\ell\in\mathbb{N}_{\ge 2}$, the feasibility at all sampling instants $k_j,k_{j+1},k_{j+2},\dots$ can be ensured if β_i^M in (12) is chosen as the maximum element of the ith column of (13), thereby proving (i).

For (ii), from Lemma 3.1, constraints (12) ensure that $Pr\{g^Tx(k_j+i|k_j)\le h\}\ge p$ is satisfied for all $i\in\mathbb{N}_{[1,M]}$, which implies the satisfaction of (2) for all $k\in\mathbb{N}_{[k_j+1,k_{j+1}]}$. By recursive feasibility of (12) in (i), it holds that (2) is satisfied for all $k\in\mathbb{N}_{[k_j+1,k_{j+1}]}$ and all $j\in\mathbb{N}$.

Parameters γ_i^M in (9) and β_i^M in (12) are determined by not only the length i of the predicted time steps but also the length M of the open-loop steps. By setting $M=1$, the results in Lemma 3.1 and Theorem 3.1 are reduced to Theorems 1 and 3 in Kouvaritakis et al. (2010). In the following, two properties of the sequence β_i^M, $i\in\mathbb{N}_{\ge 1}$, are established.

Lemma 3.2. *For all* $M\in\mathbb{N}_{[1,N-1]}$*, it holds that*

$$\beta_i^M=\begin{cases}\gamma_i^M,\ i\in\mathbb{N}_{[1,M]},\\ b_i^M+\sum_{\ell=M+2}^{i}d_\ell^M+\gamma_1^M,\ \ i\in\mathbb{N}_{\ge M+1}.\end{cases} \quad (16)$$

Proof. For $i\in\mathbb{N}_{[1,M]}$, (16) holds directly. For $i\in\mathbb{N}_{\ge M+1}$, we have $\gamma_i^M\le b_i^M+\xi_i^M$. Further, it holds for $i\in\mathbb{N}_{\ge M+2}$ that $\xi_i^M\le d_i^M+\xi_{i-1}^M$. Then, it can be concluded that β_i^M is equal to the last non-zero element in the ith column of (13), which gives (16).

Lemma 3.3. *For all* $M\in\mathbb{N}_{[1,N-1]}$ *and all* $i\in\mathbb{N}_{\ge 1}$*, it holds that* $\beta_{i+M}^M=b_{i+M}^M+\beta_i^1$.

Proof. From Lemma 3.2, β_i^1 can be rewritten as $\beta_i^1=\sum_{\ell=1}^{i-1}\max_{w\in\mathcal{W}}g^T\Phi^\ell w+\gamma_1^1,\ i\in\mathbb{N}_{\ge 2}$. Then, from the fact that $\gamma_1^1=\gamma_1^M$, it follows for $i\in\mathbb{N}_{\ge 1}$ that $\beta_{i+M}^M=b_{i+M}^M+\sum_{\ell=1}^{i-1}\max_{w\in\mathcal{W}}g^T\Phi^\ell w+\gamma_1^M=b_{i+M}^M+\beta_i^1$.

3.2. Terminal set

To ensure that constraints (12) are satisfied over an infinite prediction horizon, a terminal set is used. First, due to $c(k_j+N+i|k_j)=\mathbf{0}$ for all $i\in\mathbb{N}$, the terminal dynamics of the nominal system can be rewritten as $z(k_j+N+i+1|k_j)=\Phi z(k_j+N+i|k_j),\ i\in\mathbb{N}$. Define a constraint set for $z(k_j+N|k_j)$ as

$$\{z\mid g^T\Phi^iz\le h-\beta_{N+i}^M,\ i\in\mathbb{N}\}. \quad (17)$$

Then given some $\hat{N}\in\mathbb{N}$, split the infinite prediction horizon in (17) into two stages $i\in\mathbb{N}_{\le M+\hat{N}}$ and $i\in\mathbb{N}_{\ge M+\hat{N}+1}$. In the second stage, an upper bound of β_i^M is used, which is introduced through the following lemma.

Lemma 3.4. *For all* $M\in\mathbb{N}_{[1,N-1]}$*, there exist a scalar* $0<\rho<1$ *and a positive definite matrix* S *such that the sequence* β_i^M *for* $i\in\mathbb{N}_{\ge M+1}$ *is upper bounded by*

$$\beta_i^M\le\bar{\beta}^M\triangleq\bar{b}^M+\sum_{\ell=M+2}^{v-1}d_\ell^M+\frac{\rho^v}{1-\rho}\|g\|_S+\gamma_1^M, \quad (18)$$

with any integer $v\in\mathbb{N}_{\ge M+3}$ *and* $\bar{b}^M\triangleq\max_{i\in\mathbb{N}_{\ge M+1},w\in\mathcal{W}}g^T\Phi^{i-M}\sum_{\ell=0}^{M-1}A^\ell w$.

Proof. In (16), the existence of the upper bound $\bar{b}^M$ on b_i^M can be ensured by the strict stability of Φ. Furthermore, it holds that $\sum_{\ell=M+2}^{i}d_\ell^M\le\sum_{\ell=M+2}^{\infty}d_\ell^M$. From Lemma 5 of Cannon, Cheng, Kouvaritakis, and Raković (2012), the bound on $\sum_{\ell=M+2}^{\infty}d_\ell^M$ is given by $\sum_{\ell=M+2}^{v-1}d_\ell^M+\frac{\rho^v}{1-\rho}\|g\|_S$, thereby completing the proof.

The scalar ρ and matrix S can be obtained by solving semidefinite programs as (16) in Cannon et al. (2012). Replace β_{N+i}^M in (17) by the bound $\bar{\beta}^M$ for $i\in\mathbb{N}_{\ge M+\hat{N}+1}$ and define an inner approximation of (17) as

$$\{z\mid g^T\Phi^iz\le h-\beta_{N+i}^M,\ i\in\mathbb{N}_{\le M+\hat{N}},\\ g^T\Phi^iz\le h-\bar{\beta}^M,\ i\in\mathbb{N}_{\ge M+\hat{N}}+1\}. \quad (19)$$

Finally, using Theorem 2.3 of Gilbert and Tan (1991), there exists $n^*\in\mathbb{N}_{\ge 1}$ such that the infinite number of constraints in (19) are ensured through the first $M + \hat{N} + n^*$ constraints. Hence, the terminal set for $z(k_j + N|k_j)$ is constructed as follows:

$$\chi_f^M \triangleq \{z \mid g^T \Phi^i z \le h - \beta_{N+i}^M,\ i \in \mathbb{N}_{\le M+\hat{N}},\\ g^T \Phi^i z \le h - \bar{\beta}^M,\ i \in \mathbb{N}_{[M+\hat{N}+1, M+\hat{N}+n^*]}\}, \tag{20}$$

where the smallest allowable n^* can be computed offline by solving a finite number of linear programs (see Gilbert & Tan, 1991).

3.3. Optimization problem

Given a state $x(k_j)$ and $M \in \mathbb{N}_{[1,N-1]}$, the constraints at sampling instant k_j are summarized as follows.

$$z(k_j|k_j) = x(k_j) \tag{21a}$$

$$\forall i \in \mathbb{N}_{\le N-1}:$$
$$z(k_j + i + 1|k_j) = \Phi z(k_j + i|k_j) + Bc(k_j + i|k_j) \tag{21b}$$

$$\forall i \in \mathbb{N}_{\le M-1}:$$
$$u(k_j + i|k_j) = Kz(k_j + i|k_j) + c(k_j + i|k_j) \tag{21c}$$

$$\forall i \in \mathbb{N}_{[1,N-1]}:$$
$$g^T \Phi^i x(k_j) + g^T H_i \mathbf{c}(k_j) \le h - \beta_i^M \tag{21d}$$

$$z(k_j + N|k_j) \in \chi_f^M. \tag{21e}$$

Note that there is only a finite number of deterministic constraints in (21), which can be computed for the predictions of the nominal model. Define the set of all $\mathbf{c}(k_j)$ satisfying (21) as

$$\mathcal{F}^M(x(k_j)) \triangleq \{\mathbf{c}(k_j) \mid (21) \text{ holds}\},$$

and the state $x(k_j)$ feasible if $\mathcal{F}^M(x(k_j)) \ne \emptyset$.

At sampling instant k_j, an MPC optimization problem $\mathcal{P}^M(\mathbf{c}(k_j))$, which is the tractable version of $\mathcal{P}_o^M(\mathbf{c}(k_j))$, can now be formulated as

$$V^M(k_j) \triangleq \min_{\mathbf{c}(k_j) \in \mathcal{F}^M(x(k_j))} J^M(\mathbf{c}(k_j)),$$

$$\mathbf{c}^*(k_j) \triangleq \operatorname*{argmin}_{\mathbf{c}(k_j) \in \mathcal{F}^M(x(k_j))} J^M(\mathbf{c}(k_j)),$$

where $V^M(k_j)$ denotes the optimal value function and $\mathbf{c}^*(k_j)$ the corresponding optimal solution.

Remark 3.3. Clearly constraints (21) are all affine functions in $\mathbf{c}(k_j)$ and $J^M(\mathbf{c}(k_j))$ is a quadratic cost function, see Remark 3.2. Thus, $\mathcal{P}^M(\mathbf{c}(k_j))$ is a quadratic programming problem. More importantly, the computational complexity of $\mathcal{P}^M(\mathbf{c}(k_j))$ is not increased compared with the periodically-triggered MPC scheme in Kouvaritakis et al. (2010) regarding the number of constraints and decision variables.

4. Stochastic self-triggered MPC

Using the above MPC optimization problem as a basis, a SSMPC algorithm is designed in this section. In the self-triggered setup, the goal at each sampling instant k_j is to decide not only $\mathbf{c}(k_j)$ but also the next sampling instant k_{j+1}. To reduce the computation and communication, we need to find the largest M_j such that $\mathcal{P}^{M_j}(\mathbf{c}(k_j))$ is feasible for some $\mathbf{c}(k_j) \in \mathcal{F}^{M_j}(x(k_j))$ while still maintaining certain performance of the closed-loop system. Define the self-triggered MPC problem $\mathcal{S}(x(k_j))$ as

$$M_j^* \triangleq \max\{M \in \mathbb{N}_{[1,M_{\max}]} \mid \mathcal{F}^M(x(k_j)) \ne \emptyset,\\ V^M(k_j) \le V^1(k_j)\}, \tag{22}$$

$$\mathbf{c}^*(k_j) \triangleq \operatorname*{argmin}_{\mathbf{c}(k_j) \in \mathcal{F}^{M_j^*}(x(k_j))} J^{M_j^*}(\mathbf{c}(k_j)), \tag{23}$$

where $M_{\max} \in \mathbb{N}_{[1,N-1]}$ is an a priori maximum of the inter-sampling time and $V^1(k_j)$ is the optimal value function corresponding to the MPC scheme in which control updates take place at every time instant.

Remark 4.1. The idea adopted in (22) is similar to Aydiner et al. (2015), Barradas Berglind et al. (2012) and Brunner et al. (2016), in which, by introducing a tuning parameter α as in (7), the optimal value function of the M-step open-loop MPC scheme is required to be not worse than that of a periodically-triggered MPC scheme. The inherent differences are that in Barradas Berglind et al. (2012) the system is undisturbed and in Aydiner et al. (2015) and Brunner et al. (2016) the system is subject to bounded disturbances and hard constraints. As a result, the choice of the cost function and the design of the tightened constraint sets are significantly different from that in (22).

Remark 4.2. By employing the triggering mechanism in (22), parameter α may be used to trade off the control performance and the frequency of control updates. If we choose $\alpha = 1$, the optimal value function $V^M(k_j)$ for $M \in \mathbb{N}_{[1,M_{\max}]}$ might be in general no smaller than $V^1(k_j)$, thereby leading to a small inter-sampling time M_j^*. To obtain a larger one, we can increase the value of α to counter the effect of the open-loop control, while possibly sacrificing slightly the control performance.

The resulting SSMPC algorithm is summarized below.

Algorithm 1 SSMPC

Offline:

Determine α, Q, R, K, N, $\hat{N}$, $M_{\max}$, and n^*. For all $M \in \mathbb{N}_{[1,M_{\max}]}$, compute γ_i^M, $i \in \mathbb{N}_{[1,M]}$, b_{M+i}^M, $i \in \mathbb{N}_{[1,N+\hat{N}]}$, d_{M+i}^M, $i \in \mathbb{N}_{[2,N+\hat{N}]}$, ξ_{M+1}^M, and $\bar{\beta}^M$.

Online:

1: Initialize $k = 0$, measure the initial state $x(k)$ and obtain M^* and $\mathbf{c}^*(k)$ by solving $\mathcal{S}(x(k))$.
2: For all $i \in \mathbb{N}_{\le M^*-1}$, apply the input $u(k + i) = Kz(k + i|k) + c^*(k + i|k)$ to the system.
3: Take the next sampling instant as $k + M^*$ and set $k = k + M^*$.
4: Measure the current state $x(k)$.
5: Solve $\mathcal{S}(x(k))$ to obtain M^* and $\mathbf{c}^*(k)$.
6: Go to step 2.

Under Algorithm 1, the resulting closed-loop system is

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + w(k), \\ u(k) = Kz(k|k_j) + c^*(k|k_j),\ k \in \mathbb{N}_{[k_j, k_{j+1}-1]}, \end{cases} \tag{24}$$

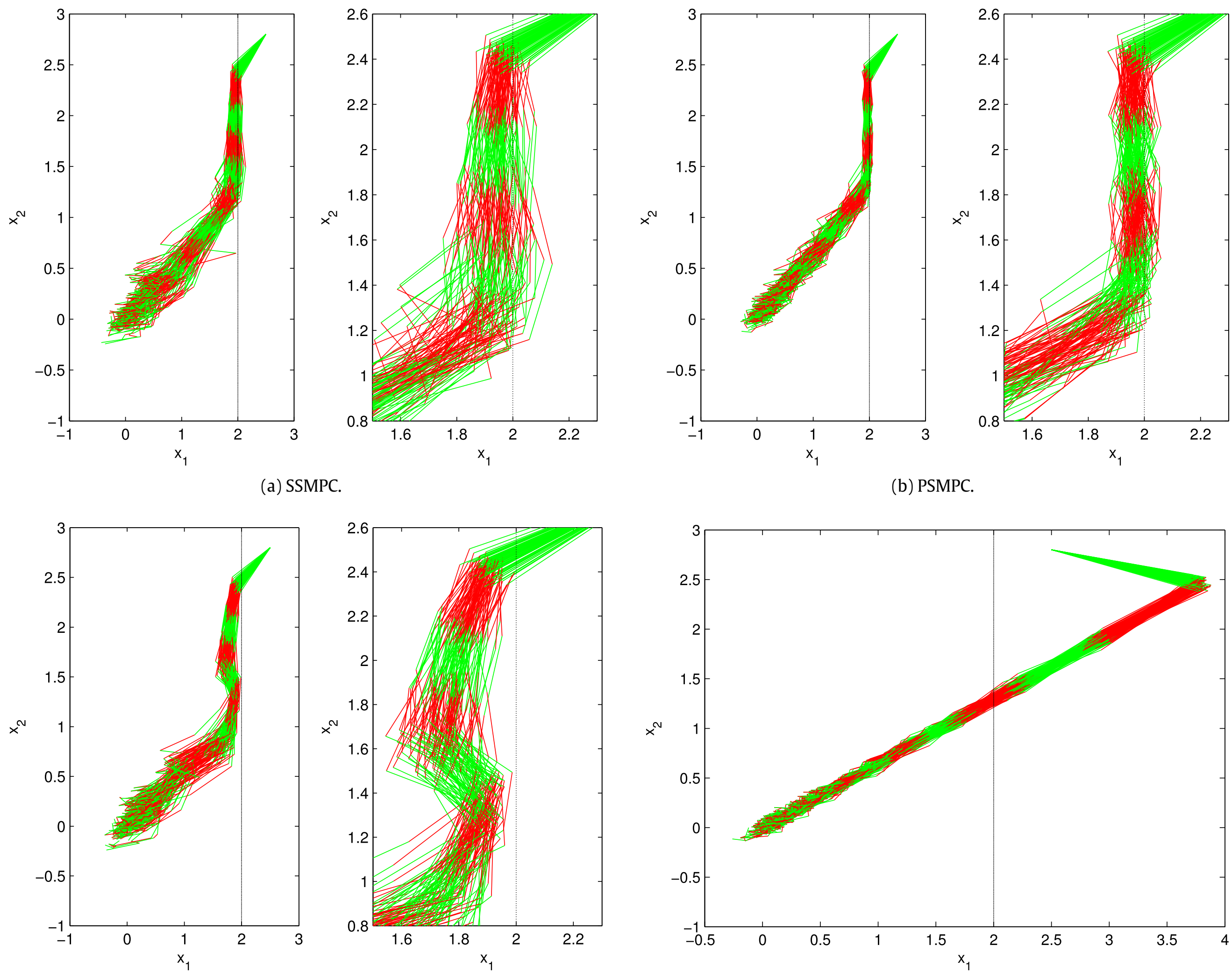
$$k_{j+1} = k_j + M_j^*, \tag{25}$$

for $j \in \mathbb{N}$ and $k_0 = 0$, which has the following properties.

Theorem 4.1 (*Recursive Feasibility and Constraint Satisfaction*)**.** *If $\mathcal{S}(x(k_0))$ is feasible at time k_0, then the feasibility of $\mathcal{S}(x(k_j))$ can be ensured at every sampling instant k_j, $j \in \mathbb{N}$, for (24)–(25) under Algorithm 1. Furthermore, for all $k \in \mathbb{N}$, constraints (2) are satisfied.*

Proof. Consider two successive sampling instants k_j and k_{j+1}. Let M_j and $\mathbf{c}(k_j)$ be solutions of $\mathcal{S}(x(k_j))$ at time k_j. Define $\tilde{\mathbf{c}}(k_{j+1}) = T^{M_j}\mathbf{c}(k_j)$. We will show that $\tilde{\mathbf{c}}(k_{j+1})$ and $M_{j+1} = 1$ are the feasible solutions of $\mathcal{S}(x(k_{j+1}))$ at time k_{j+1}, i.e., $\tilde{\mathbf{c}}(k_{j+1}) \in \mathcal{F}^1(x(k_{j+1}))$.

Constraints (21a)–(21c) in $\mathcal{F}^1(x(k_{j+1}))$ are trivially satisfied. The satisfaction of (21d) by $\tilde{\mathbf{c}}(k_{j+1})$ is ensured from Theorem 3.1.



(a) SSMPC.

(b) PSMPC.

(c) RSMPC.

(d) Unconstrained LQR control.

Fig. 2. Closed-loop trajectories under four different control schemes for 100 realizations of the uncertainty sequence.

Further, for all $i \in \mathbb{N}$, we have

$$
\begin{aligned}
&g^T \Phi^i z(k_j + M_j + N|k_j + M_j)\\
&= g^T \Phi^{M_j+i} z(k_j + N|k_j) + g^T \Phi^{N+i} \sum_{\ell=0}^{M_j-1} A^{\ell} w\\
&\leq h - \beta^{M_j}_{N+i+M_j} + b^{M_j}_{N+i+M_j} \leq h - \beta^1_{N+i},
\end{aligned}
$$

where the last inequality follows from Lemma 3.3. Therefore, we immediately obtain that $z(k_{j+1} + N|k_{j+1}) \in \chi^1_f$, i.e., (21e) in $\mathcal{F}^1(x(k_{j+1}))$ is satisfied. From all of the above, it can be concluded that at time k_{j+1}, $\mathcal{S}(x(k_{j+1}))$ is feasible and further by induction, $\mathcal{S}(x(k_j))$ are feasible at all sampling instants $k_j, j \in \mathbb{N}$.

For all $k \in \mathbb{N}$, the satisfaction of probabilistic constraints (2) is guaranteed directly from Theorem 3.1.

Theorem 4.2 (*Quadratic Stability Property*). *The closed-loop system (24)–(25) under Algorithm 1 satisfies the quadratic stability condition*

$$
\lim_{k_r \to \infty} \frac{1}{k_r} \sum_{k=k_0}^{k_r-1} \mathbb{E}[\|x(k)\|^2_Q + \|u(k)\|^2_R] \leq \ell_{ss}. \tag{26}
$$

Proof. For any $j \in \mathbb{N}$, let M_j and $\mathbf{c}(k_j)$ be the optimal solutions at sampling instant k_j and the corresponding optimal value function $V^{M_j}(k_j)$ be a Lyapunov function candidate. As in the proof of Theorem 4.1, $\tilde{\mathbf{c}}(k_{j+1}) = T^{M_j}\mathbf{c}(k_j)$ together with $M_{j+1} = 1$ is a feasible solution at sampling instant k_{j+1}. Define $\tilde{V}^1(k_{j+1})$ as the value function associated with this feasible solution. Using the fact that $\alpha \geq 1$, it holds for the closed-loop system that

$$
\begin{aligned}
\mathbb{E}_{k_j}[\tilde{V}^1(k_{j+1})] &\leq V^{M_j}(k_j) - \frac{1}{\alpha} \sum_{i=0}^{M_j-1} \mathbb{E}_{k_j}[\|x(k_j+i|k_j)\|^2_Q\\
&\quad + \|u(k_j+i|k_j)\|^2_R - \ell_{ss}]\\
&\leq V^1(k_j) - \frac{1}{\alpha} \sum_{i=0}^{M_j-1} \mathbb{E}_{k_j}[\|x(k_j+i|k_j)\|^2_Q\\
&\quad + \|u(k_j+i|k_j)\|^2_R - \ell_{ss}],
\end{aligned}
$$

where the second inequality follows from (22). The optimality of the solution leads to

$$
\begin{aligned}
\mathbb{E}_{k_j}[V^1(k_{j+1})] &\leq V^1(k_j) - \frac{1}{\alpha} \sum_{i=0}^{M_j-1} \mathbb{E}_{k_j}[\|x(k_j+i|k_j)\|^2_Q\\
&\quad + \|u(k_j+i|k_j)\|^2_R - \ell_{ss}].
\end{aligned}
$$

Summing the inequality for $j \in \mathbb{N}_{[0,r-1]}$ and taking expectation on both sides,

$$\sum_{j=0}^{r-1}\frac{1}{\alpha}\sum_{i=0}^{M_j-1}\mathbb{E}[\|x(k_j+i|k_j)\|_Q^2+\|u(k_j+i|k_j)\|_R^2-\ell_{ss}]$$
$$\le \mathbb{E}[V^1(k_0)]-\mathbb{E}[V^1(k_r)].$$

Since $\mathbb{E}[V^1(k_0)]$ is finite by assumption and $\mathbb{E}[V^1(k_r)]$ is lower bounded due to Remark 3.2, it holds that

$$\lim_{r\to\infty}\frac{1}{k_r}\sum_{j=0}^{r-1}\sum_{i=0}^{M_j-1}\mathbb{E}[\|x(k_j+i|k_j)\|_Q^2+\|u(k_j+i|k_j)\|_R^2]$$
$$\le \ell_{ss},$$

which implies the quadratic stability condition (26).

5. Numerical example

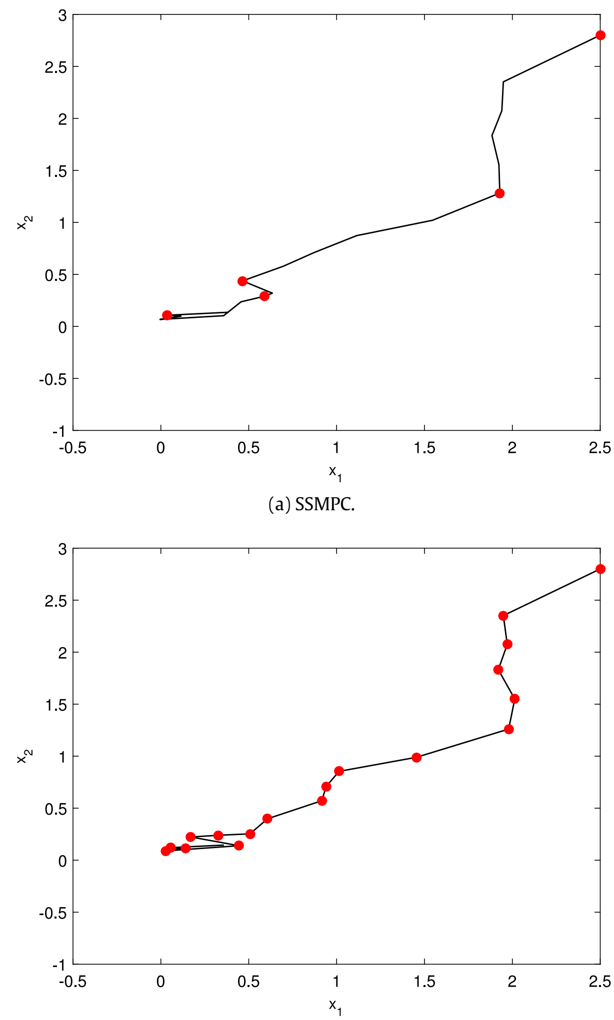
Simulation studies are provided to show the effectiveness and the advantages of the proposed SSMPC in comparison with PSMPC (by setting $M_j = 1$), RSMPC (by setting $p = 1$), and the unconstrained LQR control. Consider a linearized DC–DC converter system as in Lorenzen, Dabbene, Tempo, and Allgöwer (2017),

$$x(k+1)=\begin{bmatrix}1 & 0.0075\\ -0.143 & 0.996\end{bmatrix}x(k)+\begin{bmatrix}4.798\\ 0.115\end{bmatrix}u(k)+w(k)$$

subject to $Pr\{[1\ 0]x(k)\le 2\}\ge 0.8$. Elements of $w(k)$ are assumed to be i.i.d. truncated Gaussian random variables with zero mean, variance 0.04^2, and bounded by $|w_i(k)|\le 0.1$ for $i=1,2$. In (7), $Q=\mathrm{diag}\{1,3.5\}$, $R=0.1$, $\alpha=1.2$, and $\ell_{ss}=0.37$. $K=[0.263\ -0.329]$ is chosen as the unconstrained LQR gain. The prediction horizon and horizons in (21e) are $N=8$, $\hat{N}=12$, and $n^*=1$. The maximal open-loop length is $M_{\max}=8$. To obtain β_i^M in (21d), γ_i^M, $i\in\mathbb{N}_{[1,M]}$, and ξ_{M+1}^M are calculated according to Remark 3.2 of Dai et al. (2017). In (21e), we choose $v=13$ and calculate ρ and S of $\bar{\beta}^M$ by (16) in Cannon et al. (2012). Simulations for four control schemes are performed with 1000 realizations of the uncertainty sequence, initial condition $[2.5\ 2.8]^T$, and a simulation length of $T_{\mathrm{run}}=18$ steps. The simulations are implemented in Matlab R2012b with Yalmip and SeDuMi solver.

Stability and constraint violation: The state trajectories $\{x(k),\ k=0,1,\ldots\}$ for 100 realizations of the uncertainty sequence are depicted in Fig. 2 with the black dotted lines being the constraint bounds. The right plots of subfigure (a)–(c) in Fig. 2 enlarge the region of constraint bound to show the constraint violation. As it turns out, with SSMPC, the observed probabilities of constraint violation in the first 5 steps are 19.7%, 20.4%, 19.8%, 20.2%, and 16.3%, while by PSMPC, the violation rates are 19.8%, 20.1%, 19.9%, 16.8%, and 9% for the same 1000 realizations. Furthermore, as expected, RSMPC achieves no constraint violations, whereas violation rate is 100% in the first 3 steps under the unconstrained LQR control. The simulation results indicate that by the proposed SSMPC, the closed-loop state converges to a neighborhood of the origin and the constraint violation is tight to the specified violation value 20%.

Average inter-sampling time and performance: To illustrate the decreased communication achieved by SSMPC, Fig. 3 shows the state trajectories under SSMPC and PSMPC for 1 realization of the uncertainty sequence. The sampling instants are highlighted by red solid circles. It can be observed that the number of the sampling instants is significantly reduced. The associated average number of steps between sampling instants is compared. For each scheme, the same uncertainty sequences are used and the average is taken over 1000 realizations and 18 steps. Under the self-triggered scheme, the average inter-sampling time is $\bar{M}=2.9$, which leads to an average



(a) SSMPC.

(b) PSMPC.

Fig. 3. State trajectories under SSMPC and PSMPC for 1 realization of the uncertainty sequence.

reduction in communication by 65.5% compared to the scheme with updates at every time instant. In addition, let us compare the performance measure

$$J_{\mathrm{perf}}=\frac{1}{T_{\mathrm{run}}}\sum_{k=0}^{T_{\mathrm{run}}-1}\{\|x(k)\|_Q^2+\|u(k)\|_Q^2-\ell_{ss}\}.$$

It is 6.82 for SSMPC, as compared with 6.75 for PSMPC. It can be concluded that by the proposed SSMPC, communication is decreased without much loss in performance, which can also be illustrated by Fig. 3.

6. Conclusion

We proposed a SSMPC strategy for the stabilization of systems with additive disturbances and probabilistic constraints. It was shown that the required amount of communication was reduced while simultaneously guaranteeing a specific performance loss when compared with a periodically-triggered scheme. By taking the disturbances occurring during the inter-sampling period into account and making use of their probability distribution, a set of deterministic constraints and terminal sets were constructed to formulate a computationally tractable MPC optimization problem.

Probabilistic constraints were ensured at each time instant despite the open-loop operation between any two sampling instants. Moreover, recursive feasibility and stability were proved for the closed-loop system. The results were compared in simulations with other MPC methods from the literature.

Acknowledgments

This work of Li Dai and Yuanqing Xia was supported by the National Natural Science Foundation of China under Grant 61603041, the Beijing Natural Science Foundation under Grant 4161001, the National Natural Science Foundation Projects of International Cooperation and Exchanges under Grant 61720106010, and Foundation for Innovative Research Groups of the National Natural Science Foundation of China under Grant 61621063. The work of Yulong Gao and Karl Henrik Johansson was supported in part by the Knut and Alice Wallenberg Foundation, the Swedish Foundation for Strategic Research, and the Swedish Research Council. The work of Lihua Xie was supported by the National Natural Science Foundation of China under Grant 61633014.

References

Aydiner, E., Brunner, F. D., & Heemels, W. P. M. H. (2015). Robust self-triggered model predictive control for constrained discrete-time LTI systems based on homothetic tubes. In *Proceedings of european control conference* (pp. 1587–1593).

Barradas Berglind, J. D. J., Gommans, T. M. P., & Heemels, W. P. M. H. (2012). Self-triggered MPC for constrained linear systems and quadratic costs. In *Proceedings of IFAC conference on nonlinear model predictive control* (pp. 342–348).

Brunner, F. D., Heemels, W. P. M. H., & Allgöwer, F. (2014). Robust self-triggered MPC for constrained linear systems. In *Proceedings of european control conference* (pp. 472–477).

Brunner, F. D., Heemels, W. P. M. H., & Allgöwer, F. (2016). Robust self-triggered MPC for constrained linear systems: A tube-based approach. *Automatica*, *72*, 73–83.

Cannon, M., Cheng, Q., Kouvaritakis, B., & Raković, S. V. (2012). Stochastic tube MPC with state estimation. *Automatica*, *48*(3), 536–541.

Cannon, M., Kouvaritakis, B., & Wu, X. (2009). Probabilistic constrained MPC for multiplicative and additive stochastic uncertainty. *IEEE Transactions on Automatic Control*, *54*(7), 1626–1632.

Chisci, L., Rossiter, J. A., & Zappa, G. (2001). Systems with persistent disturbances: predictive control with restricted constraints. *Automatica*, *37*(7), 1019–1028.

Dai, L., Xia, Y., Gao, Y., & Cannon, M. (2017). Distributed stochastic MPC of linear systems with additive uncertainty and coupled probabilistic constraints. *IEEE Transactions on Automatic Control*, *62*(7), 3474–3481.

Gilbert, E. G., & Tan, K. T. (1991). Linear systems with state and control constraints: The theory and application of maximal output admissible sets. *IEEE Transactions on Automatic Control*, *36*(9), 1008–1020.

Hashimoto, K., Adachi, S., & Dimarogonas, D. V. (2017). Self-triggered model predictive control for nonlinear input-affine dynamical systems via adaptive control samples selection. *IEEE Transactions on Automatic Control*, *62*(1), 177–189.

Heemels, W. P. M. H., Johansson, K. H., & Tabuada, P. (2012). An introduction to event-triggered and self-triggered control. In *Proceedings of IEEE conference on decision and control* (pp. 3270–3285).

Henriksson, E., Quevedo, D. E., Sandberg, H., & Johansson, K. H. (2012). Self-triggered model predictive control for network scheduling and control. In *Proceedings of IFAC symposium on advanced control of chemical processes* (pp. 432–438).

Hetel, L., Fiter, C., Omran, H., Seuret, A., Fridman, E., Richard, J. P., et al. (2017). Recent developments on the stability of systems with aperiodic sampling: an overview. *Automatica*, *76*, 309–335.

Kouvaritakis, B., Cannon, M., Raković, S. V., & Cheng, Q. (2010). Explicit use of probabilistic distributions in linear predictive control. *Automatica*, *46*(10), 1719–1724.

Long, Y., Liu, S., Xie, L., & Johansson, K. H. (2014). A scenario-based distributed stochastic MPC for building temperature regulation. In *Proceedings of 2014 IEEE international conference on automation science and engineering* (pp. 1091–1096).

Lorenzen, M., Dabbene, F., Tempo, R., & Allgöwer, F. (2017). Constraint-tightening and stability in stochastic model predictive control. *IEEE Transactions on Automatic Control*, *62*(7), 3165–3177.

Mayne, D. Q., Seron, M. M., & Raković, S. V. (2005). Robust model predictive control of constrained linear systems with bounded disturbances. *Automatica*, *41*(2), 219–224.

Qin, S. J., & Badgwell, T. A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, *11*(7), 733–764.

Raković, S. V., Kouvaritakis, B., Findeisen, R., & Cannon, M. (2012). Homothetic tube model predictive control. *Automatica*, *48*(8), 1631–1638.

Li Dai was born in Beijing, China, in 1988. She received the B.S. degree in Information and Computing Science in 2010 and the Ph.D. degree in Control Science and Engineering in 2016 from Beijing Institute of Technology, Beijing, China. Now she is an assistant professor at the School of Automation of Beijing Institute of Technology. Her research interests include model predictive control, distributed control, data-driven control, stochastic systems, and networked control systems.

Yulong Gao received the B.S. degree in Automation in 2013 and the M.S. degree in Control Science and Engineering in 2016 from Beijing Institute of Technology, Beijing, China. Now he is a Ph.D. student at the School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology. His research interests include model predictive control and stochastic control with application in traffic systems and networked control systems.

Lihua Xie received the B.E. and M.E. degrees in electrical engineering from Nanjing University of Science and Technology in 1983 and 1986, respectively, and the Ph.D. degree in electrical engineering from the University of Newcastle, Australia, in 1992. Since 1992, he has been with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, where he is currently a professor and Director, Delta-NTU Corporate Laboratory for Cyber-Physical Systems. He served as the Head of Division of Control and Instrumentation from July 2011 to June 2014. He held teaching appointments in the Department of Automatic Control, Nanjing University of Science and Technology from 1986 to 1989.

Dr. Xie's research interests include robust control and estimation, networked control systems, multi-agent networks, localization and unmanned systems. He is an Editor-in-Chief for Unmanned Systems and an Associate Editor for IEEE Transactions on Network Control Systems. He has served as an editor of IET Book Series in Control and an Associate Editor of a number of journals including IEEE Transactions on Automatic Control, Automatica, IEEE Transactions on Control Systems Technology, and IEEE Transactions on Circuits and Systems-II. He is an elected member of Board of Governors, IEEE Control System Society (Jan 2016–Dec 2018). Dr. Xie is a Fellow of IEEE and Fellow of IFAC.

Karl Henrik Johansson is Director of the Stockholm Strategic Research Area ICT The Next Generation and Professor at the School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology. He received M.Sc. and Ph.D. degrees in Electrical Engineering from Lund University. He has held visiting positions at UC Berkeley, Caltech, NTU, HKUST Institute of Advanced Studies, and NTNU. His research interests are in networked control systems, cyber-physical systems, and applications in transportation, energy, and automation. He is a member of the IEEE Control Systems Society Board of Governors and the European Control Association Council. He has received several best paper awards and other distinctions, including a ten-year Wallenberg Scholar Grant, a Senior Researcher Position with the Swedish Research Council, the Future Research Leader Award from the Swedish Foundation for Strategic Research, and the triennial Young Author Prize from IFAC. He is member of the Royal Swedish Academy of Engineering Sciences, Fellow of the IEEE, and IEEE Distinguished Lecturer.

Yuanqing Xia was born in Anhui Province, China, in 1971. He graduated from the Department of Mathematics, Chuzhou University, Chuzhou, China, in 1991. He received the M.S. degree in Fundamental Mathematics from Anhui University, Anhui, China, in 1998 and the Ph.D. degree in Control Theory and Control Engineering from the Beijing University of Aeronautics and Astronautics, Beijing, China, in 2001. From 1991 to 1995, he was with Tongcheng Middle-School, Anhui, where he worked as a Teacher. During January 2002–November 2003, he was a Postdoctoral Research Associate with the Institute of Systems Science, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing, China, where he worked on navigation, guidance and control. From November 2003 to February 2004, he was with the National University of Singapore as a Research Fellow, where he worked on variable structure control. From February 2004 to February 2006, he was with the University of Glamorgan, Pontypridd, UK, as a Research Fellow, where he worked on networked control systems. From February 2007 to June 2008, he was a Guest Professor with Innsbruck Medical University, Innsbruck, Austria, where he worked on biomedical signal processing. Since 2004, he has been with the Department of Automatic Control, Beijing Institute of Technology, Beijing, first as an Associate Professor, then, since 2008, as a Professor. In 2012, he was appointed as Xu Teli Distinguished Professor at the Beijing Institute of Technology and obtained a National Science Foundation for Distinguished Young Scholars of China. His current research interests are in the fields of networked control systems, robust control and signal processing, active disturbance rejection control and flight control. He has published eight monographs with Springer and Wiley, and more than 100 papers in journals. He has obtained Second Award of the Beijing Municipal Science and Technology (No. 1) in 2010, Second National Award for Science and Technology (No. 2) in 2011, and Second Natural Science Award of The Ministry of Education (No. 1) in 2012. He is a Deputy Editor of the Journal of the Beijing Institute of Technology, Associate Editor of Acta Automatica Sinica, Control Theory and Applications, the International Journal of Innovative Computing, Information and Control, and the International Journal of Automation and Computing.