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# Multi-room occupancy estimation through adaptive gray-box models

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***Abstract*— We consider the problem of estimating the occupancy level in buildings using indirect information such as $CO_2$ concentrations and ventilation levels. We assume that one of the rooms is temporarily equipped with a device measuring the occupancy. Using the collected data, we identify a gray-box model whose parameters carry information about the structural characteristics of the room. Exploiting the knowledge of the same type of structural characteristics of the other rooms in the building, we adjust the gray-box model to capture the $CO_2$ dynamics of the other rooms. Then the occupancy estimators are designed using a regularized deconvolution approach which aims at estimating the occupancy pattern that best explains the observed $CO_2$ dynamics. We evaluate the proposed scheme through extensive simulation using a commercial software tool, IDA-ICE, for dynamic building simulation.**

***Index Terms*— Occupancy estimation, Maximum Likelihood, $CO_2$ dynamics, inference, building automation**

## I. INTRODUCTION

The estimation of occupancy levels in buildings has important implications in efficient control of Heating, Venting and Air Conditioning (HVAC) systems [1], [2], [3], [4], [5], [6]. Instrumenting buildings with dedicated hardware such as cameras may raise privacy concerns and be economically disadvantageous, in particular when this requires retrofitting old structures. On the other hand, there is an increasing interest in understanding the effectiveness of estimating occupancy using non-dedicated information sources, such as $CO_2$ concentration and air inlet actuation levels.

There are two main strategies to estimate occupancy in buildings. The first utilizes direct occupancy measurements collected by people-counting devices (see [7], [8]). The second strategy exploits non-dedicated sensor and devices, and typically estimate the occupancy levels by inverting the $CO_2$ dynamics. The model relating the $CO_2$ concentration with occupancy can be derived either using physics-based concepts (e.g., mass-balance equations) or by employing data-based modeling techniques. As for the physics-based $CO_2$ models, assuming well-mixed air in the room, authors in [9] derived a bilinear model which has similarities with the model presented in this paper. Still assuming well-mixed air, [10], [11] and [4] make use of mass-balance equations and linear models for the $CO_2$ dynamics. More detailed models are considered in [12]. Regarding data-based modeling techniques, [13] uses methods of moments, while [14] proposes both linear parametric and nonparametric identified models, from which estimators based on deconvolution are designed. A novel approach is proposed in [15], where, using blind system identification techniques [16], no training sets including occupancy measurements are required. Other types of estimators use black box models identified using, e.g., neural networks or hidden Markov models, and potentially including several sources of information (e.g., temperature, humidity, door and light status, electricity consumption patterns) [17], [18], [19], [20], [21], [22]. This literature focuses on occupancy estimation in single rooms. Besides a few studies dealing with modeling and estimation of occupancy movements across buildings (see e.g., [23], [17]), the multi-room case has not received as much attention as the single-room case.

In this paper we take an important step towards the extension of single-room occupancy estimators to the multi-room case. Our fundamental question is whether the information on the $CO_2$ dynamics gathered in one room can be exploited to design occupancy estimators for other rooms of the same building. To answer such a question, we assume that one room of the building is temporarily equipped with an occupancy measurement device. We use the data collected by this device, together with $CO_2$ concentration and ventilation data, to identify a nonlinear gray-box model via Maximum Likelihood (ML) [24]. The structure of the gray-box model is derived from first principles [25] and permits to define a one-to-one correspondence between the model parameter vector and the physical parameters characterizing the room (i.e., room volume and size of the ventilation system). Exploiting this correspondence, we adapt the gray-box model to the characteristics of the other rooms, and we design an occupancy estimation based on regularized environmental signal deconvolution, similarly to the strategy proposed in [14]. The role of regularization here is to promote piecewise constant occupancy patterns.

We evaluate the proposed estimation scheme simulating, via the commercial software IDA-ICE [26], a building on the KTH campus. The generated data are validated by

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comparison with a dataset available in [27].

The paper is structured as follows: in Section II we derive and identify the physics-based gray-box model which models the $CO_2$ dynamics. In Section III we introduce the occupancy estimator based on the identified model. In Section IV we present the problem of extending the occupancy estimator to the multi-room case. In Section V we report our experiments. We eventually end the paper with some conclusions.

## II. Modeling and identification of the $CO_2$ dynamics

### A. A physics-based model

Assuming well-mixed air (see [25]), the $CO_2$ concentration of room $j$, denoted by $\overline{c}_j(t)$, can be modeled from mass-conservation considerations as

$$v_j \frac{d\overline{c}_j(t)}{dt} = \big(\dot{Q}_j^{\text{vent,sup}} + \dot{Q}_j^{\text{leak,in}}\big)c - \big(\dot{Q}_j^{\text{vent,exh}} + \dot{Q}_j^{\text{leak,out}}\big)\overline{c}_j(t) + g\ o_j(t). \quad (1)$$

In (1) $v_j$ is the volume of the room; $c$ is the outdoor air $CO_2$ concentration, which we assume constant and equal to 420 $\mathrm{ppm}$; $\dot{Q}_j^{\text{vent,sup}}$ and $\dot{Q}_j^{\text{vent,exh}}$ are supply and exhaust mechanical ventilation rates; $\dot{Q}_j^{\text{leak,in}}$ and $\dot{Q}_j^{\text{leak,out}}$ are the inflow and outflow air leakages through doors and windows. The term $g\,o_j(t)$ models the occupants $CO_2$ generation in the room, where $g$ is the $CO_2$ generation rate per person and $o_j(t)$ is the number of occupants at time $t$. In the case of balanced ventilation it is reasonable to assume that $\dot{Q}_j^{\text{vent,sup}} \approx \dot{Q}_j^{\text{vent}} \approx \dot{Q}_j^{\text{vent,exh}}$ and that $\dot{Q}_j^{\text{leak,in}} \approx \dot{Q}^{\text{leak}} \approx \dot{Q}_j^{\text{leak,out}}$. Assuming balanced ventilation, dynamics (1) can be simplified to

$$\frac{d\overline{c}_j(t)}{dt} = \frac{\dot{Q}_j^{\text{vent}}}{v_j}\big(c - \overline{c}_j(t)\big) + \frac{\dot{Q}^{\text{leak}}}{v_j}\big(c - \overline{c}_j(t)\big) + \frac{g}{v_j}o_j(t). \quad (2)$$

We also consider the case for which the ventilation system keeps a constant ventilation flow in the considered zones, and that this flow can be increased, if the indoor $CO_2$ concentration is above a certain threshold, by means of an opportune control signal $u_j(t)$. Under these assumptions, (2) can be rewritten as

$$\frac{d\overline{c}_j(t)}{dt} = \frac{\dot{Q}_j^{\text{vent,min}} + (\dot{Q}_j^{\text{vent,max}} - \dot{Q}_j^{\text{vent,min}})u_j(t)}{v_j}\big(c - \overline{c}_j(t)\big) + \frac{\dot{Q}^{\text{leak}}}{v_j}\big(c - \overline{c}_j(t)\big) + \frac{g}{v_j}o_j(t), \quad (3)$$

where $\dot{Q}_j^{\text{vent,max}}$ and $\dot{Q}_j^{\text{vent,min}}$ are the maximum and minimum airflow through the ventilation system. Since $\dot{Q}_j^{\text{vent,min}}$ does not depend on the ventilation control signal $u_j(t)$, we rewrite (3) as

$$\frac{d\overline{c}_j(t)}{dt} = \frac{\dot{Q}_j^{\text{u}}u_j(t)}{v_j}\big(c - \overline{c}_j(t)\big) + \frac{\dot{Q}_j^{\text{c}}}{v_j}\big(c - \overline{c}_j(t)\big) + \frac{g}{v_j}o_j(t), \quad (4)$$

with $\dot{Q}_j^{\text{u}} = \dot{Q}_j^{\text{vent,max}} - \dot{Q}_j^{\text{vent,min}}$ and $\dot{Q}_j^{\text{c}} = \dot{Q}_j^{\text{vent,min}} + \dot{Q}^{\text{leak}}$.

We discretize the continuous-time model (4) using the backward Euler discretization[1], so we obtain

$$\frac{\overline{c}_j(k) - \overline{c}_j(k-1)}{T} = \frac{\Big(\dot{Q}_j^{\text{u}}u_j(k) + \dot{Q}_j^{\text{c}}\Big)}{v_j}\big(c - \overline{c}_j(k)\big) + \frac{g}{v_j}o_j(k), \quad (5)$$

where $T$ is the sampling time. We define $c_j(k) := \overline{c}_j(k) - c$ and the parameter vector $\boldsymbol{\theta}_j^T := \big[\theta_j' \ \theta_j'' \ \theta_j'''\big]$, where

$$\theta_j' := \frac{v_j}{v_j + T\dot{Q}_j^{\text{u}}}, \theta_j'' := \frac{Tg}{v_j + T\dot{Q}_j^{\text{u}}}, \theta_j''' := \frac{T\dot{Q}_j^{\text{c}}}{v_j + T\dot{Q}_j^{\text{u}}}. \quad (6)$$

We assume that the measurements of $c_j(k)$ are corrupted by additive noise. Then, the measured $CO_2$ concentration, denoted by $y_j(k)$, can be expressed through the measurement model $y_j(k) = c_j(k) + e_j(k)$, where $e_j(k)$ is the measurement noise, assumed white and Gaussian. The overall model for the $CO_2$ dynamics can be rewritten as the nonlinear Output Error (OE) system

$$\begin{cases} c_j(k) = \dfrac{\theta_j'}{1 + \theta_j'''u_j(k)}c_j(k-1) + \dfrac{\theta_j''}{1 + \theta_j'''u_j(k)}o_j(k) \\ y_j(k) = c_j(k) + e_j(k). \end{cases} \quad (7)$$

We remark that (7) is a simplified model that does not account for changes on the parameters due to human activities (e.g., opening doors or windows) and the operational status of the HVAC system (e.g., Variable Air Volume (VAV) systems that enter or leave economic cycles). Due to space constraints we do not analyze here the influence of these non idealities on the estimation scheme and consider this issue as a future work.

### B. Identification of the gray-box model

In this section we describe a procedure for identifying the parameter vector $\boldsymbol{\theta}_j$ characterizing the model (7). Here we assume that we have collected the dataset of information from room $j$, say

$$\mathcal{D}_j := \{y_j(k), u_j(k), o_j(k)\}_{k \in \mathcal{K}_j}, \quad (8)$$

containing recorded occupancy levels plus environmental information from the building supervisory control and data acquisition (SCADA) system for a set of time indexes $\mathcal{K}_j$.

To use model (7) with the purpose of inferring $o_j(k)$ we estimate its unknown parameters $\boldsymbol{\theta}_j$ exploiting the knowledge on $\mathcal{D}_j$. To this aim we introduce the auxiliary notation

$$a_j(k) := \frac{\theta_j'}{1 + \theta_j'''u_j(k)}, \qquad b_j(k) := \frac{\theta_j''}{1 + \theta_j'''u_j(k)}, \quad (9)$$

so that (7) becomes

$$c_j(k) = a_j(k)c_j(k-1) + b_j(k)o_j(k). \quad (10)$$

Expanding recursively (10) back in time, and defining

$$\widetilde{c}_j(k) := c_j(k) - c_j(0)\prod_{\tau=0}^{k-1} a_j(k-\tau), \quad (11)$$

[1] This choice is motivated by the fact that the backward Euler discretization led to better identification and estimation performance than the forward Euler discretization.

$$B_j(k, k-h) := b_j(k-h) \prod_{\tau=0}^{h-1} a_j(k-\tau) \qquad (12)$$

(with the convention that $\prod_{\tau=0}^{-1} \star = 1$ for every possible $\star$), it follows that

$$\begin{bmatrix} \widetilde{c}_j(1) \\ \vdots \\ \widetilde{c}_j(k) \end{bmatrix} = \begin{bmatrix} B_j(1,1) & & 0 \\ \vdots & \ddots & \\ B_j(k,1) & \cdots & B_j(k,k) \end{bmatrix} \begin{bmatrix} o_j(1) \\ \vdots \\ o_j(k) \end{bmatrix}. \qquad (13)$$

Given $u_j(1), \ldots, u_j(k)$, $o_j(1), \ldots, o_j(k)$ and our Gaussian assumptions on the noise $e_j(k)$ in (7), we have that

$$\widehat{c}_j\left(k; \boldsymbol{\theta}_j\right) := y_j(0) \prod_{\tau=0}^{k-1} a_j(k-\tau) + \begin{bmatrix} B_j(k,1) & \cdots & B_j(k,k) \end{bmatrix} \begin{bmatrix} o_j(1) \\ \vdots \\ o_j(k) \end{bmatrix}, \qquad (14)$$

is the Minimum Variance Unbiased (MVU) estimator of $c_j(k)$ for a given parameter guess $\boldsymbol{\theta}_j$. This estimator can be used for defining the ML estimator for the parameters $\boldsymbol{\theta}_j$ given the dataset $\mathcal{D}_j$, say $\widehat{\boldsymbol{\theta}}_j$, that is obtained solving

$$\widehat{\boldsymbol{\theta}}_j := \arg \min_{\widetilde{\boldsymbol{\theta}}_j \in \mathbb{R}^3} \sum_{k \in \mathcal{K}_j} \left( y_j(k) - \widehat{c}_j\left(k; \widetilde{\boldsymbol{\theta}}_j\right) \right)^2. \qquad (15)$$

Even if problem (15) is nonlinear, it involves only three decision variables and can be efficiently solved using standard interior point methods [28].

## III. Estimating occupancy levels by regularized deconvolution

In this section we revise the occupancy estimation approach proposed in [14], and propose some modifications so to adjust it to the novel nonlinear gray-box model (7). We assume that we have the estimate $\widehat{\boldsymbol{\theta}}_j$ of the parameters of room $j$, that for each time instant $k$ we have access to $y_j(k)$, $y_j(k-1)$, and $u_j(k)$, and that we want to estimate $o_j(k)$ (assumed unavailable) from this information.

From the assumption of Gaussianity of the measurement noise $e_j(k)$ in (7), the best unbiased estimator of $o_j(k)$ corresponds to a Least Squares (LS) estimator. However, since we know that candidate occupancy patterns are piecewise constant, more effective estimators can be obtained by applying regularized estimators. Let

$$\widetilde{y}_j(k) := y_j(k) - y_j(0) \prod_{\tau=0}^{k-1} a_j(k-\tau), \qquad (16)$$

$$\widetilde{\boldsymbol{y}}_j := \begin{bmatrix} \widetilde{y}_j(1) \\ \vdots \\ \widetilde{y}_j(k) \end{bmatrix}, \quad \boldsymbol{o}_j := \begin{bmatrix} o_j(1) \\ \vdots \\ o_j(k) \end{bmatrix}, \quad B_j := \begin{bmatrix} B_j(h, \tau) \end{bmatrix}.$$

Furthermore, let the discrete derivative of $o_j(k)$ be

$$\Delta o_j(\tau) := o_j(\tau) - o_j(\tau - 1), \quad \tau = 1, \ldots, k-1,$$

$$\Delta \boldsymbol{o}_j := \left[ \Delta o_j(1), \ldots, \Delta o_j(k-1) \right].$$

[14] proposes to estimate occupancy levels as

$$\widehat{\boldsymbol{o}}_j = \left\lfloor \arg \min_{\widetilde{\boldsymbol{o}}_j \in \mathbb{R}_+^k} \ \left\| \widetilde{\boldsymbol{y}}_j - B_j \widetilde{\boldsymbol{o}}_j \right\|_2^2 + \lambda_j \left\| \Delta \widetilde{\boldsymbol{o}}_j \right\|_1 \right\rceil, \qquad (17)$$

where the vector-wise rounding operator $\lfloor \cdot \rceil$ is used to obtain integer solutions. This estimator[2] is composed of a LS-type penalty favoring adherence to data, and a $\ell_1$ penalty favoring guesses for which the occupancy is piecewise constant. The parameter $\lambda_j$ trades-off between the two components (see [14] for further details).

It is straightforward to modify (17) to obtain an online estimator which considers only a fixed-length (say, $N$) data window of the past. At each time instant, the estimator is run by constructing the vectors $\widetilde{\boldsymbol{y}}_j$ and $\boldsymbol{o}_j$ using the latest $N$ data of the past. The length $N$ is chosen so that the computational complexity is low enough to allow a real-time solution of (17) and so that the discarded information does not influence significantly the outcomes of the estimator. A reasonable choice for tuning $\lambda_j$ in (17) is to use the value $\widehat{\lambda}_j$ that leads to the best estimation performance on the dataset used for training the parameters $\widehat{\boldsymbol{\theta}}_j$ in Section II-B. The performance index can be chosen as $\left\| \boldsymbol{o}_j - \widehat{\boldsymbol{o}}_j(\lambda_j) \right\|_2$, where $\boldsymbol{o}_j$ is constructed from the dataset (8) and $\widehat{\boldsymbol{o}}_j(\lambda_j)$ is the occupancy pattern obtained using $\lambda_j$ in the estimator (17).

## IV. From single-room to multi-rooms estimators

In this section we extend estimator (17) so that it can be applied to a generic room that has never been instrumented with occupancy sensors. This extension exploits the information on the $CO_2$ dynamics obtained in Section II-B, i.e., for that room of the same building which has instead been temporarily instrumented with occupancy sensors. We assume that the sampling time $T$ and the volume $v_j$ are known for all the rooms of interest[3].

Assume that every single room is instrumented with sensors measuring $CO_2$ and HVAC actuation levels (which are generally available in standard HVAC systems). We also assume that room $j = 0$ is the one that has been instrumented with occupancy sensors for a short period. For this room the dataset $\mathcal{D}_0$ defined in (8) is available and thus it is possible to first identify the $CO_2$ dynamics of the room by estimating the unknown parameters $\boldsymbol{\theta}_0$ through (15), and second estimate the occupancy levels invoking (17).

However, the rooms $j \neq 0$ are without occupancy measurements, i.e., they lack of training sets; for these rooms the identification strategy (15) cannot be used to find the $CO_2$ dynamics. Call these rooms *untrained rooms*. The question is then: how could estimator (17) be extended to these untrained rooms?

To answer this question we notice that to implement (17) one needs to know the $CO_2$ dynamics of the room (7) or, alternatively, $\boldsymbol{\theta}_j$ and the regularization parameter $\lambda_j$. Finding

[2]Problem (17) is usually called fused-lasso. More elaborated theoretical analysis on the performance of these estimators can be found in [29] and [30].

[3]Here $T$ is assumed to be 5 minutes for all the rooms.

the variables $\boldsymbol{\theta}_j$ for a room, in turn, requires either a training set $\mathcal{D}_j$ or the knowledge of $\left(\dot{Q}_j^{\mathrm{c}}, \dot{Q}_j^{\mathrm{u}}, g\right)$, since there is a one-to-one correspondence between $\boldsymbol{\theta}_j$ and $\left(\dot{Q}_j^{\mathrm{c}}, \dot{Q}_j^{\mathrm{u}}, g\right)$. Since for the rooms $j \neq 0$ the set $\mathcal{D}_j$ is not available, we need to infer the triplet $\left(\dot{Q}_j^{\mathrm{c}}, \dot{Q}_j^{\mathrm{u}}, g\right)$ and the regularization parameter $\lambda_j$ from the training room and other available information.

### A. Estimating $\left(\dot{Q}_j^{u}, \dot{Q}_j^{c}, g\right)$

We assume that

$$\frac{\dot{Q}_j^{\text{vent,max}}}{\dot{Q}_0^{\text{vent,max}}} = \frac{M_j}{M_0} \qquad \frac{\dot{Q}_j^{\text{vent,min}}}{\dot{Q}_0^{\text{vent,min}}} = \frac{M_j}{M_0} \qquad \forall j \neq 0, \tag{18}$$

where $M_j$ and $M_0$ are parameters proportional to the ventilation inlet area serving rooms $j$ and 0, respectively. We notice that $M_j$ and $M_0$ can be obtained easily by physical inspection of these two rooms. According to assumption (18), the maximum and minimum ventilation air flows are proportional to the total inlet areas. However, this assumption is made for the purpose of this paper and might not be always applicable, since the design of the ventilation system also depends on the room usage. We study the implications of assumption (18) through a simulated example in Section V.

Assume furthermore that the value of $\dot{Q}^{\text{leak}}$ is negligible compared to $\dot{Q}_j^{\text{vent,min}}$, so that $\dot{Q}_j^{\mathrm{c}} \approx \dot{Q}_j^{\text{vent,min}}$. We can write

$$\dot{Q}_j^{\mathrm{u}} = M_j \dot{Q}^{\mathrm{u}} \qquad \dot{Q}_j^{\mathrm{c}} = M_j \dot{Q}^{\mathrm{c}} \qquad \forall j \neq 0, \tag{19}$$

where $\dot{Q}^{\mathrm{u}}$ and $\dot{Q}^{\mathrm{c}}$ are constant values. Based on (19), we can re-adapt the gray-box model of the training room 0 to the characteristics of every other room $j \neq 0$. In other words, we can start by estimating $\widehat{\theta}_0$ through the single-room estimator (17), i.e., find the triplet $\left(\dot{Q}_0^{\mathrm{c}}, \dot{Q}_0^{\mathrm{u}}, g\right)$. After this, one can use the information on $M_j$ and $v_j$ together with (6) to find $\widehat{\theta}_j$ for all $j \neq 0$, i.e., the model for the $CO_2$ dynamics for every untrained room $j \neq 0$. It is straightforward to estimate the occupancy levels in untrained rooms $j$ using (17) with the opportune parameters.

### B. Estimating $\lambda_j$

The regularization parameter $\lambda_j$ is connected to the usage and structural characteristics of the room $j$. For instance, rooms for which people enter and exit frequently would require a small $\lambda_j$ (and vice versa). The problem of generalizing $\lambda_j$ to untrained rooms thus cannot be answered without additional assumptions on the usage of the room, and is at the best of our knowledge an open problem. In the following we analyze two different cases, corresponding to two specific hypotheses on the usage patterns in buildings:

*1) Assuming the same usage pattern for all the rooms:* this corresponds to assume $\lambda_j = \lambda$ for all $j$. In this case $\lambda$ should be estimated by coupling the tuning procedures described in Section III by finding the best $\widehat{\lambda}$ in the occupancy estimator for the training room.

*2) Assuming the usage patterns to depend on the size of the room:* this corresponds to assume $\lambda_j = \lambda v_j$, i.e., the usage pattern depends linearly on the size (for simplicity, the volume, assuming that the ceilings heights are equal among different rooms). This simple assumption leads to the strategy

$$\frac{\lambda_j}{\lambda_0} = \frac{v_j}{v_0}, \tag{20}$$

where $\lambda_0$ is obtained by solving the tuning problem in Section III for the training room. Once $\lambda_0$ has been found, generalizing to other untrained rooms is immediate, as soon as one knows the ratio of the room volumes $v_j/v_0$.

## V. Assessing the extended occupancy estimators

We evaluate the effectiveness of our derivations through a building simulations tool and the following experiments.

### A. Simulation software environment

Simulations have been performed using IDA-ICE 4.6, a commercial program for dynamic simulations of energy and comfort in buildings [26]. The program features equation-based modeling (NMF-language [31], [32] or Modelica language [33]) and is equipped with a variable time step differential-algebraic solver [34].

### B. Geometry of the simulated building

The simulated indoor environment in Figure 1 represents the ground floor of a seven-storey building in the KTH main campus in Stockholm. The rooms considered for our simulations are the labeled ones, and have different dimensions and uses. The rooms are equipped with VAV ventilation units with mechanical ventilation airflow $u_j(k)$ varying with the current $CO_2$ concentration in the room. In all the rooms the ventilation is provided by a central fan active between 8:00 and 18:00. Room dimensions, reported in Table I, range from the 40 $m^2$ of a small workshop (A:231) to the 130 $m^2$ of a lecturing room (A:213). The rooms have different usage patterns, as reflected in the specific ventilation flows in the rooms. For instance, the project room (A:235) has more regular occupancy patterns than the conference hall (B:213), where periods of zero and high occupancies are alternated.

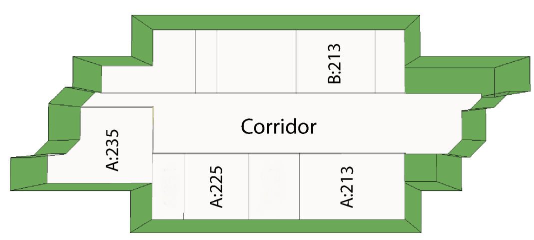


Fig. 1. Floor plan modeled in IDA-ICE.

### C. Simulation setup

Room environment simulations were carried out for a period of two weeks between July 13 and July 26 in 2014. The climate file used to drive IDA-ICE was a weather file for the Bromma airport in Stockholm.

| Room name | Room size $[m^2]$ | Min air flow $[l/(s\ m^2)]$ | Max air flow $[l/(s\ m^2)]$ |
|---|---|---|---|
| A:235 | 125 | 0.6 | 2.16 |
| A:225 | 81 | 0.93 | 3.34 |
| A:213 | 130 | 0.58 | 2.1 |
| B:213 | 96 | 1.05 | 13.13 |

TABLE I

ROOM FOOTPRINT, MINIMUM AND MAXIMUM ROOM MECHANICAL VENTILATION AIR FLOWS PER UNIT AREA. AIR FLOWS PER UNIT AREA ARE INDICATIVE OF THE NEED FOR VENTILATION AND THE LEVEL OF ACTIVITY IN THE ROOM.

Air infiltrates through the windows and doors depending on the external wind speed and the air leakage area; Table II gathers the main infiltration parameters used in the simulations. The air tightness of the building is assumed to be 0.5 Air Changes per Hour (ACH) at 50 Pa.

| Room name | Windows surface $[m^2]$ | Air leakage area $[m^2]$ |
|---|---|---|
| A:235 | 10.6 | 0.015 |
| A:225 | 2.3 | 0.008 |
| A:213 | 3.4 | 0.014 |
| B:213 | 0 | 0.009 |

TABLE II

TOTAL EXTERNAL WINDOWS AREA AND AIR LEAKAGE AREA IN THE ROOMS.

Each room has a different profile for the occupants; the level of activity of the occupants was set to 1.8 Metabolic Equivalent of Task (MET), corresponding to a light physical activity, such as typical office working conditions; $CO_2$ emissions per person (parameter $g$), which is proportional to the activity, resulted in 15.4 $\mathrm{mg_{CO_2}/s}$, corresponding to $8 \cdot 10^{-6}\ \mathrm{m^3_{CO_2}/s}$.

### D. Validation of the data generation mechanism

To assess the accuracy of the IDA-ICE physical model with respect to the real room dynamics, we compare measured and simulated $CO_2$ data in Figure 2, under the same conditions of occupancy and ventilation levels [35]. The real data are collected from the laboratory room A:225 [27] with a sampling time of $T = 1$ minute. The two sets of measured and simulated data show that the physical model is capable to capture the main $CO_2$ dynamics. The mismatch between the two curves is attributed to events whose effect, though minor, is not simple to account for; examples of such events are doors kept open and non-logged window openings.

### E. Assessing the single-room occupancy estimation algorithm

Here we compare the predictive capabilities of the single-room model (7) against numerical representations of the rooms. This assessment is performed to check out whether the proposed model reproduces the internal and not accessible $CO_2$ model of IDA-ICE. To this aim we:

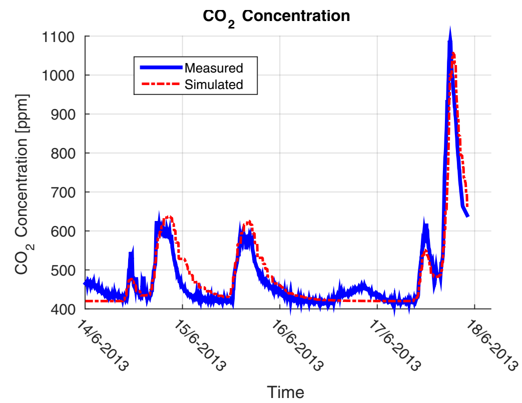


Fig. 2. Validation of the IDA-ICE model. $CO_2$ levels from room measurements and from simulation are compared, from [35].

1) collect the dataset $\mathcal{D}_j = \{c_j(k)u_j(k), o_j(k)\}_{k\in\mathcal{K}_j}$ for each room from the virtual building simulated with IDA-ICE;
2) add to $c_j(k)$ some artificial white Gaussian noise (whose variance is estimated from the real data used in SectionV-D and is equal to 35) and build the dataset
$$\mathcal{D}_j := \{y_j(k), u_j(k), o_j(k)\}_{k\in\mathcal{K}_j}\,; \tag{21}$$
3) identify the model, i.e., estimate the unknown part of $\boldsymbol{\theta}$ through the ML strategy discussed in Section II-B. This step corresponds to estimate the parameters $\widehat{\boldsymbol{\theta}}$ solving (15) and thus to obtain both the $CO_2$ estimator $\widehat{y}_j\left(k;\widehat{\boldsymbol{\theta}}\right)$ through (14) and the occupancy estimator $\widehat{o}_j(k)$ through (17);

In Figure 3 the $CO_2$ simulated by the estimated model is validated and compared to the generated one by IDA-ICE, where the main focus of the validation is on the simulation. It is possible to see that the proposed model is able to reproduce the $CO_2$ generated by IDA-ICE. Realizations of the true occupancy and the estimated one for the same room is depicted in Figure 4. From Figure 4 it can be seen that the proposed occupancy estimator for a single-room model can give accurate results in reproducing the true occupancy. To quantitatively evaluate the estimation capabilities of the estimators, Table III provides some performance indexes for all rooms[4]. The Mean Square Error (MSE) of the estimations is small for all rooms and the algorithm has good detection of occupied rooms (small FNs).

| Room | MSE | Accuracy | FP | FN |
|---|---|---|---|---|
| A:213 | 0.125 | 0.496 | 0.144 | 0.022 |
| A:225 | 0.247 | 0.563 | 0.280 | 0.000 |
| A:235 | 0.109 | 0.636 | 0.063 | 0.011 |
| B:213 | 0.075 | 0.750 | 0.047 | 0.021 |

TABLE III

SUMMARY OF THE PERFORMANCE INDEXES OF THE COMPLETE SINGLE-ROOM ESTIMATORS.

[4]For description of the performance indexes see [14].

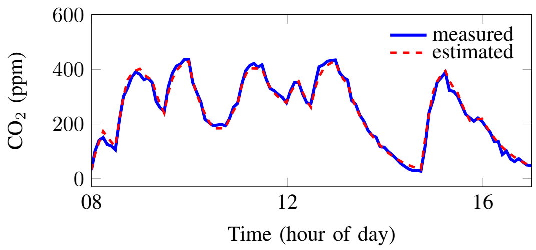


Fig. 3. Validation of model (7) against IDA-ICE for room A:225.

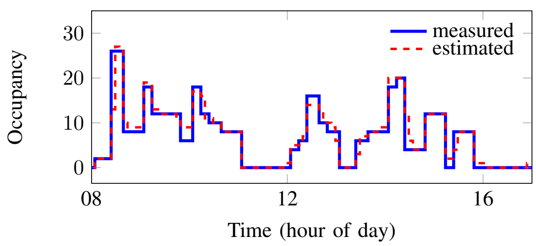


Fig. 4. Realizations of the true and estimated occupancy through the single-room estimator (17) for room A:213.

### *F. Assessing the multi-room occupancy estimation algorithm*

To evaluate the effectiveness of the proposed multi-room occupancy estimator we collect data from IDA-ICE for all the rooms mentioned in Table I and we apply the occupancy estimation algorithm of Section IV.

In Figure 5 we provide the estimation results in one of the untrained rooms. It can be seen that the estimator is able to estimate the number of occupiers with fairly good precision, even though not as well as the single-room estimator. To have a better evaluation of the estimation performance, Table IV reports the achieved performance indexes for every untrained room, assumed to have all the same usage pattern (so that the $\lambda$ obtained during the training step can be used for every room). The suggested multi-room estimator tends to have good detection abilities, even if suffering a slight performance degradation compared to the single-room case. This can be considered as a consequence of the assumptions made in (18), which do not hold for this simulation example (see Table I).

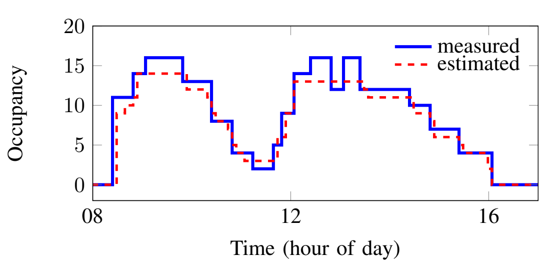


Fig. 5. Realizations of the true and estimated occupancy through the multi-room estimator for the untrained room A:213 when the model is trained on room A:225.

| trained r. | untrained r. | MSE | accuracy | FP | FN |
|---|---|---|---|---|---|
| A:225 | A:235 | 0.179 | 0.413 | 0.364 | 0.001 |
| A:225 | A:213 | 0.232 | 0.276 | 0.399 | 0.000 |
| A:225 | B:213 | 0.104 | 0.489 | 0.062 | 0.012 |

TABLE IV

SUMMARY OF THE PERFORMANCE INDEXES OF THE COMPLETE ESTIMATORS .

## VI. CONCLUSIONS

In this paper we have studied the problem of estimating the occupancy levels in buildings using available environmental and actuation signals. Our proposed method is centered on the $CO_2$ dynamics which, starting from first principles, are modeled using a nonlinear gray-box model. The parameters of this model are identified on one of the rooms using a Maximum Likelihood (ML) approach. The resulting model is utilized to construct an occupancy estimator based on regularized deconvolution; this estimator is then adapted to other rooms of the building by exploiting the knowledge of the characteristics of the rooms and their relation with the room where the model is first identified. We have built a simulated environment where we have tested the estimation scheme, showing the effectiveness of the proposed scheme.

A natural extension of the current work is the application of blind system identification techniques to the proposed scheme, so to remove the need of a training phase. More extensions may consider improving the estimations by using the knowledge of interconnection of the rooms and the locations of exits and entrances.

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## APPENDIX I NOTATION

| *parameter* | *description* | *unit* |
|---|---|---|
| $j \in \mathbb{N}_+$ | room index | adim. |
| $t \in \mathbb{R}$ | time index (continuous) | adim. |
| $k \in \mathbb{N}_+$ | time index (discrete) | adim. |
| $o_j(k)$ | occupancy at time $k$ in room $j$ | adim. |
| $g$ | $CO_2$ generation rate per person (assumed constant and known) | $m^3_{CO_2}$ / s |
| $\overline{c}_j(k)$ | $CO_2$ concentration level at time $k$ in room $j$ | ppm |
| $c$ | $CO_2$ concentration level of the air injected by the ventilation system (assumed constant and known) | ppm |
| $c_j(k) := \overline{c}_j(k) - c$ | normalized $CO_2$ concentration level at time $k$ in room $j$ | ppm |
| $y_j(k)$ | noisy measurement of $c_j(k)$ | ppm |
| $u_j(k) \in [0, 1]$ | actuation levels of the ventilation system at time $k$ in room $j$ | adim., % |
| $\dot{Q}_j^{\max}$ | nominal maximum airflow of the ventilation system for room $j$ | $m^3$ / s |
| $\dot{Q}_j^{\min}$ | nominal minimum airflow of the ventilation system for room $j$ | $m^3$ / s |
| $\dot{Q}^{\text{leak}}$ | leaking air flow (e.g., from windows and doors; assumed constant for each room) | $m^3$ / s |
| $v_j$ | volume of room $j$ | $m^3$ |

TABLE V
SUMMARY OF THE MOST IMPORTANT PARAMETERS.